EE 649: Speech Processing By Computer
Notes on Computing the IDFT and Cepstrum

In general the IDFT can be obtained using DFT (FFT) algorithms as follows:

$$x(n) = \frac{1}{N}\left[\sum_{k=0}^{N-1} X(k)^* e^{-j\frac{2\pi}{N}nk}\right]^*$$

where * denotes complex conjugate.

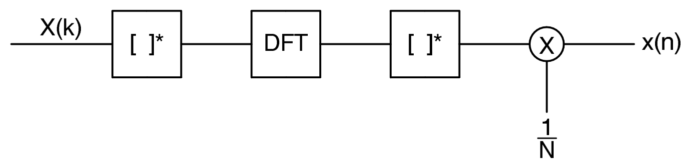

Figure 1: Computations in the IDFT.

Obviously if $X(k)$ is real, then $X(k)^* = X(k)$. Moreover, if $X(k)$ is an even function (i.e., $X(k) = X(-k)$) then DFT [X(k)] is real, so no complex conjugate operation need be performed. This will be the case when $X(k) = |S(k)|$ or $log|S(k)|$, where $S(k) = DFT\{s(n)\}$ for signal {s(n)}, and all Npoints of the signal (i.e., positive and negative frequencies) are preserved. For such cases

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{-j\frac{2\pi}{N}nk}$$

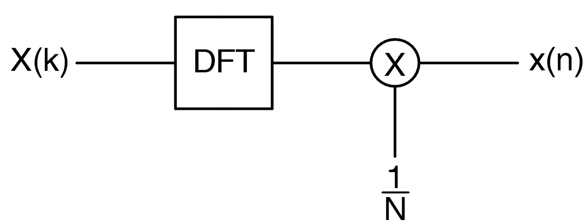

Figure 2: Computations in the IDFT when $X(k)$ is an even function.

Note that if $X(k)$ is filtered, as in the cepstrum, the filter must preserve the "even-ness".

Practical Notes:

1. To use the simplifications above, you want to preserve "even-ness" throughout the computation. Therefore, use both positive and negative frequencies throughout. "Lowpass filtering" can be done to preserve even-ness as shown graphically below; be sure your indexing does in fact preserve even-ness. Your final plots may show all (positive and negative) or only positive cepstrum and frequency values.

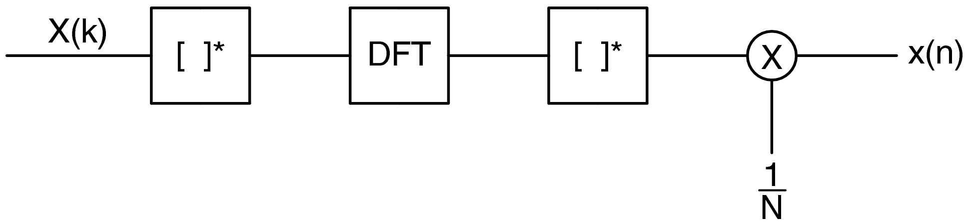

Figure 3: Preserving even-ness in the cepstrum.

2. Note that for the IDFT and for the final DFT (following the "lowpass filtering") in the cepstrum analysis, you want to preserve sign information in the real part, so a function that computes only the magnitude FFT (e.g., FFTMAG) should not be used.