

EE302 Homework #1

Assigned 8/24/09, Due 9/4/09 (by 4:30 in dropbox in MSEE 330)

1. A space S is defined as $S = \{1, 3, 5, 7, 9, 11\}$, and three subsets as $A = \{1, 3, 5\}$, $B = \{7, 9, 11\}$, $C = \{1, 3, 9, 11\}$. Assume that each element has probability $1/6$. Find the following probabilities:
 - (a) $\Pr(A)$
 - (b) $\Pr(B)$
 - (c) $\Pr(C)$
 - (d) $\Pr(A \cup B)$
 - (e) $\Pr(A \cup C)$
 - (f) $\Pr[(A - C) \cup B]$

2. A pair of dice are rolled.
 - (a) What is the probability space S ?
 - (b) Determine the probability of each of the following events:
 - i. The sum of the rolls is even.
 - ii. The first roll is equal to the second.
 - iii. The first roll is larger than the second.
 - (c) Are the events {the sum of the rolls is even} and {the first roll is equal to the second} mutually exclusive? collectively exhaustive? Are they independent? You must justify your answers..

3. Three intermittent switches A, B, C are in a box. Switches A, B and C work 75%, 50% and 25% of the time, respectively. Different switches work independently. Suppose a switch is selected at random with equal probability from the box.
 - (a) Find the probability that the selected switch works
 - (b) Find the probability that the selected switch is not switch C , given that the selected switch works.

4. A binary digit, 0 or 1, is transmitted over a noisy channel. The channel either correctly transmits the digit or it erases the digit, and the receiver knows when an erasure occurs, but not the value of the digit that was erased. Hence a 0 or a 1 is transmitted, and if a 0 is transmitted then a 0 or an e is received, while if a 1 is transmitted then a 1 or an e is received (e stands for erasure). Assume that the probability that a 0 is received given a 0 is transmitted is 0.8, and the probability that a 1 is received given a 1 is transmitted is 0.9. Also assume that a 0 is transmitted with probability 0.6.
 - (a) Find the probability that an erasure occurs
 - (b) Find the probability that a 1 is transmitted given an erasure occurs.
 - (c) Are the events {an erasure occurs} and {a 1 is transmitted} independent? You must justify your answer.

5. Determine whether each of the following is true or false (note: the statement is true if it is always true, otherwise it is false). If you say it is true then give a proof, while if you say it is false then give a counterexample, i.e., a particular case where it fails.

(a) $\overline{A - B} = A \cup \overline{B}$

(b) $\Pr(A \cup B) \leq \Pr(A) + \Pr(B)$

(c) $\Pr(A|B) \geq \Pr(A)$ if $B \supset A$

(d) If A_1, \dots, A_n are mutually exclusive, then $\Pr(B) = \sum_{i=1}^n \Pr(B|A_i) \Pr(A_i)$