Automatic Induction of Bellman-Error Features for Probabilistic Planning

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Abstract

Domain-specific features are important in representing problem structure throughout machine learning and decision-theoretic planning. In planning, once state features are provided, domain-independent algorithms such as approximate value iteration can learn weighted combinations of those features that often perform well as heuristic estimates of state value (e.g., distance to the goal). Successful applications in real-world domains often require features crafted by human experts. Here, we propose automatic processes for learning useful domain-specific feature sets with little or no human intervention. Our methods select and add features that describe state-space regions of high inconsistency in the Bellman equation (statewise Bellman error) during approximate value iteration. Our method can be applied using any real-valued-feature hypothesis space and corresponding learning method for selecting features from training sets of state-value pairs. We evaluate the method with hypothesis spaces defined by both relational and propositional feature languages, using nine probabilistic planning domains. We show that approximate value iteration using a relational feature space performs at the state-of-the-art in domain-independent stochastic relational planning.

1. Introduction

There is a substantial gap in performance between domain-independent planners and domain-specific planners. Domain-specific human input is able to produce very effective planners in all competition planning domains as well as many game applications such as backgammon, chess, and Tetris. In deterministic planning, work on TLPLAN (Bacchus & Kabanza, 2000) has shown that simple depth-first search with domain-specific human input, in the form of temporal logic formulas describing acceptable paths, yields an effective planner for a wide variety of competition domains. In stochastic planning, feature-based value-function representations have been used with human-selected features with great success in applications such as backgammon (Sutton & Barto, 1998; Tesauro, 1995) and Tetris (Bertsekas & Tsitsiklis, 1996). The usage of features provided by human experts is often critical to the success of systems using such value-function approximations. Here, we consider the problem of automating the transition from domain-independent planning to domain-specific performance, replacing the human input with automatically learned domain properties. We thus study a style of planner that learns from encountering problem instances to improve performance on subsequently encountered problem instances from the same domain.

We focus on stochastic planning using machine-learned value functions represented as linear combinations of state-space features. Our goal then is to augment the state-space representation during planning with new machine-discovered features that facilitate accurate representation of the value function. The resulting learned features can be used in representing the value function for other problem instances from the same domain, allowing amortization of the learning costs across solution of multiple problem instances. Note that this property is in contrast to most competition planners, especially in deterministic planning, which retain no useful information between problem instances. Thus, our approach to solving planning problems can be regarded as automatically constructing domain-specific planners, using domain-independent techniques.

We learn features that correlate well to the statewise Bellman error of value functions encountered during planning, using any provided feature language with corresponding learner to select features from the space. We evaluate this approach using both relational and propositional feature spaces. There are other recent approaches to acquiring features in stochastic planning with substantial differences from our approach which we discuss in detail in Section 5 (Patrascu, Poupart, Schuurmans, Boutilier, & Guestrin, 2002; Gretton & Thiébaux, 2004; Sanner & Boutilier, 2006; Keller, Mannor, & Precup, 2006; Parr, Painter-Wakefield, Li, & Littman, 2007). No previous work has evaluated the selection of relational features by correlation to statewise Bellman error. Recent theoretical results (Parr et al., 2007) for uncontrolled Markov processes show that exactly capturing statewise Bellman error in new features, repeatedly, will lead to convergence to the uncontrolled optimal value for the value function selected by linear-fixed-point methods for weight training. Unfortunately for machine-learning approaches to selecting features, these results have not been transferred to approximations of statewise Bellman-error features: for this case, the results in (Parr et al., 2007) are weaker and do not imply convergence. Also, none of this theory has been transferred to the controlled case of interest here, where the analysis is much more difficult because the effective (greedy) policy under consideration during value-function training is changing. We consider the controlled case, where no known theoretical properties similar to those of (Parr et al., 2007) have been shown. Our purpose is to demonstrate the capability of statewise Bellman error features empirically, and with rich representations that require machine learning techniques that lack approximation guarantees. Next, we give an overview of our approach, introducing Markov decision processes, value functions, Bellman error, feature hypothesis languages and our feature learning methods.

We use Markov decision processes (MDPs) to model stochastic planning problems. An MDP is a formal model of a single agent facing a sequence of action choices from a pre-defined action space, and transitioning within a pre-defined state space. We assume there is an underlying stationary stochastic transition model for each available action from which state transitions occur according to the agent's action choices. The agent receives reward after each action choice according to the state visited (and possibly the action chosen), and has the objective of accumulating as much reward as possible (possibly favoring reward received sooner, using discounting, or averaging over time, or requiring that the reward be received by a finite horizon).

MDP solutions can be represented as state-value functions assigning real numbers to states. Informally, in MDP solution techniques, we desire a value function that respects the action transitions in that "good" states will either have large immediate rewards or have actions available that lead to other "good" states; this well-known property is formalized in *Bellman equations* that recursively characterize the optimal value function (see Section 2). The degree to which a given value function fails to respect action transitions in this way, to be formalized in the next section, is referred to as the *Bellman error* of that value function, and can be computed at each state.

Intuitively, statewise Bellman error has high magnitude in regions of the state space which appear to be undervalued (or overvalued) relative to the action choices available. A state with high

Bellman error has a locally inconsistent value function; for example, a state is inconsistently labeled with a low value if it has an action available that leads only to high-value states. Our approach is to use machine learning to fit new features to such regions of local inconsistency in the current value function. If the fit is perfect, the new features guarantee we can represent the "Bellman update" of the current value function. Repeated Bellman updates, called "value iteration", are known to converge to the optimal value function. We add the learned features to our representation and then train an improved value function, adding the new features to the available feature set.

Our method for learning new features and using them to approximate the value function here can be regarded as a *boosting-style* learning approach. A linear combination of features can be viewed as a weighted combination of an ensemble of simple hypotheses. Each new feature learned can be viewed as a simple hypothesis selected to match a training distribution focused on regions that the previous ensemble is getting wrong (as reflected in high statewise Bellman error throughout the region). Growth of an ensemble by sequentially adding simple hypotheses selected to correct the error of the ensemble so far is what we refer to as "boosting style" learning.

Our approach can be considered for selecting features in any feature-description language for which a learning method exists to effectively select features that match state-value training data. We consider two very different feature languages in our empirical evaluation. Human-constructed features are typically compactly described using a relational language (such as English) wherein the feature value is determined by the relations between objects in the domain. Likewise, we consider a relational feature language, based on domain predicates from the basic domain description. (The domain description may be written, for example, in a standard planning language such as PPDDL (Younes, Littman, Weissman, & Asmuth, 2005).) Here, we take logical formulas of one free variable to represent features that count the number of true instantiations of the formula in the state being evaluated. For example, the "number of holes" feature that is used in many Tetris experiments (Bertsekas & Tsitsiklis, 1996; Driessens, Ramon, & Gärtner, 2006) can be interpreted as counting the number of empty squares on the board that have some other filled squares above them. Such numeric features provide a mapping from states to natural numbers.

In addition to this relational feature language, we consider using a propositional feature representation in our learning structure. Although a propositional representation is less expressive than a relational one, there exist very effective off-the-shelf learning packages that utilize propositional representations. Indeed, we show that we can reformulate our feature learning task as a related classification problem, and use a standard classification tool, the decision-tree learner C4.5 (Quinlan, 1993), to create binary-valued features. Our reformulation to classification considers only the sign, not the magnitude, of the statewise Bellman error, attempting to learn features that characterize the positive-sign regions of the state space (or likewise the negative-sign regions). A standard supervised classification problem is thus formulated and C4.5 is then applied to generate a decision-tree feature, which we use as a new feature in our value-function representation. This propositional approach is easier to implement and may be more attractive than the relational one when there is no obvious advantage in using relational representation, or when computing the exact statewise Bellman error for each state is significantly more expensive than estimating its sign. In our experiments, however, we find that our relational approach produces superior results than our propositional learner. The relational approach also demonstrates the ability to generalize features to larger problems from the same domain, an asset of relational representation that is not readily available in propositional representations.

We present experiments in nine domains. Each experiment starts with a single, constant feature, mapping all states to the same number, forcing also a constant value function that makes no distinctions between states. We then learn domain-specific features and weights from automatically generated sampled state trajectories, adjusting the weights after each new feature is added. We evaluate the performance of policies that select their actions greedily relative to the learned value functions. We evaluate our learners using the stochastic computer-game **Tetris** and seven planning domains from the two international probabilistic planning competitions (Younes et al., 2005; Bonet & Givan, 2006). We demonstrate that our relational learner generates superior performance in Tetris as compared to the best previous domain-independent system (called "Relational Reinforcement Learning", or RRL (Driessens et al., 2006)). Our relational learner also demonstrates superior success ratio in the probabilistic planning-competition domains as compared both to our propositional approach and to the probabilistic planners FF-Replan (Yoon, Fern, & Givan, 2007) and FOALP (Sanner & Boutilier, 2006). Additionally, we show that our propositional learner outperforms a previous method of Patrascu et al. (Patrascu et al., 2002) on the same **SysAdmin** domain used for evaluation there.

2. Background

2.1 Markov Decision Process

We define here our terminology for Markov decision processes. For a more thorough discussion of Markov decision processes, see (Bertsekas & Tsitsiklis, 1996) and (Sutton & Barto, 1998). A Markov decision process (MDP) M is a tuple (S, A, R, T, s_0). Here, S is a finite state space containing initial state s_0, and A selects a non-empty finite available action set $A(s)$ for each state s in S. The reward function R assigns a real reward to each state-action-state triple (s, a, s') where action a is enabled in state s, i.e., a is in $A(s)$. The transition probability function T maps state-action pairs (s, a) to probability distributions over S, $\mathcal{P}(S)$, where a is in $A(s)$.

