ECE302 Homework #2

Assigned 9/08/09, Due 9/18/09 (by 4:30 in dropbox in MSEE 330)

1. Text, problem 2.97, page 91
2. Text, problem 3.18, page 132. *Hint*: Let I be the index of the first refresh request which is not lost, i.e., $I = i$ if the first $i - 1$ requests are lost, and request i is not lost. Now express and evaluate the probability of extending the reservation time due to j refresh requests in terms of the pmf for I.
3. Text, problem 4.14, page 216.
4. Text, problem 4.24, page 218
5. A random variable X has a pdf

$$f_X(x) = k \exp(-x), \quad 0 \leq x \leq 5.$$

   (a) Find the value of k

   (b) Find the probability that $1 \leq X < 3$, and the probability that $1 < X \leq 5$.

   (c) Find the probability that $X^3 < 5$.

   (d) Find the cdf of X. Plot the pdf and the cdf of X.

6. A random variable X has a pmf

$$p_X(x_i) = k \left(\frac{1}{2}\right)^{x_i}, \quad x_i = 0, 1, 2, 3, 4, 5.$$

   (a) Find the value of k.

   (b) Find the probability that $1 \leq X < 3$, and the probability that $1 < X \leq 5$.

   (c) Find the probability that $X^3 < 5$.

   (d) Find the pdf and cdf of X. Plot the pmf, pdf and cdf of X.

7. Determine whether each of the following is true or false (note: the statement is true if it is always true, otherwise it is false). If you say it is true then refer to a known result or give a proof, while if you say it is false then give a counterexample, i.e., a particular case where it fails.

   (a) If A, B, and C are independent, then $\Pr(A \cap B|C) = \Pr(A|C)\Pr(B|C)$

   (b) The events S, ϕ, A are independent (S is the certain event, ϕ is the impossible event and A is an arbitrary event here)

   (c) If the events A and B are mutually exclusive, then they cannot be independent

   (d) If the events A and B are independent, then they cannot be mutually exclusive

   (e) $\Pr(A_1 \cap A_2 \cap \cdots \cap A_n) = \Pr(A_n)\Pr(A_{n-1}|A_n) \cdot \cdots \cdot \Pr(A_1|A_n \cap \cdots \cap A_2)$

   (f) $\sum_{k=1}^{\infty} \binom{n}{k} p^k (1-p)^{n-k} = 1$ for $0 \leq k \leq n$, $n \geq 1$

   (g) Let X be a continuous random variable with pdf $f_X(x)$. Then $\Pr(X = x_0) = f_X(x_0)$.

(h) Let X be a continuous random variable with pdf $f_X(x)$. Then (for $a \leq b$) $\Pr(a \leq X \leq b) = \int_a^b f_X(x)\,dx$

(i) Let X be a discrete random variable with pmf $p_X(x_i)$. Then $\Pr(X = x_0) = p_X(x_0)$.

(j) Let X be a discrete random variable with pmf $p_X(x_i)$. Then (for $a \leq b$) $\Pr(a \leq X \leq b) = \sum_{x_i : a \leq x_i \leq b} p_X(x_i)$

(k) Let X be a random variable with cdf $F_X(x)$. Then (for $a \leq b$) $\Pr(a \leq X \leq b) = F_X(b) - F_X(a)$