Poisson Process

The Poisson process consists of various r.v.'s which model events which occur at random points in continuous time $(t \geq 0)$.

The exponential r.v. has pdf

$$f_T(t) = \lambda e^{-\lambda t}, \quad t \geq 0$$
$$= 0, \quad t < 0$$

where λ is a parameter with $\lambda > 0$.
Mean and variance are

$$\bar{T} = \frac{1}{\lambda}, \quad \sigma_T^2 = \frac{1}{\lambda^2}$$

The cdf is

$$F_T(t) = 1 - e^{-\lambda t}, \quad t \geq 0$$
$$= 0, \quad t < 0$$

The Erlang (also called gamma) r.v. of order k has pdf

$$f_{T_k}(t) = \frac{\lambda (\lambda t)^{k-1} e^{-\lambda t}}{(k-1)!}, \quad t \geq 0$$
$$= 0, \quad t < 0$$

where $\lambda > 0$ and $k = 1, 2, \ldots$ are parameters. Note that Erlang of order 1 is exponential.

The mean and variance are

$$\bar{T}_k = \frac{k}{\lambda}, \quad \sigma^2_{T_k} = \frac{k}{\lambda^2}$$

The Erlang cdf is

$$F_{T_k}(t) = 1 - \sum_{n=0}^{k-1} \frac{(\lambda t)^n e^{-\lambda t}}{n!}, \quad t \geq 0$$

$$= 0, \quad t < 0$$

The Poisson r.v. has pmf

$$p_{K_t}(k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, \ldots$$

where $\lambda > 0$ and $t \geq 0$ are parameters.

The mean and variance are

$$\bar{K}_t = \lambda t, \quad \sigma^2_{K_t} = \lambda t$$

All of these r.v.'s come up in modelling the behavior of a sequence of events which occur at random points in continuous time ($t \geq 0$).

These could be times that

- packets arrive at a node in a network
- particles impinge on a detector
- customers arrive at a service station
- devices or components break down

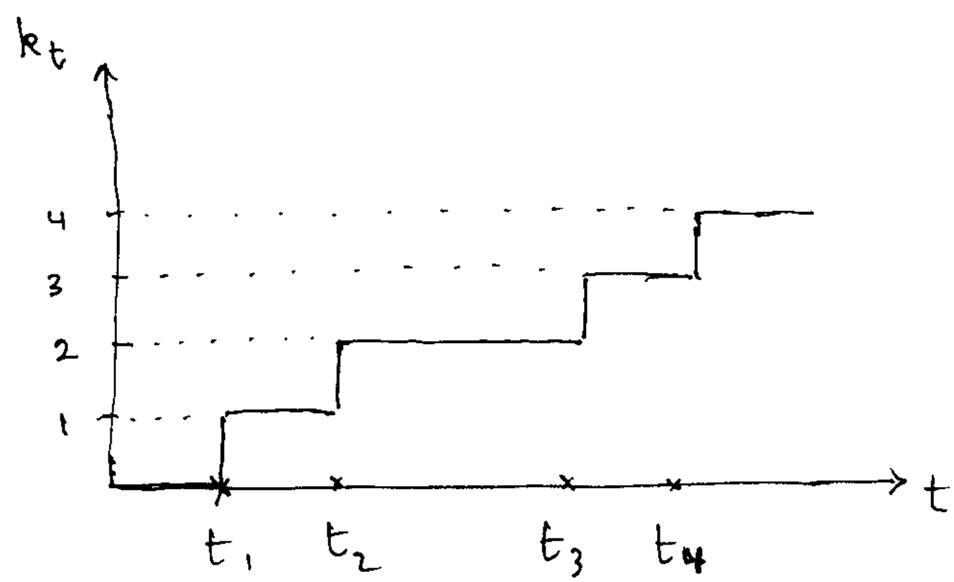
λ is the mean rate of occurence

Specifically:

An exponential r.v. is used to model the time up to first event and/or the time between the n^{th} and $n+1^{th}$ events (for $n = 0, 1, \ldots$)

An Erlang r.v. of order k is used to model the time up to the k^{th} event and/or the time between the n^{th} and $n+k^{th}$ events (for $n=0,1,\ldots$)

The Poisson r.v. with parameter t is used to model the number of events which occur in the interval $[0,t]$ and/or in an interval $[s, s+t]$ for $s \geq 0$.