Given discount factor $0 \leq \gamma < 1$ and *policy* π mapping each state $s \in S$ to an action in $A(s)$, the value function $V^\pi(s)$ gives the expected discounted reward obtained from state s selecting action $\pi(s)$ at each state encountered and discounting future rewards by a factor of γ per time step. There is at least one optimal policy π^* for which $V^{\pi^*}(s)$, abbreviated $V^*(s)$, is no less than $V^\pi(s)$ at every state s, for any policy π. The following "Q function" evaluates an action a with respect to a future-value function V,

$$Q(s, a, V) = \sum_{s' \in S} T(s, a, s')[R(s, a, s') + \gamma V(s')].$$

Recursive Bellman equations use $Q()$ to describe V^* and V^π as follows. First, $V^\pi(s) = Q(s, \pi(s), V^\pi)$. Then, $V^*(s) = \max_{a \in A(s)} Q(s, a, V^*)$. Also using $Q()$, we can select an action greedily relative to any value function. The policy Greedy(V) selects, at any state s, the action $\arg\max_{a \in A(s)} Q(s, a, V)$.

Value iteration iterates the operation

$$\text{update}(V)(s) = \max_{a \in A(s)} \sum_{s' \in S} T(s, a, s')[R(s, a, s') + \gamma V(s')],$$

computing the " Bellman update" update(V) from V, producing a sequence of value functions converging in the sup-norm to V^*, regardless of the initial V used.

We define the statewise Bellman error $B(V, s)$ for a value function V at a state s to be update$(V)(s) - V(s)$. We will be inducing new features based on their correlation to the statewise Bellman error, or based on the sign of the statewise Bellman error. The sup-norm distance of a value function V from the optimal value function V^* can be bounded using the Bellman error magnitude, which is defined as $\max_{s \in S} |B(V, s)|$ (e.g., see (Williams & Baird, 1993)).

We note that computing update(V), and thus statewise Bellman error, can involve a summation over the entire state space, whereas our fundamental motivations require avoiding such summations. In many MDP problems of interest, the transition matrix T is sparse in a way that set of states reachable in one step with non-zero probability is small, for any current state. In such problems, statewise Bellman error can be computed effectively using an appropriate representation of T. More generally, when T is not sparse in this manner, the sum can be effectively approximately evaluated by sampling next states according to the distribution represented by T.

2.2 Modeling Goal-oriented Problems

Stochastic planning problems can be goal-oriented, where the objective of solving the problem is to guide the agent toward a designated state region (i.e., the goal region). We model such problems by structuring the reward and transition functions R and T so that any action in a goal state leads with positive reward to a zero-reward absorbing state, and reward is zero everywhere else. We retain discounting to represent our preference for shorter paths to the goal. Alternatively, such problems can be modeled as stochastic shortest path MDPs without discounting (Bertsekas, 1995). Our techniques can easily be generalized to formalisms which allow varying action costs as well, but we do not model such variation in this work.

More formally, we define a goal-oriented MDP to be any MDP meeting the following constraints. Here, we use the variables s and s' for states in S and a for actions in $A(s)$. We require that S contain a zero-reward absorbing state $\perp$, i.e., such that $R(\perp, a, s) = 0$ and $T(\perp, a, \perp) = 1$ for all s and a. The transition function T must assign either one or zero to triples $(s, a, \perp)$, and we call the region of states s for which $T(s, a, \perp)$ is one *t*he goal region. The reward function is constrained so that $R(s, a, s')$ is zero unless $s' = \perp$. In constructing goal-oriented MDPs from other problem representations, we may introduce dummy actions to carry out the transitions involving $\perp$ described here.

2.3 Compactly Represented MDPs

In this work, we consider both propositional and relational state representations.

In relational MDPs, the spaces S and $A(s)$ for each s are relationally represented, i.e., there is a finite set of objects O, state predicates P, and action names N used to define these spaces as follows. A *state fact* is an application $p(o_1, \ldots, o_n)$ of an n-argument state predicate p to object arguments o_i. A state is any set of state facts, representing exactly the true facts in that state. An *action instance* $a(o_1, \ldots, o_n)$ is an application of an n-argument action name to n objects o_i. The action space $A = \bigcup_{s \in S} A(s)$ is the set of all action instances.

In propositional problems (which can be derived automatically from relational problems by grounding), the action space is explicitly specified and the state space is compactly specified by providing a finite sequence of basic state properties called *state attributes*, with Boolean, integer, or real values. A propositional state is then any vector of values for the state attributes.

2.4 Representing PPDDL Planning Problems using MDPs

We discuss how to represent goal-oriented stochastic planning problems defined in standardized planning language such as PPDDL (Younes et al., 2005) as goal-oriented MDPs. We limit our focus to problems in which the goal regions can be described as (conjunctive) sets of state facts. We reference and follow the approach used in (Fern, Yoon, & Givan, 2006) here regarding converting from planning problems to compactly represented MDPs in a manner that facilitates generalization between problem instances.

A PPDDL problem definition defines a planning problem instance. A *planning domain* is a distribution over problem instances sharing the same state predicates P_W, same action names N and corresponding action definitions. Each problem instance in the domain will provide a finite object set O, initial state s_i and goal condition G. The initial state is given as a set of state facts and the goal condition is given as a conjunction of state facts, each constructed from the predicates in P_W. Actions can take objects as parameters, and are defined by giving discrete finite probability distributions over action outcomes, each of which is specified using add and delete lists of state facts about the action parameters. Conditional effects and quantified preconditions are allowed. For details of PPDDL, please see (Younes et al., 2005).

In planning competitions, it has been customary to specify planning domains by providing *problem generators* that accept size parameters as input and then output PPDDL problem instances. These generators thus specify size-parameterized planning domains. It is important to note, however, that not all problem generators provided in the recent planning competitions specify planning domains according to the definition used here. In particular, some problem generators vary the action set or the state predicates between the instances generated. The relationship between the different problem instances generated by such generators is much looser than that required by our definition, and as such these "domains" are more like arbitrary collections of planning problems.

Because our logical language allows generalization between problems only if those problems share the same state and action language, we limit our empirical evaluation in Section 7 to domains that were provided with problem generators that specify planning domains as just defined here, i.e., without varying the action definitions between instances (or for which we can easily code such a generator). We refer to domains with such generators as *planning domains with fixed action spaces*.

Generalization between problems of varying size Because the object set varies in size, without bound, across the problem instances of a domain, there are infinitely many possible states within the different instances of a single domain. Each MDP we analyze has a finite state space, and so we model a planning domain as an infinite set of MDPs for which we are seeking a good policy (in the form of a good value function), one for each problem instance[1].

A value function for an infinite set of MDPs is a mapping from the disjoint union of the state spaces of the MDPs to the real numbers. Such a value function can be used greedily as a policy in any of the MDPs in the set. However, explicit representation of such a value function would have infinite size. Here, we will use knowledge representation techniques to compactly represent value functions over the infinite set of problem instance MDPs for any given planning domain. The compact representation derives from generalization across the domains, and our approach is funda-

1. In this paper we consider two candidate representations for features; only one of these, the relational representation, is capable of generalizing between problem sizes. For the propositional representation, we restrict all training and testing to problem instances of the same size.

mentally about finding good generalizations between the MDPs within a single planning domain. Our representation for value functions over planning domains is given below in Sections 2.5 and 4.

In this section, we discuss how to represent as a single finite MDP any single planning problem instance. However, we note that our objective in this work is to find good value functions for the infinite collections of such MDPs that represent planning domains. Throughout this paper, we assume that each planning domain is provided along with a means for sampling example problems from the domain, and that the sampling is parameterized by difficulty (generally, problem size) so that easy example problems can be selected. Although, PPDDL does not provide any such problem distributions, benchmark planning domains are often provided with problem generators defining such distributions: where such generators are available, we use them, and otherwise we code our own distributions over problem instances.

Generalizing between problems with varying goals To facilitate generalization between problem instances with different goals, and following (Martin & Geffner, 2004) and (Fern et al., 2006), we translate a PPDDL instance description into an MDP where each state specifies not only what is true in the state but also what the goal is. Action transitions in this MDP will never change the "goal", but the presence of that goal within the state description allows value functions (that are defined as conditioning only on the state) to depend on the goal as well. The goal region of the MDP will simply be those MDP states where the specified current state information matches the specified goal information.

Formally, in translating PPDDL problem instances into compact MDPs, we enrich the given set of world-state predicates P_W by adding a copy of each predicate indicating the desired state of that predicate. We name the goal-description copy of a predicate p by prepending the word "goal-" to the name. The set of all goal-description copies of the predicates in P_W is denoted P_G, and we take $P_W \cup P_G$ to be the state predicates for the MDP corresponding to the planning instance. Intuitively, the presence of goal-p(a,b) in a state indicates that the goal condition requires the fact $p(a, b)$ to be part of the world state. The only use of the goal predicates in constructing a compact MDP from a PPDDL description is in constructing the initial state, which will have the goal conditions true for the goal predicates.

We use the domain **Blocksworld** as an example here to illustrate the reformulation (the same domain is also used as an example in (Fern et al., 2006)). The goal condition in a Blocksworld problem can be described as a conjunction of ground **on-top-of** facts. The world-state predicate **on-top-of** is in P_W. As discussed above, this implies that the predicate **goal-on-top-of** is in P_G. Intuitively, one ground instance of that predicate, **goal-on-top-of(b1,b2)**, means that for a state in the goal region, the block **b1** has to be directly on the top of the block **b2**.

