ECE302 Homework #3

Assigned 9/21/09, Due 10/2/09 (by 4:30 in dropbox in MSEE 330)

1. Text, problem 4.77, page 222

2. A random variable Y is related to a random variable X by $Y = X^3$.

   (a) Suppose X is a continuous random variable with pdf

   $$\begin{aligned} f_X(x) &= 2\exp(-2x), \qquad 0 < x < \infty. \\ &= 0, \qquad \text{else} \end{aligned}$$

   Find the pdf of Y, and the probability that Y is greater than $2X$.

   (b) Suppose X is a discrete random variable with pmf

   $$\begin{aligned} p_X(x_i) &= \left(\frac{1}{2}\right)^{x_i+1}, \qquad x_i = 0, 1, \ldots, \\ &= 0, \qquad \text{else.} \end{aligned}$$

   Find the pmf of Y and the probability that Y is greater than $2X$.

   (c) Explain how the answers to the above two parts change if $Y = |X|^3$

3. Consider the limiter $Y = g(X)$ shown in Figure P4.3 on p. 220 of the text with $a = 1$ and $b = 2$. . Assume that

   $$f_X(x) = \tfrac{1}{2}e^{-|x|}, \qquad -\infty < x < \infty$$

   (a) What kind of random variable is Y?

   (b) Find $f_Y(y)$ using the density method

   (c) Find $f_Y(y)$ using the distribution method

   (d) Find $\mathrm{E}[Y]$

4. Text, problem 3.29, page 133

5. Text, problem 4.39, page 219

6. Text, problem 4.56, page 220

7. Determine whether each of the following is true or false (note: the statement is true if it is always true, otherwise it is false). If you say it is true then refer to a known result or give a proof, while if you say it is false then give a counterexample, i.e., a particular case where it fails.

   (a) Let X be a discrete random variable and $Y = g(X)$. Then $p_Y(y_j) = \sum_{x_i : g(x_i) = y_j} p_X(x_i)$.

   (b) Let X be a continuous random variable and $Y = g(X)$. Then $f_Y(y) = \frac{d}{dy}\int_{x:g(x)\le y} f_X(x)\,dx$

   (c) Let X be a continuous random variable and $Y = g(X)$. Then $f_Y(y) = \sum_{i=1}^{n} f_X(x_i)\left|\frac{dx_i}{dy}\right|$ for $y = g(x_i)$, $i = 1, \ldots, n$.

   (d) Let X be a random variable and $Y = g(X)$. Then $\int_{-\infty}^{\infty} y f_Y(y)\,dy = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$

   (e) Let X be a random variable and a and b be constants. Then $\mathrm{E}[aX+b] = a\,\mathrm{E}[X] + b$.

   (f) Let X be a random variable and a and b be constants. Then $\mathrm{Var}[aX+b] = a^2\,\mathrm{Var}[X]$

   (g) Let X be a random variable. Then $\mathrm{Var}[X] = 0$ if and only if $X = 0$.