ECE 302 HW1 Solution

1. (a) $P_r(A) = P_r(\{1\}) + P_r(\{3\}) + P_r(\{5\})$
$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$
$= \frac{1}{2}$

or $P_r(A) = \frac{\text{The number of outcomes in } A}{\text{The total number of outcomes in } S}$
$= \frac{3}{6} = \frac{1}{2}$

(b) $P_r(B) = P_r(\{1\}) + P_r(\{9\}) + P_r(\{11\})$
$= \frac{1}{2}$

(c) $P_r(C) = P_r(\{1\}) + P_r(\{3\}) + P_r(\{9\}) + P_r(\{11\})$
$= \frac{2}{3}$

(d) $A \cup B = \{1, 3, 5, 7, 9, 11\}$
$\Rightarrow P_r(A \cup B) = \frac{6}{6} = 1$

(e) $A \cup C = \{1, 3, 5, 9, 11\}$
$\Rightarrow P_r(A \cup C) = \frac{5}{6}$

(f) $A - C = \{5\}$ $\quad (A-C) \cup B = \{5, 7, 9, 11\}$
$\Rightarrow P_r[(A-C) \cup B] = \frac{4}{6} = \frac{2}{3}$

2. (a) $S = \{(i, j) : i, j \in \{1, 2, 3, 4, 5, 6\}\}$
the total number of the outcomes in $S$ is 36
$i$ is the outcome of the 1st roll
$j$ - - - - - - - - - - - - 2nd roll

(b) i. Actually, there are 18 outcomes statisfy the requirement that "the sum is even"
$\Rightarrow P_r(\{i+j = \text{even}\}) = \frac{1}{2}$

ii. there are 6 outcomes statisfy the requirment
$\Rightarrow P_r(\{i = j\}) = \frac{6}{36} = \frac{1}{6}$

iii.

| $i$ | number of outcomes |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| | total 15 |

$\Rightarrow Pr(\{i>j\}) = \frac{15}{36} = \frac{5}{12}$

(c) let $A = \{$the sum of the rolls is even$\}$.

$B = \{$the first roll is equal to the second roll$\}$

$A \cap B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\} = B \neq \phi$

Hense they are not mutually exclusive.

$A \cup B = A \neq S$

Hence they are not collectively exhaustive.

$Pr(A) = \frac{1}{2}$ $\quad Pr(B) = \frac{1}{6}$

$Pr(A \cap B) = Pr(B) = \frac{1}{6} \neq Pr(A)\,Pr(B)$

Hense they are not independent.

3. let $W = \{$the selected swith works$\}$

$A = \{$swith A is selected$\}$

$B = \{$ --- B --- $\}$

$C = \{$ --- C --- $\}$

We know: $Pr(W|A) = 0.75$

$Pr(W|B) = 0.5$

$Pr(W|C) = 0.25$

$Pr(A) = Pr(B) = Pr(C) = \frac{1}{3}$

(a) $Pr(W) = Pr(W|A)Pr(A) + Pr(W|B)Pr(B) + Pr(W|C)Pr(C)$

$= 0.75 \times \frac{1}{3} + 0.5 \times \frac{1}{3} + 0.25 \times \frac{1}{3}$

$= \frac{1}{2}$

(b) $Pr(\bar{C}|W) = 1 - Pr(C|W)$

$= 1 - \frac{Pr(W|C)Pr(C)}{Pr(W)} = 1 - \frac{\frac{1}{4} \times \frac{1}{3}}{\frac{1}{2}} = \cancel{\frac{5}{6}}$

$= \frac{5}{6}$

4. let. $T_0 = \{0 \text{ is transmitted}\}$

$T_1 = \{1$ ~~is transmitted~~$\}$

$R_0 = \{0 \text{ is received}\}$

$R_1 = \{1 \text{ is received}\}$

$E = \{\text{erasure occurs}\}$

We know $Pr\{T_0\} = 0.6$

$Pr\{T_1\} = 0.4$

$Pr\{R_0|T_0\} = 0.8$

$Pr\{R_1|T_1\} = 0.9$

$Pr\{E|T_0\} = 1 - Pr\{R_0|T_0\} = 0.2$

$Pr\{E|T_1\} = 1 - Pr\{R_1|T_1\} = 0.1$

(a) $Pr(E) = Pr(E|T_0) Pr(T_0) + Pr(E|T_1)Pr(T_1)$

$= 0.2 \times 0.6 + 0.1 \times 0.4$

$= 0.16$

(b) $P_r(T_1 \mid E) = \dfrac{P_r(T_1 \cap E)}{P_r(E)} = \dfrac{P_r(E \mid T_1)\, P_r(T_1)}{P_r(E)}$

$= \dfrac{0.1 \times 0.4}{0.16} = 0.25$

(c) $P_r(T_1 \cap \bar{E}) = P_r(\bar{E} \mid T_1)\, P_r(T_1) = 0.1 \times 0.4 = 0.04$

$\neq P_r(T_1)\, P_r(\bar{E}) = 0.16 \times 0.4 = 0.064$

Hense, they are independent.

5. (a) false

$\overline{A-B} = \overline{A \cap \bar{B}} = \bar{A} \cup B$

~~S~~ counterexample:

$S = \{1, 2, 3, 4, 5, 6\}$ $A = \{1, 2, 3\}$ $B = \{1, 3, 5\}$

$\overline{A-B} = \{1, 3, 4, 5, 6\}$ $A \cup \bar{B} = \{1, 2, 3, 4, 6\}$

Hense $\overline{A-B} \neq A \cup \bar{B}$

(b) true.

$P_r(A \cup B) = P_r(A) + P_r(B) - P_r(A \cap B)$

$P_r(A \cap B) \geq 0$

Hense $P_r(A \cup B) \leq P_r(A) + P_r(B)$

(c) ~~$Pr(A|B) =$~~ Pr true.

$P_r(A \mid B) = \dfrac{P_r(A \cap B)}{P_r(B)}$

since $A \subset B$

Hence $A \cap B = A$

$P_r(A \mid B) = \dfrac{P_r(A)}{P_r(B)}$

since $P_r(B) \geq 1$

Hence $P_r(A \mid B) \geq P_r(A)$

(d) false.

counterexample; let $S = \{1, 2, 3, 4, 5, 6\}$ $B = \{1, 2, 3\}$ $A_1 = \{1, 3, 5\}$

$A_2 = \{4, 6\}$

$Pr(B) = \frac{1}{2}$ $Pr(A_1) = \frac{1}{2}$ $Pr(A_2) = \frac{1}{3}$

$$Pr(B|A_1) = \frac{Pr(B \cap A_1)}{Pr(A_1)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

$$Pr(B|A_2) = \frac{Pr(B \cap A_2)}{Pr(A_2)} = \frac{0}{\frac{1}{3}} = 0$$

$$Pr(B|A_1)\,Pr(A_1) + Pr(B|A_2)\,Pr(A_2) = \frac{2}{3} \times \frac{1}{2} + 0 \times \frac{1}{3} = \frac{1}{3} + 0 = \frac{1}{3} \neq Pr(B)$$

(Total probability theorem. requires. $A_1 \cdots A_n$ are mutually exclusive and collectively exhaustive.)