States with no available actions PPDDL allows the definition of domains where some states do not meet the preconditions for any action to be applied. However, our MDP formalism requires at least one available action in every state. In translating a PPDDL problem instance to an MDP we define the action transitions so that any action taken in such a "dead" state transitions deterministically to the absorbing $\perp$ state. Because we consider such states undesirable in plan trajectories, we give these added transitions a reward of negative one unless the source state is a goal state.

The resulting MDP We now formally describe an MDP $M = (S, A, R, T, s_0)$ given a planning problem instance. As discussed in Section 2.3, the sets S and $A(s)$ are defined by specifying the predicates and objects available. The PPDDL description specifies the sets N of action names and

O of objects, as well as a set P_W of world predicates. We construct the enriched set $P = P_W \cup P_G$ of state predicates and define the state space as all sets of applications of these predicates to the objects in O. The set $A(s)$ for any state s is the set of PPDDL action instances built from N and O for which s satisfies the preconditions, except that if this set is empty, $A(s)$ is the set of all PPDDL action instances built from N and O. In the latter case, we say the state is "dead." The reward function R is defined as discussed previously in Section 2.2; i.e., $R(s, a, s') = 1$ when the goal condition G is true in s, $R(s, a, s') = -1$ when s is a non-goal dead state, and zero otherwise. We define $T(s, a, s')$ according to the semantics of PPDDL augmented with the semantics of $\perp$ from Section 2.2—$T(s, a, \perp)$ will be one if s satisfies G, s is dead, or $s = \perp$, and zero otherwise.[2] Transiting from one state to another never changes the goal condition description in the states given by predicates in P_G. The MDP initial state s_0 is just the PPDDL problem initial state s_i augmented by the goal condition G using the goal predicates from P_G. If a propositional representation is desired, it can be easily constructed directly from this relational representation by grounding.

2.5 Linear Approximation of Value Functions

As many previous authors have done (Patrascu et al., 2002; Sanner & Boutilier, 2006; Bertsekas & Tsitsiklis, 1996; Tesauro, 1995; Tsitsiklis & Roy, 1997), we address very large compactly represented S and/or A by implicitly representing value functions in terms of state-space *features* $f : S \to \mathbb{R}$. Our features f must select a real value for each state. We describe two approaches to representing and selecting such features in Section 4.

Recall from Section 1 that our goal is to learn a value function for a family of related MDP problems. We assume that our state-space features are defined across the union of the state spaces in the family.

We represent value functions using a linear combination of l features extracted from s, i.e., as $\tilde{V}(s) = \sum_{i=0}^{l} w_i f_i(s)$. Our goal is to find features f_i (each mapping states to real values) and weights w_i so that $\tilde{V}$ closely approximates V^*.

Various methods have been proposed to select weights w_i for linear approximations (see, e.g., (Sutton, 1988) or (Widrow & Hoff, 1960)). Here, we review and use a trajectory-based approximate value iteration (AVI) approach. Other training methods can easily be substituted. AVI constructs a finite sequence of value functions $V^1, V^2, \ldots, V^T$, and returns the last one. Each value function is represented as $V^\beta(s) = \sum_{i=0}^{l} w_i^\beta f_i(s)$. To determine weights $w_i^{\beta+1}$ from V^β, we draw a set of training states $s_1, s_2, \ldots, s_n$ by following policy Greedy(V^β) in different example problems selected using the provided problem distribution at the current level of problem difficulty. (See Section 3 for discussion of the control of problem difficulty.) The number of trajectories drawn and the maximum length of each trajectory are parameters of the AVI method. For each training state s, we compute the Bellman update update$(V^\beta)(s)$ from the MDP model of the problem instance. We can then compute $w_i^{\beta+1}$ from the training states using

$$w_i^{\beta+1} = w_i^\beta + \frac{1}{n_i} \sum_j \alpha f_i(s_j)(\text{update}(V^\beta)(s_j) - V^\beta(s_j))$$

, where α is the learning rate and n_i is the number of states s in $s_1, s_2, \ldots, s_n$ for which $f_i(s)$ is non-zero. Weight updates using this weight-update formula descend the gradient of the L_2 distance

2. Note that according to our definitions in Section 2.2, the dead states are now technically "goal states", but have negative rewards.

between V^β and update(V^β) on the training states, with the features first rescaled to normalize the effective learning rate to correct for feature values with rare occurrence in the training set.[3]

Scaling step-size during AVI For the complex domains addressed in this paper, simple gradient descent has many potential pitfalls. One such pitfall is that the gradient surface may be extremely steep at some points. Because the weight changes in AVI are proportional to the gradient, arbitrarily large gradients result in arbitrarily large single-step weight changes that are rarely desirable (and can also cause floating-point overflow). There is a substantial literature on dynamically adjusting step size during gradient descent (Jacobs, 1988; Kwong & Johnston, 1992; Harris, Chabries, & Bishop, 1986; Mathews & Xie, 1993); however, gradient descent is not the main topic of this paper and so we resort only to a simple work-around for arbitrarily large gradients: rather than step proportional to the gradient, we compress the unbounded space of possible step sizes to a finite interval using a sigmoidal function, as described next. Large gradients here are due to large statewise Bellman error averages over the training set, as can be seen by examining the weight update equation, Equation 1. Here we compress large weight updates by a sigmoidal scaling of the average statewise Bellman error, as described formally in the next three equations:

$$\begin{aligned} B_{avg} &= \frac{1}{n}\sum_j(\text{update}(V^\beta)(s_j) - V^\beta(s_j)) \\ \tau &= \frac{1}{1+\exp(-4(1-|B_{avg}|/r_{\text{scale}}))} \\ w_i^{\beta+1} &= w_i^\beta + \tau\frac{1}{n_i}\sum_j \alpha f_i(s_j)(\text{update}(V^\beta)(s_j) - V^\beta(s_j)) \end{aligned} \tag{1}$$

In our experiments, we use this approach to computing $w^{\beta+1}$ rather than the direct approach given by equation 1. The scaling factor τ will be close to one unless the average statewise Bellman error B_{avg} grows large, and thus significant differences between the direct approach and the scaled approach appear only in that case. The domain-specific parameter r_{scale} represents the reward scaling of the problem domain. We note that any MDP problem can be rescaled by multiplying all rewards by the same positive scalar with consequent rescaling of the value of any policy at any state by the same scalar. Our method here is not invariant to this rescaling and thus requires a hand-set domain parameter to represent the reward scaling. It is an interesting topic of future research to automatically, possibly dynamically, find the value of this reward scaling parameter.

Sign restriction in weight adjustment Another pitfall in using gradient descent with complex gradient surfaces is that dramatic increases in error can result from one step of weight update. In our AVI setting, this can result in dramatic drops in the success rate of the resulting greedy policy. Because in goal-oriented domains a useful gradient is computed only from successful trajectories, such dramatic drops in success rate can result in an uninformative gradient from which AVI often cannot recover. Various mechanisms can be designed for detecting dramatic drops in policy quality during AVI and revisiting the weight updates that lead to them; here we focus only on revisiting weight updates that change the sign of a weight, and only when the immediately resulting policy performs much worse than the policy before the weight update.

3. In deriving this gradient-descent weight-update formula, each feature f_i is scaled by $r_i = \sqrt{\frac{n}{n_i}}$, giving $f_i' = r_i f_i$.

It is fairly intuitive that weight updates changing the sign of a weight are particularly suspect. If the weight for a feature has been tuned to a positive value, it is hopefully because that feature has been seen to correlate to the desired value function; however, this immediately implies that the negation of that feature anti-correlates with the desired value. Changing the sign of a weight is a form of rejecting previous training regarding the entire direction of the importance of the corresponding feature. Empirically, we have found that AVI on complex error surfaces often makes damaging mistakes by stepping too far in weight update to the degree that the sign of a feature is reversed and the resulting policy is suddenly severely degraded.

In our experiments in goal-oriented planning problems, we implement a mechanism to detect and avoid weight sign changes that must be avoided to preserve policy quality, as follows. First, we define a method for empirically comparing policies: we say that a policy π_1 "tests as significantly better" than a policy π_2 if Student's t-test confirms the hypothesis that the success rate of π_2 is at most 0.9 times the success rate of π_1 with significance 0.025 based upon 100 sample trajectories of each. Second, each time we construct an AVI training set by drawing trajectories, we measure the success rate of the policy Greedy$V$ used over the trajectories drawn to create the training set—we call this the training success rate of the value function V. If the training success of the current value function V_2 is lower than the training success of the previous value function V_1, we then test if the the policy Greedy(V_1) tests as significantly better than the policy Greedy(V_2). If so, we reconsider any weight sign changes (including changes to or from zero) made during the intervening weight update as follows. Suppose that V_1 is described by weights w^{β} and V_2 by weights $w^{\beta+1}$. For each weight w_i that changed sign from w_i^{β} to $w_i^{\beta+1}$, we test if reversing the update of just that weight, using w_i^{β} in place of $w_i^{\beta+1}$, yields a greedy policy that tests significantly better than Greedy(V_2). Any such weights that yield significant improvements when their $\beta+1$-iteration updates are reversed are then restored to their β-iteration values and their sign is locked for the remainder of this run of AVI. In other words, any future weight update to that weight which would change the sign of that weight is replaced with no change to that weight.