$T_k = t_k$
$K_t = k_t$

Refer to collections of all these r.v.'s ($\{T_k : k=1,2,\ldots\}$, $\{K_t : t \geq 0\}$) as Poisson process

Bernoulli Process

The Bernoulli process consists of various r.v.'s which model events which occur at random points in discrete time ($t = 1, 2, \ldots$)

The geometric r.v. has pmf

$$p_T(t) = (1-p)^{t-1} p, \quad t = 1, 2, \ldots$$

where p is a parameter with $p \in [0,1]$

Mean and variance are

$$\bar{T} = \frac{1}{p}, \quad \sigma_T^2 = \frac{1-p}{p^2}$$

The negative binomial (also called Pascal) r.v. of order k has pmf

$$p_{T_k}(t) = \binom{t-1}{k-1} (1-p)^{t-k} p^k, \quad t = k, k+1, \ldots$$

where $p \in [0,1]$ and $k = 1, 2, \ldots$ are parameters. Note that negative binomial of order 1 is geometric

Mean and variance are

$$\bar{T}_k = \frac{k}{p}, \quad \sigma_{T_k}^2 = \frac{k(1-p)}{p^2}$$

The binomial r.v. has pmf

$$p_{K_t}(k) = \binom{t}{k}(1-p)^{t-k}p^k, \quad k=0,1,\ldots,t$$

where $p \in [0,1]$ and $t = 1,2,\ldots$ are parameters.

Mean and variance are

$$\bar{K}_t = pt, \quad \sigma_{K_t}^2 = p(1-p)t$$

All of these r.v.'s come up in modelling the behavior of a sequence of events which occur at random points in discrete time $(t = 1,2,\ldots)$

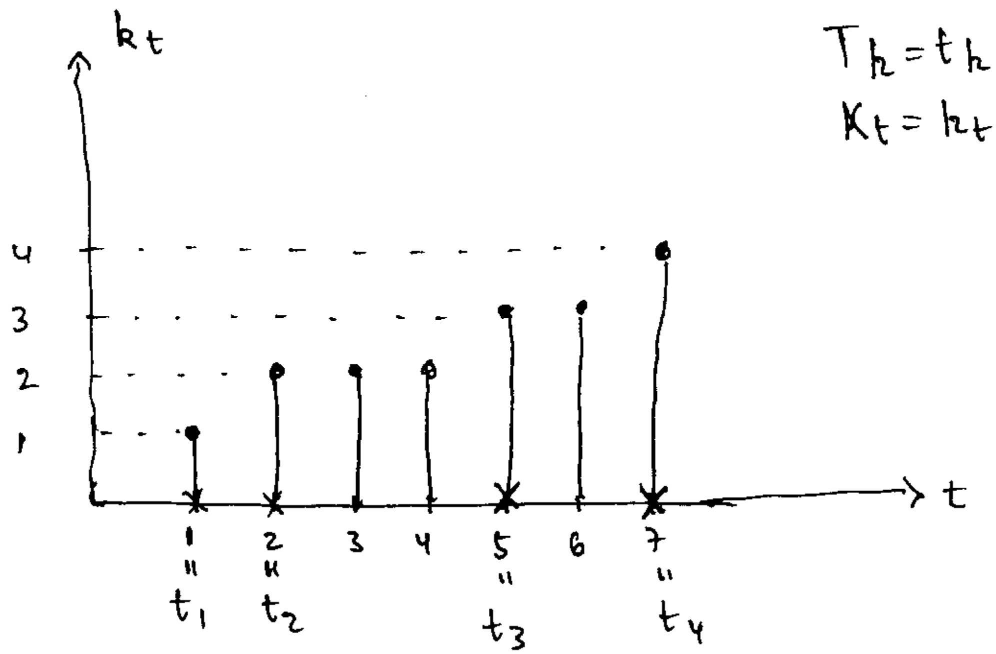
p is the mean rate of occurence

Specifically:

A geometric r.v. is used to model the time up to the first event and/or time after n^{th} event up to and including the time of $n+1^{th}$ event.

A negative binomial r.v. of order k is used to model the time up to the k^{th} event and/or the time after the k^{th} event up to and including the $n+k^{th}$ event (for $n = 0, 1, \ldots$)

The binomial r.v. with parameter t is used to model the number of events which occur in $\{1, \ldots, t\}$ or during $\{s, \ldots, s+t\}$ (for $s = 1, 2, \ldots$)



Refer to collections of all these r.v.'s ($\{T_k : k = 1, 2, \ldots\}$, $\{K_t : t = 1, 2, \ldots\}$) as <u>Bernoulli process</u>