3. Feature-discovering Value-function Construction

As noted above, we use a "boosting style" learning approach in finding value functions, iterating between selecting weights and generating new features by focusing on the Bellman error in the current value function. Our value function representation can be viewed as a weighted ensemble of single-feature hypotheses. We start with a value function that has only a trivial feature, a constant feature always returning the value one, with initial weight zero. We iteratively both retrain the weights and select new features matching regions of states for which the current weighted ensemble has high statewise Bellman error.

We take a "learning from small problems" approach and learn features first in problems with relatively lower difficulty, and increasing problem difficulty over time, as discussed below. Learning initially in small problems (Martin & Geffner, 2004; Yoon, Fern, & Givan, 2002) is more effective due to the smaller state space and the ability to obtain positive feedback (i.e., reach the goal) in a smaller number of steps. We show experimentally in Section 7 that good value functions for high difficulty problems can indeed be learned in this fashion from problems of lower, increasing difficulties.

Our approach relies on two assumed subroutines, and can be instantiated in different ways by providing different algorithms for these subroutines. First, a method of weight selection is assumed;

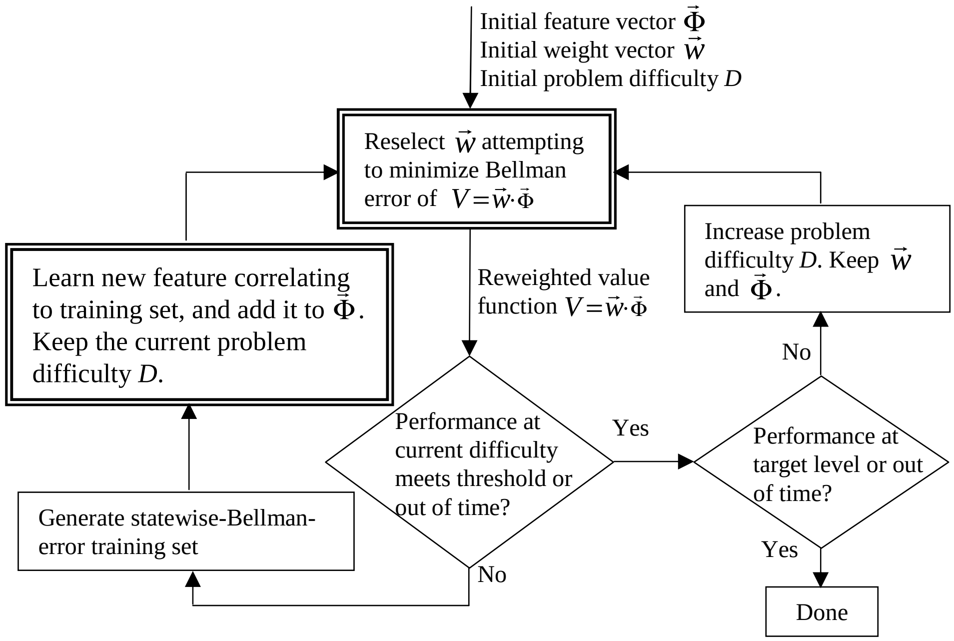


Figure 1: Control flow for feature learning. Boxes with double borders represent assumed subroutines for our method. We assume that the targeted class of problems is parameterized by problem difficulty (such as problem size). When this is not so, all problems are treated as having the same difficulty, and the two performance tests are the same; in this case, the step named ”Increase problem difficulty” is never reached.

this method takes as input a problem domain and a fixed set of features, and selects a weight vector for a value function for the problem domains using the provided features. We intend this method to heuristically or approximately minimize L_∞ Bellman error in its choice of weight vector. Second, a feature hypothesis space and corresponding learner are assumed to be provided by the system designer.

The control flow for our approach is shown in Figure 1. Each iteration at a fixed problem difficulty selects weights for the current feature set (using any method attempting to minimize L_∞ Bellman error), computes the statewise Bellman error of the resulting value function for a training set of states, and learns a new feature matching that training set, adding that feature to the feature set.

For the experiments reported in Section 7, we evaluate the following choices for the assumed subroutines. For all experiments we use AVI to select weights for feature sets. We evaluate two choices for the feature hypothesis space and corresponding learner, one relational and one propositional, as described in Section 4.

Separate training sets are drawn for weight selection and for the feature learning; the former will depend on the weight selection method, and is described for AVI in Section 2.5, and the latter is described in this section.

Problem difficulty is increased when sampled performance of the greedy policy at the current difficulty exceeds user-specified performance thresholds. In our planning-domain experiments, the performance parameters measured are success ratio (percentage of trials that find the goal) and average successful plan length (the average number of steps to the goal among all successful trials). The non-goal-oriented domains of **Tetris** and **SysAdmin** use different performance measures: average total reward for Tetris and Bellman error for SysAdmin (to facilitate comparison with (Patrascu et al., 2002)).

We also assume a user-provided schedule for problem difficulty increases in problems where difficulty is parameterized by more than one parameter (e.g., size may be measured in by the number of objects of each type); further domain-independent automation of the increase in difficulty is a topic for future research. We give the difficulty-increase schedules and performance thresholds for our experiments in the section presenting the experiments, Section 7.

Training set generation The training set for selection of a new feature is a set of state-value pairs. The training set is constructed by repeatedly sampling an example problem instance from the problem distribution at the current level of difficulty, and applying the current greedy policy Greedy(V) to that problem instance to create a trajectory of states encountered. Every state (removing duplicates) encountered is added to the training set, paired with its statewise Bellman error computed in the problem instance that generated it. The size of the feature-selection training set and the maximum length of each training trajectory are specified by the user as parameters of the algorithm.

A possible problem occurs when the current greedy policy cannot reach enough states to complete the desired training set. If 200 consecutive trajectories are drawn without visiting a new state before the desired training set size is reached, the process is modified as follows. At that point, the method attempts to complete the training set by drawing trajectories using random walk (again using sampled example problems from the current problem distribution). If this process again leads to 200 consecutive trajectories without a new state, the method terminates training-set generation and uses the current training set even though it is smaller than the target size.

Applicability of the method Feature-discovering value-function construction as just described does not require complete access to the underlying MDP model. The AVI updates and the training set generation are both based on the following computations on the model:

1. Given a state s the ability to compute the action set $A(s)$.
2. Given a state s, action $a \in A(s)$, and value function V, the ability to compute the Q-value $Q(s, a, V)$.
3. Given a state s and action $a \in A(s)$, the ability to draw a state from the next state distribution defined by $T(s, a, s')$.
4. Given a state s, the ability to compute the features in the selected feature language on s and any computations on the state required for the selected feature learner.

The first three items enable the computation of the Bellman update of s and the last item enables computation of the estimated value function given the weights and features defining it as well as the selection of new features by the feature learner.

While the PPDDL planning domains studied provide all the information needed to perform these computations, our method also applies to domains that are not natural to represent in PPDDL. These can be analyzed by our method once the above computations can be implemented. For instance, in our **Tetris** experiments in Section 7.2, the underlying model is represented by providing hand-coded routines for the above computations within the domain.

Analysis MDP value iteration is guaranteed to converge to the optimal value function if conducted with a tabular value-function representation in the presence of discounting (Bertsekas, 1995). Although weight selection in AVI is designed to mimic value iteration, while avoiding a tabular representation, there is no general guarantee that the weight updates will track value iteration and thus converge to the optimal value function. In particular, there may be no weighted combination of features that represents the optimal value function, and likewise none that represents the Bellman update update(V) for some value function V produced by AVI weight training process. Our learning system introduces new features to the existing feature ensemble in response to this problem: the training set used to select the new feature pairs states with their statewise Bellman error. If the learned feature exactly captures the statewise Bellman-error concept (by exactly capturing the training set and generalizing successfully) then the new feature space will contain the Bellman update of the value function used to generate the training data.

We aim to find features that approximate the "Bellman error feature," which we take to be a function mapping states to their statewise Bellman error. Theoretical properties of Bellman error features in the uncontrolled Markov processes (i.e., without the max operator in the Bellman equation) have recently been discussed in (Parr et al., 2007), where the addition of such features (or close approximations thereof) is proven to reduce the weighted L_2-norm distance between the best weight setting and the the true (uncontrolled) value V^*, when linear fixed-point methods are used to train the weights before feature addition. Prior to that work (in (Wu & Givan, 2005)), and now in parallel to it, we have been empirically exploring the effects of selecting Bellman error features in the more complex controlled case, leading to the results reported here.

It is clear that if we were to simply add the Bellman error feature directly, and set the corresponding weight to one, the resulting value function would be the desired Bellman update update(V) of the current value function V Adding such features at each iteration would thus give us a way to conduct value iteration exactly, without enumerating states. But each such added feature would describe the Bellman error of a value function defined in terms of previously added features, posing a serious computational cost issue when evaluating the added features. In particular, each Bellman error feature for a value function V can be estimated at any particular state with high confidence by evaluating the value function V at that state and at a polynomial-sized sample of next states for each action (based on Chernoff bounds). However, if the value function V is based upon a previously added Bellman-error feature, then each evaluation of V requires further sampling (again, for each possible action) to compute. In this manner, the amount of sampling needed for high confidence grows exponentially with the number of successive added features of this type. The levels of sampling do not collapse into one expectation because of intervening choices between actions, as is often the case in decision-theoretic sampling. Our feature selection method is an attempt to tractably approximate this exact value iteration method by learning concise and efficiently computable descriptions of the Bellman-error feature at each iteration.

Our method can thus be viewed as a heuristic approximation to exact value iteration. Exact value iteration is the instance of our method obtained by using an explicit state-value table as the feature representation and generating training sets for feature learning containing all states — to obtain exact value iteration we would also omit AVI training but instead set each weight to one.

When the feature language and learner can be shown to approximate explicit features tightly enough (so that the resulting approximate Bellman update is a contraction in the L_∞ norm), then it is easy to prove that tightening approximations of V^* will result if all weights are set to one. However, for the more practical results in our experiments, we use feature representations and learners for which no such approximation bound relative to explicit features is known.

4. Two Candidate Hypothesis Spaces for Features

In this section we describe two hypothesis spaces for features, a relational feature space and a propositional feature space, along with their respective feature learning methods. For each of the two feature spaces, we assume the learner is provided with a training set of states paired with their statewise Bellman error values.

Note that these two feature-space-learner pairs lead to two instances of our general method and that others can easily be defined by defining new feature spaces and corresponding learners. In this paper we empirically evaluate the two instances presented here.

4.1 Relational Features

A relational MDP is defined in terms of a set of state predicates. These state predicates are the basic elements from which we define a feature-representation language. Below, we define a general-purpose means of enriching the basic set of state predicates. The resulting enriched predicates can be used as the predicate symbols in standard first-order predicate logic. We then consider any formula in that logic with one free variable as a feature, as follows.

A state in a relational MDP is a first-order interpretation. A first-order formula with one free variable is then a function from such states to natural numbers which maps each state to the number of objects in that state that satisfy the formula. We take such first-order formulas to be real-valued features by normalizing to a real number between zero and one—this normalization is done by dividing the feature value by the maximum value that the feature can take, which is typically the total number of objects in the domain, but can be smaller than this in domains where objects (and quantifiers) are typed. A similar feature representation is used in (Fawcett, 1996).

This feature representation is used for our relational experiments, but the learner we describe in the next subsection only considers existentially quantified conjunctions of literals (with one free variable) as features. The space of such formulas is thus the effective features space for our relational experiments.

> **Example 4.1:** Take **Blocksworld** with the table as an object for example, **on**(x, y) is a predicate in the domain that asserts the block x is on top of the object y, where y may be a block or the table. A possible feature for this domain can be described as $\exists y$ **on**(x, y), which is a first-order formula with x as the one free variable. This formula means that there is some other object immediately below the block object x, which essentially excludes the table object and the block being held by the arm (if any) from the object set described by the feature. For n blocks problems, the un-normalized value of this feature is n for states with no block being held by the arm, or $n - 1$ for states with a block being held by the arm.

The enriched predicate set More interesting examples are possible with the enriched predicate set that we now define. To enrich the set of state predicates P, we add for each binary predicate p a transitive closure form of that predicate $p+$ and predicates min-p and max-p identifying minimal and maximal elements under that predicate. In goal-based domains, recall that our problem representation (from Section 2.4) includes, for each predicate p, a goal version of the predicate called goal-p to represent the desired state of the predicate p in the goal. Here, we also add a means-ends analysis predicate correct-p to represent p facts that are present in both the current state and the goal.

So, for objects x and y, correct-p(x,y) is true if and only if both $p(x, y)$ and goal-p(x,y) are true. $p{+}(x, y)$ is true of objects x and y connected by a path in the binary relation p. The relation max-p(x) is true if object x is a maximal element with respect to p, i.e., there exists no other object y such that $p(x, y)$ is true. The relation min-p(x) is true if object x is a minimal element with respect to p, i.e., there exists no other object y such that $p(y, x)$ is true.

> **Example 4.1 (cont.):** The feature $\exists y$ correct-**on**(x, y) means that x is stacked on top of some object y both in the current state and in the goal state. The feature $\exists y$ **on**$+(x, y)$ means that in the current state, x is directly above some object y, i.e., there is a sequence of **on** relations traversing a path between x and y, inclusively. The feature max-**on**$+(x)$ means that x is the table object when all block-towers are placed on the table, since the table is the only object that is not **on** any other object. The feature min-**on**$+(x)$ means that there is no other object on top of x, i.e., x is clear.

4.2 Learning Relational Features

We select first-order formulas as candidate features using a beam search with a beam width W. The search starts with basic features derived automatically from the domain description and repeatedly derives new candidate features from the best scoring W features found so far, adding the new features as candidates and keeping only the best scoring W features at all times. After new candidates have been added a fixed depth d of times, the best scoring feature found overall is selected to be added to the value-function representation. Candidate features are scored for the beam search by their correlation to the Bellman error feature as formalized below.

Specifically, we score each candidate feature f with its correlation coefficient to the Bellman error feature f_{BE} as estimated by this training set. The correlation coefficient between f and f' is defined as corr-coef$(f, f') = \frac{E\{f(s)f'(s)\}-E\{f(s)\}E\{f'(s)\}}{\sigma_f \sigma_{f'}}$. Instead of using a known distribution to compute this value, we use the states in the training set and compute a sampled version instead. Note that our features are non-negative, but can still be well correlated to the Bellman error (which can be negative), and that the presence of a constant feature in our representation allows a non-negative feature to be shifted automatically as needed. The scoring function for feature selection is then a regularized version of the correlation coefficient between the feature and the Bellman error feature

$$\text{score}(f) = |\text{corr-coef}(f, f_{BE})|(1 - \lambda \text{depth}(f)),$$

where the "depth" of a feature is the depth in the beam search at which it first occurs, and λ is a parameter of the learner representing the degree of regularization (bias towards low-depth features).

It remains only to specify which features in the hypothesis space will be considered initial, or basic, features for the beam search, and to specify a means for constructing more complex features from simpler ones for use in extending the beam search. We first take the state predicate set P in a domain and enrich P as described in Section 4.1. After this enrichment of P, we take as basic features the existentially quantified applications of (possibly negated) state predicates to variables with zero or one free variable[4]. A feature with no free variables is treated technically as a one-free-variable feature where that variable is not used; this results in a "binary" feature value that

4. If the domain distinguishes any objects by naming them with constants, we allow these constants as arguments to the predicates here as well.

is either zero or the total number of objects, because instantiating the free variable different ways always results in the same truth value. We assume throughout that every existential quantifier is automatically renamed away from every other variable in the system. We can also take as basic features any human-provided features that may be available, but we do not add such features in our experiments in this paper in order to clearly evaluate our method's ability to discover domain structure on its own.

At each stage in the beam search we add new candidate features (retaining the W best scoring features from the previous stage). The new candidate features are created as follows. Any feature in the beam is combined conjunctively with any other, or with any basic feature. The method of combination of two features is described in Figure 2. This figure shows non-deterministic pseudo-code for combining two input features, such that any way of making the non-deterministic choices results in a new candidate feature. The pseudo-code refers to the feature formulas f_1 and f_2 describing the two features. In some places, these formulas and others are written with their free variable exposed, as $f_1(x)$ and $f_2(y)$. Also substitution for that variable is notated by replacing it in the notation, as in $f_1(z)$.

The combination is by conjoining the feature formulas, as shown in line 2 of Figure 2; however, there is additional complexity resulting from combining the two free variables and possibly equating bound variables between the two features. The two free variables are either equated (by substitution) or one is existentially quantified before the combination is done, in line 1. Up to two pairs of variables, chosen one from each contributing feature, may also be equated, with the resulting quantifier at the front, as described in line 3. Every such combination feature is a candidate.

Example 4.2: Assume we have two basic features $\exists z\ p(x, z)$ and $\exists w\ q(y, w)$. The set of the possible candidates that can be generated by combining these two features are:
When line 3 in Figure 2 runs zero times,

1. $(\exists x\ \exists z\ p(x, z)) \wedge (\exists w\ q(y, w))$, from $\exists x f_1(x) \wedge f_2(y)$
2. $(\exists z\ p(x, z)) \wedge (\exists y\ \exists w\ q(y, w))$, from $f_1(x) \wedge \exists y f_2(y)$, and
3. $(\exists z\ p(x, z)) \wedge (\exists w\ q(x, w))$, from $f_1(x) \wedge f_2(x)$

and when line 3 runs one time,

4. $\exists u\ ((\exists z\ p(u, z)) \wedge (q(y, u)))$, from equating x and w in item 1 above,
5. $\exists u\ (\exists x\ p(x, u)) \wedge (q(y, u))$, from equating x and z in item 1 above,
6. $\exists u\ (p(x, u) \wedge (\exists w\ q(u, w)))$, from equating z and y in item 2 above,
7. $\exists u\ (p(x, u) \wedge (\exists y\ q(y, u)))$, from equating z and w in item 2 above, and
8. $\exists u\ (p(x, u) \wedge (\ q(x, u)))$, from equating z and w in item 3 above.

The first three are computed using cases 1a, 1b, and 1c, respectively. The remaining five derive from the first three by equating bound variables from f_1 and f_2.

Features generated at a depth k in this language can easily require enumerating all k-tuples of domain objects. Since the cost of this evaluation grows exponentially with k, we bound the maximum number of quantifiers in scope at any point in any feature formula to q, and refuse to consider any feature violating this bound.

The values W, λ, d, and q are the parameters controlling the relational learner we evaluate in this paper. How we set these parameters is discussed further in the experimental setup description in Section 6.

Input: $f_1(x)$, $f_2(y)$

1. Perform one of
   a. $f_1 = (\exists x) f_1(x)$
   b. $f_2 = (\exists y) f_2(y)$
   c. $f_2 = f_2(x)$

2. $o_1 = f_1 \wedge f_2$

3. Perform the following zero, one, or two times:

   a. Let v be a variable occurring in f_1 and o_1.
   Let e_1 be the expression of the form $(\exists v)\phi_1(v)$ that occurs in o_1

   b. Let w be a variable occurring in f_2 and o_1.
   Let e_2 be the expression of the form $(\exists w)\phi_2(w)$ that occurs in o_1

   c. Let u be a new variable, not used in o_1
   d. $o_2 =$ replace e_1 with $\phi_1(u)$ and replace e_2 with $\phi_2(u)$ in o_1
   e. $o_1 = (\exists u) o_2$

4. return o_1

Figure 2: A non-deterministic method for combining two feature formulas. The choice between 1a, 1b, and 1c, the choice of number of iterations of step 3, and the choices of e_1 and e_2 in steps 3a and 3b are all non-deterministic choices. Any feature that can be produced by any run of this non-deterministic method is a candidate. Note: it is assumed that f_1 and f_2 have no variables in common, by renaming if necessary before this operation.

4.3 Propositional Features

Here we discuss a second candidate hypothesis space for features, using a propositional representation. We use decision trees to represent these propositional features. A detailed discussion of classification using decision trees can be found in (Mitchell, 1997). A decision tree is a binary tree with internal nodes labeled by binary tests on states, edges labeled "yes" and "no" representing results of the binary tests, and leaves labeled with classes (in our case, either zero or one). A path through the tree from the root to a leaf with label l identifies a labeling of some set of states—each state consistent with the state-test results on the path is viewed as labeled l by the tree. In this way, a decision tree with real number labels at the leaves is viewed as labeling all states with real numbers, and is thus a feature.

We learn decision trees from training sets of labeled states using the well known C4.5 algorithm (Quinlan, 1993). This algorithm induces a tree greedily matching the training data from the root down. We use C4.5 to induce new features—the key to our algorithm is how we construct suitable training sets for C4.5 so that the induced features are useful in reducing Bellman error.

We include as possible state tests for the decision trees we induce every grounded predicate application[5] from the state predicates, as well as every previously selected decision-tree feature (each of which is a binary test because all leaf labels are zero or one).

4.4 Learning Propositional Features

To construct binary features, we use only the sign of the "Bellman error feature," not the magnitude. The sign of the statewise Bellman error at each state serves as an indication of whether the state is undervalued or overvalued by the current approximation, at least with respect to exactly representing the Bellman update of the current value function. If we can identify a collection of "undervalued" states as a new feature, then assigning an appropriate positive weight to that feature will increase their value. Similarly, identifying "overvalued" states with a new feature and assigning a negative weight will decrease their value. We note that the domains of interest are generally too large for state-space enumeration, so we will need classification learning to generalize the notions of overvalued and undervalued across the state space from training sets of sample states.

To avoid blurring the concepts of overvalued and undervalued with each other, we discard states with statewise Bellman error near zero from either training set. Specifically, among the states with negative statewise Bellman error, we discard any state with such error closer to zero than the median within that set; we do the same among the states with positive statewise Bellman error. More sophisticated methods for discarding training data near the intended boundary can be considered in future research; these will often introduce additional parameters to the method. Here, we seek an initial and simple evaluation of our overall approach. After this discarding, we define Σ_+ to be the set of all remaining training pairs with states having positive statewise Bellman error, and Σ_- likewise those with negative statewise Bellman error.

We then use Σ_+ as the positive examples and Σ_- as the negative examples for a supervised classification algorithm; in our case, C4.5 is used. The hypothesis space for classification the space of decision trees built with tests selected from the primitive attributes defining the state space and goal; in our case, we also use previously learned features that are decision trees over these attributes. The concept resulting from supervised learning is then treated as a new feature for our linear approximation architecture, with an initial weight of zero.

4.5 Discussion

Generalization across varying domain sizes The propositional feature space described above varies in size as the number of objects in a relational domain is varied. As a result, features learned at one domain size are not generally meaningful (or even necessarily defined) at other domain sizes. The relational approach above is, in contrast, able to generalize naturally between different domains sizes. Our experiments report on the ability of the propositional technique to learn within each domain size directly, but do not attempt to use that approach for learning from small problems to gain performance in large problems. This is a major limitation in producing good results for large domains.

Learning time The primary motivation for giving up generalization over domain sizes in order to employ a propositional approach is that the resulting learner can use highly efficient, off-the-

5. A grounded predicate application is a predicate applied to the appropriate number of objects from the problem instance.

shelf classification algorithms. The learning times reported in Section 7 show that our propositional learner learns new features orders of magnitude faster than the relational learner.

5. Related Work

5.1 Previous research on feature-learning value-function construction

Automatic learning of relational features for approximate value-function representation has surprisingly not been frequently studied until quite recently, and remains poorly understood. Here, we review recent work that is related on one or more dimensions to our contribution.

Feature selection based on Bellman error magnitude Feature selection based on Bellman error has recently been studied in the uncontrolled (policy-evaluation) context in (Keller et al., 2006) and (Parr et al., 2007), with attribute-value or explicit state spaces rather than relational feature representations. Here, we extend this work to the controlled decision-making setting and study the incorporation of relational learning and the selection of appropriate knowledge representation for value functions that generalize between problems of different sizes within the same domain.

The main contribution of (Parr et al., 2007) is formally showing, for the uncontrolled case of policy evaluation, that using (possibly approximate) Bellman-error features "provably tightens approximation error bounds," i.e., that adding an exact Bellman error-feature provably reduces the (weighted L_2-norm) distance from the optimal value function that can be achieved by optimizing the weights in the linear combination of features. This result is extended in a weaker form to approximated Bellman-error features, again for the uncontrolled case. The limitation to the uncontrolled case is a substantial difference from the setting of our work. The limited experiments shown use explicit state-space representations, and the technique learns a completely new set of features for each policy evaluation conducted during policy iteration. In contrast, our method accumulates features during value iteration, at no point limiting the focus to a single policy. Constructing a new feature set for each policy evaluation is a procedure more amenable to formal analysis than retaining all learned features throughout value iteration because the policy being implicitly considered during value iteration (the greedy policy) is potentially changing throughout. However, when using relational feature learning, the runtime cost of feature learning is currently too high to make constructing new feature sets repeatedly practically feasible.

(Parr et al., 2007) builds on the prior work in (Keller et al., 2006) that also studied the uncontrolled setting. That work provides no theoretical results nor any general framework, but provides a specific approach to using Bellman error in attribute value representations (where a state is represented as a real vector) in order to select new features. The approach provides no apparent leverage on problems where the state is not a real vector, but a structured logical interpretation, as is typical in planning benchmarks.

Feature discovery via goal regression Other previous methods (Gretton & Thiébaux, 2004; Sanner & Boutilier, 2006) find useful features by first identifying goal regions (or high reward regions), then identifying additional dynamically relevant regions by regressing through the action definitions from previously identified regions. The principle exploited is that when a given state feature indicates value in the state, then being able to achieve that feature in one step should also indicate value in a state. Regressing a feature definition through the action definitions yields a definition of the states that can achieve the feature in one step. Repeated regression can then identify many regions of states that have the possibility of transitioning under some action sequence to a high-reward region.

Because there are exponentially many action sequences relative to plan length, there can be exponentially many regions discovered in this way, as well as an exponential increase in the size of the representation of each region. Both exponentials are in terms of the number of regression steps taken. To control this exponential growth in the number of features considered, regression has been implemented with pruning optimizations that control or eliminate overlap between regions when it can be detected inexpensively as well as dropping of unlikely paths. However, without a scoring technique (such as the fit to the Bellman-error used in this paper) to select features, regression still generates a very large number of useless new features. The currently most effective regression-based first-order MDP planner, described in (Sanner & Boutilier, 2006) is only effective when disallowing overlapping features to allow optimizations in the weight computation, yet clearly most human feature sets in fact have overlapping features.

Our inductive technique avoids these issues by considering only compactly represented features, selecting those which match sampled statewise Bellman error training data. We provide extensive empirical comparison to the First-Order Approximate Linear Programming technique (FOALP) from (Sanner & Boutilier, 2006) in our empirical results. Our empirical evaluation yields stronger results across a wide range of probabilistic planning benchmarks than the goal-regression approach as implemented in FOALP (although aspects of the approaches other than the goal-regression candidate generation vary in the comparison as well).

Regression-based approaches to feature discovery are related to our method of fitting Bellman error in that both exploit the fact that states that can reach valuable states must themselves be valuable, i.e. both seek local consistency. In fact, regression from the goal can be viewed as a special case of iteratively fitting features to the Bellman error of the current value function. Depending on the exact problem formulation, for any k, the Bellman error for the k-step-to-go value function will be non-zero (or otherwise nontrivially structured) at the region of states that reach the goal first in $k + 1$ steps. Significant differences between our Bellman error approach and regression-based feature selection arise for states which can reach the goal with different probabilities at different horizons. Our approach fits the magnitude of the Bellman error, and so can smoothly consider the degree to which each state reaches the goal at each horizon. Our approach also immediately generalizes to the setting where a useful heuristic value function is provided before automatic feature learning, whereas the goal-regression approach appears to require goal regions to begin regression. In spite of these issues, we believe that both approaches are appropriate and valuable and should be considered as important sources of automatically derived features in future work.

Effective regression requires a compact declarative action model, which is not always available[6]. The inductive technique we present does not require even a PDDL action model, as the only deductive component is the computation of the Bellman error for individual states. Any representation from which this statewise Bellman error can be computed is sufficient for this technique. In our empirical results we show performance for our planner on **Tetris**, where the model is represented only by giving a program that, given any state as input, returns the explicit next state distribution for that state. FOALP is inapplicable to such representations due to dependence on logical deductive rea-

6. For example, in the Second International Probabilistic Planning Competition, the regression-based FOALP planner required human assistance in each domain in providing the needed domain information even though the standard PDDL model was provided by the competition and was sufficient for each other planner.

soning. We believe the inductive and deductive approaches to incorporating logical representation are both important and are complementary.

The goal regression approach is a special case of the more general approach of generating candidate features by transforming currently useful features. One such transformation is goal regression. Others that have been considered include abstraction, specialization, and decomposition (Fawcett, 1996) — all of which simplify the features, in contrast to goal regression. Research on human-defined concept transformations dates back at least to the landmark AI program AM (Davis & Lenat, 1982). Our work uses only one means of generating candidate features: a beam search of logical formulas in increasing depth. This means of candidate generation has the advantage of strongly favoring concise and inexpensive features, but may miss more complex but very accurate/useful features. But our approach directly generalizes to these other means of generating candidate features. What most centrally distinguishes our approach from all previous work leveraging such feature transformations is the use of statewise Bellman error to score candidate features. FOALP (Sanner & Boutilier, 2006) uses no scoring function, but includes all non-pruned candidate features in the linear program used to find an approximately optimal value function; the Zenith system (Fawcett, 1996) uses a scoring function provided by an unspecified "critic."

Previous scoring functions for MDP feature selection A method, from (Patrascu et al., 2002), selects features by estimating and minimizing the L_1 error of the value function that results from retraining the weights with the candidate feature included. L_1 error is used in that work instead of Bellman error because of the difficulty of retraining the weights to minimize Bellman error. Because our method focuses on fitting the Bellman error of the current approximation (without retraining with the new feature), it avoids this expensive retraining computation during search and is able to search a much larger feature space effectively. While (Patrascu et al., 2002) contains no discussion of relational representation, the L_1 scoring method could certainly be used with features represented in predicate logic; no work to date has tried this (potentially too expensive) approach.

5.2 Relational reinforcement learning

In (Džeroski, DeRaedt, & Driessens, 2001), a relational reinforcement learning (RRL) system learns logical regression trees to represent Q-functions of target MDPs. This work is related to ours since both use relational representations and automatically construct functions that capture state value. In addition to the Q-function trees, a policy tree learner is also introduced in (Džeroski et al., 2001) that finds policy trees based on the Q-function trees. We do not learn an explicit policy description and instead use only greedy policies for evaluation.

The logical expressions in RRL regression trees are used as decision points in computing the value function (or policy) rather than as numerically valued features for linear combination, as in our method. Generalization across problem sizes is achieved by learning policy trees; the learned value functions apply only to the training problem sizes. To date, the empirical results from this approach have failed to demonstrate an ability to represent the value function usefully in familiar planning benchmark domains. While good performance is shown for simplified goals such as placing a particular block A onto a particular block B, the technique fails to capture the structure in richer problems such as constructing particular arrangements of Blocksworld towers. RRL has not been entered into any of the international planning competitions. These difficulties representing complex relational value functions persist in extensions to the original RRL work (Driessens & Džeroski, 2004; Driessens et al., 2006), where again only limited applicability is shown to benchmark planning domains such as those used in our work.

5.3 Approximate policy iteration for relational domains

Our planners use greedy policies derived from learned value functions. Alternatively, one can directly learn representations for policies. The policy-tree learning in (Džeroski et al., 2001), discussed previously in Section 5.2, is one such example. Recent work uses a relational decision-list language to learn policies for small example problems that generalize well to perform in large problems (Khardon, 1999; Martin & Geffner, 2004; Yoon et al., 2002). Due to the inductive nature of this line of work, however, the selected policies occasionally contain severe flaws, and no mechanism is provided for policy improvement. Such policy improvement is quite challenging due to the astronomically large highly structured state spaces and the relational policy language.

In (Fern et al., 2006), an approximate version of policy iteration addressing these issues is presented. Starting from a base policy, approximate policy iteration iteratively generates training data from an improved policy (using policy rollout) and then uses the learning algorithm in (Yoon et al., 2002) to capture the improved policy in the compact decision-list language again. Similar to our work, the learner in (Fern et al., 2006) aims to take a flawed solution structure and improve its quality using conventional MDP techniques (in that case, finding an improved policy with policy rollout) and machine learning. Unlike our work, in (Fern et al., 2006) the improved policies are learned in the form of logical decision lists. Our work can be viewed as complementary to this previous work in exploring the structured representation of value functions where that work explored the structured representation of policies. Both approaches are likely to be relevant and important to any long-term effort to solve structured stochastic decision-making problems.

5.4 Automatic extraction of domain knowledge

There is a substantial literature on learning to plan using methods other than direct representation of a value function or a reactive policy, especially in the deterministic planning literature. These techniques are related to ours in that both acquire domain specific knowledge via planning experience in the domain. Much of this literature targets control knowledge for particular search-based planners (Estlin & Mooney, 1997; Kambhampati, Katukam, & Qu, 1996; Veloso, Carbonell, Perez, Borrajo, Fink, & Blythe, 1995), and is distant from our approach in its focus on the particular planning technology used and on the limitation to deterministic domains. It is unclear how to generalize this work to value-function construction or probabilistic domains.

However, the broader learning-to-plan literature also contains work producing declarative learned domain knowledge that could well be exploited during feature discovery for value function representation. In (Fox & Long, 1998), a pre-processing module called TIM is able to infer useful domain-specific and problem-specific structures, such as typing of objects and state invariants, from descriptions of domain definition and initial states. While these invariants are targeted in that work to improving the planning efficiency of a Graphplan based planner, we suggest that future work could exploit these invariants in discovering features for value function representation. Similarly, in (Gerevini & Schubert, 1998), DISCOPLAN infers state constraints from the domain definition and initial state in order to improve the performance of SAT-based planners; again, these constraints could be incorporated in a feature search like our method but have not to date.

6. Experimental Setting

We present experiments in nine stochastic planning domains, including both reward-oriented and goal-oriented domains. We use Pentium 4 Xeon 2.8GHz machines with 3GB memory. In this section, we give a general overview of our experiments before giving detailed results and discussion for individual domains in Section 7. Here, first, we briefly discuss the selection of evaluation domains in Section 6.1. Second, in Section 6.2, we give details on the parameter selection for our learning algorithms.

6.1 Domains considered

In all the evaluation domains below, it is necessary to specify a discount factor γ when modeling the domain as an MDP with discounting. The discount factor effectively specifies the tradeoff between the goals of reducing expected plan length and increasing success rate. γ is not a parameter of our method, but of the domain being studied, and our feature-learning method can be applied for any choice of γ. Here, for simplicity, we choose γ to be 0.95 throughout all our experiments. We note that this is the same discount factor used in the **SysAdmin** domain formalization that we compare to from the previous work of Patrascu et al. (Patrascu et al., 2002).

Tetris In Section 7.2 we evaluate the performance of both our relational and propositional learners using the stochastic computer-game **Tetris**, a reward-oriented domain where the goal of a player is to maximize the accumulated reward. We compare our results to the performance of a set of hand-crafted features, and the performance of randomly selected features.

Planning Competition Domains In Section 7.3, we evaluate the performance of our relational learner in seven goal-oriented planning domains from the two international probabilistic planning competitions (IPPCs) (Younes et al., 2005; Bonet & Givan, 2006). For comparison purposes, we evaluate the performance of our propositional learner on two of the seven domains (**Blocksworld** and a variant of **Boxworld** described below). Results from these two domains illustrate the difficulty of learning useful propositional features in complex planning domains. We also compare the results of our relational planner with two recent competition stochastic planners FF-Replan (Yoon et al., 2007) and FOALP (Sanner & Boutilier, 2006) that have both performed well in the planning competitions. Finally, we compare our results to those obtained by randomly selecting relational features and tuning weights for them. For a complete description of, and PPDDL source for, the domains used, please see (Younes et al., 2005; Bonet & Givan, 2006).

Every goal-oriented domain with a problem generator from the first or second IPPC was considered for inclusion in our experiments. For inclusion, we require a planning domain with a fixed action space, as defined in Section 2.4, that in addition has only ground conjunctive goal regions. Four domains have these properties directly, and we have adapted three more of the domains to have these properties as we describe in the next paragraph. The resulting selection provides seven IPPC planning domains for our empirical study. Figure 3 lists the reasons for the exclusion of the other six goal-oriented domains. In addition, four of the domains that we use in evaluation occur in both competitions in slightly different forms and we evaluate on one version of each of these four, as described in Figure 4.

The three domains we adapted for inclusion are as follows. We created our own problem generators for the first IPPC domains **Towers of Hanoi** and **Fileworld**, as none were provided in the competition. For both these domains, there is only one instance of each size. In Towers of Hanoi,

Domain name	IPPC version	Reason for exclusion
Colored blocksworld	IPPC1	Goal region is not a ground conjunction
Drive	IPPC2	Uses predicates with three or more arguments
Elevators	IPPC2	Uses predicates with three or more arguments
Pitchcatch	IPPC2	Action space not fixed throughout domain
Schedule	IPPC2	Action space not fixed throughout domain
Random	IPPC2	Action space not fixed throughout domain

Figure 3: Reasons for excluding some planning competition domains from our experiments.

Domain name	Differences	Version used	Reason for choice
Blocksworld	Many small differences – IPPC2 adds **emptyhand**, **on-table**(x), and **clear**(x) – IPPC2 removes table object – IPPC2 adds actions: **pick-up-from-table**, **put-down**, **pick-tower**, **put-tower-on-block**, and **put-tower-down** – IPPC2 allows **on**(x, x)	IPPC1	IPPC2 version inaccuracy allows **on**(x, x)
Exploding blocks	No generator in IPPC1	IPPC2	Problem generator in IPPC2
Tireworld	No generator in IPPC1	IPPC2	Problem generator in IPPC2
Zenotravel	No generator in IPPC1	IPPC2	Problem generator in IPPC2

Figure 4: Differences between IPPC1 and IPPC2 versions of planning domains present in both competitions, which version is used in our experimental evaluation, and why.

all instances share the same action set and state predicates, so that a suitable problem generator is straightforward. In Fileworld, a planning domain with a fixed action space results if we consider the collection of instances that share the same fixed number of folders, but varying the number of files. When the number of folders varies, the state predicates and actions change, so that instances with varying numbers of folders cannot be in the same fixed-action-space planning domain under our definitions (preventing natural generalization between sizes). For our experiments, we create a suitable domain by coding a problem generator restricted to three folders.

Furthermore, **Fileworld**, as written for the competition, is partially propositionalized (for unknown reasons). First, rather than have a one-argument predicate "have-folder", the competition domain has one proposition "have-f" for each folder f. Also, the competition domain duplicates and renames each action for each folder rather than take a folder object as an action argument (again for unknown reasons). Finally, the competition domain contains an apparent bug because it does not give types to the objects, so it is possible to file a folder in itself. Because we study relational generalization here, we have constructed the obvious lifted version of this domain with object types; we include the PPDDL source as Appendix A-1 of this paper. We call the resulting domain **Lifted-Fileworld3**.

Finally, for **Boxworld**, we modify the problem generator so that the goal region is always a ground conjunctive expression by replacing the goal "all boxes must be at their destinations" with a conjuction of specific box location goals. We call the resulting domain **Conjunctive-Boxworld**.

SysAdmin We conclude our experiments by comparing our propositional learner with a previous method by Patrascu et al. (Patrascu et al., 2002), using the the same **SysAdmin** domain used for evaluation there. This empirical comparison on the SysAdmin domain is shown in Section 7.4.

6.2 Parameterization of our methods

Here we describe our choice of parameters for our methods. Where possible, parameterization is done once, to apply identically to all experiments, as described here. There are some choices made once for each domain, and these are described in the subsection dedicated to each domain. The primary choices that must be made in a domain-specific way control learning from small problems: we must specify for each domain the performance threshold at which difficulty will be increased (as shown in Fig. 1) as well as the sequence of difficulties to be considered (in cases where there is more than one parameter controlling problem size). We defer to future research the topic of automated control of problem difficulty when learning from small problems.

Trajectory termination Training sets for both feature learning and for AVI weight update are drawn by drawing trajectories based on the current greedy policy in problems drawn from the problem distribution at the current level of difficulty, as detailed in Sections 3 and 2.5. It is an important and somewhat independent research topic to automatically recognize when such a trajectory is not making progress, e.g., by recognizing dead-end regions of states and/or lack of progress towards the goal. Any such research can be plugged into our methods directly by terminating all training trajectories when they fail an appropriate test.

Here, we do not address this issue in any sophisticated way, but terminate trajectories whenever one of three conditions holds:

1. a goal state is reached,
2. a dead-end state is reached,
3. the trajectory contains 1,000 steps.

Training set sizes Each feature-learning training set across all our relational-learning experiments is drawn to be 20,000 states by the method described in Section 3. Because propositional feature learning is faster than relational feature learning, we are able to allow 200,000 states in propositional feature learning training sets in the **Tetris** and **SysAdmin** experiments, but still only 20,000 states in the planning domains.

Throughout all experiments, each AVI weight-update training set is drawn by collecting the states from 30 trajectories.

Learning rate for weight updates in AVI As discussed in Section 2.5, we adjust the weights of our approximated value functions using AVI. We use a search-then-converge schedule for the learning rate of this iterative gradient descent method throughout our experiments (see (Darken & Moody, 1992)); specifically, we set the learning rate α in AVI to $\frac{3}{1+k/100}$, where k is the number of AVI iterations already executed.

Parametrization of the relational algorithm There are various parameters in the feature construction process described in this section, including the beam-width W, the beam-search depth limit d, the regularization parameter λ, and the bound on the maximum number of quantifiers in scope q. Changes to these parameters affect the quality of the constructed features by changing the feature-space regions searched and the number of candidate features considered, as well as changing the preferences expressed in scoring the features. The selection of these parameters further affects the choice of the size of feature training set, as in practice fewer training examples can be considered when the number of candidate features grows.

Throughout all our experiments we choose W to be 60, d to be 5, and λ to be 0.03 for all domains. We set q to 1 for the planning competition domains (setting q to 2 does not result in a noted improvement in the performance in these domains when using the above parameters, but results in a substantial and occasionally intolerable runtime cost), and we set q to 2 for **Tetris**. These severe limits on q are necessary to control the expense of searching the feature space. Note however that there is implicit quantification in the transitive-closure predicates and min/max predicates in the extended predicate set defining the feature space, in addition to the explicit quantifiers limited by q. See Section 4.1 for discussion of the extended predicate set.

Parametrization of the propositional algorithm Our propositional feature learning algorithm is already well defined in Section 4.4, except for how to setup the underlying C4.5 learner (Quinlan, 1993). We use the default parameters for C4.5, except for the following: we use the gain criterion instead of the gain ratio criterion. We allow the trees to grow from a node without any restriction on the minimum number of objects in the resulting branches[7]. The pruning confidence level is set to 0.9.

7. Experimental Results

7.1 How to read our results

The task of evaluating a feature-learning planning system is subtle and complex. This is particularly a factor in the relational case because generalization between problem sizes and learning from small problems must be evaluated. The resulting data is extensive and highly structured, requiring some training of the reader to understand and interpret. Here we introduce the reader to the structure of our results.

In experiments with the propositional learning (or with randomly selected propositional features), the problem size never varies within one run of the learner, because the propositional representation from Section 4.3 can't generalize between sizes. We run a separate experiment for each size considered. Each experiment is two independent trials; each trial starts with a single trivial feature and repeatedly adds features until a termination condition is met. After each feature addition, AVI is used to select the weights for combining the features to form a value function, and the performance of that value function is measured (by sampling the performance of the greedy policy). We then compute the average (of the two trials) of the performance as a function of the number of features used. Since this results in a single line plot of performance as a function of number of features, several different fixed-problem-size learners can be compared on one figure, with one line for each, as is done for example in Figures 7 and 14. The performance measure used varies appropriately with the domain as presented below.

We study the ability of relational representation from Section 4.1 to generalize between sizes. This study can only be properly understood against the backdrop of the flowchart in Figure 1. As described in this flowchart, one trial of the learner will learn a sequence of features and encounter a sequence of increasing problem difficulties. One iteration of the learner will *either* add a new feature *or* increase the problem difficulty (depending on the current performance). In either case, the weights are then retrained by AVI and a performance measurement of the resulting greedy policy is taken. Because different trials may increase the size at different points, we cannot meaningfully average the measurements from two trials. Instead, we present two independent trials separately

7. The default C4.5 parameter requires at least 2 branches from any node to contain at least 2 objects.

in two tables, such as the Figures 5 and 12. For the first trial, we also present the same data a second time as a line plot showing performance as a function of number of features, where problem size changes are annotated along the line, such as the plots in Figures 6 and 13. Note that success ratio generally increases along the line when features are added, but falls when problem size is increased. (In **Tetris**, however, we measure "rows erased" rather than success ratio, and "rows erased" generally increases with either the addition of a new feature or the addition of new rows to the available grid.)

To interpret the tables showing trials of the relational learner, it is useful to focus on the first two rows, labeled "# of features" and "Problem difficulty." These rows, taken together, show the progress of the learner in adding features and and increasing problem size. Each column in the table represents the result in the indicated problem size using the indicated number of learned features. From one column to the next, there will be a change in only one of these rows—if the performance of the policy shown in a column is high enough, it will be the problem difficulty that increases, and otherwise it will be the number of features that increases. Further adding to the subtlety in interpreting these tables, we note that when several adjacent columns increase the number of features, we sometimes splice out all but two of these columns to save space. Thus, if several features are added consecutively at one problem size, with slowly increasing performance, we may show only the first and last of these columns at that problem size, with a consequent jump in the number of features between these columns. We likewise sometimes splice out columns when several consecutive columns increase problem difficulty. We have found that these splicings not only save space but increase readability after some practice reading these tables.

Performance numbers shown in each column (success ratio and average plan length, or number of rows erased, for **Tetris**) refer to the performance of the weight-tuned policy resulting for that feature set at that problem difficulty. We also show in each column the performance of that value function (without re-tuning weights) on the target problem size. Thus, we show quality measures for each policy found during feature learning on both the current problem size at that point and on the target problem size, to illustrate the progress of learning from small problems on the target size via generalization.

In both propositional and relational experiments, trials are stopped by experimenter judgment when additional results are too expensive for the value they are giving in evaluating the algorithm. Also, in each trial, the accumulated real time for the trial is measured and shown at each point during the trial. We use real time rather than CPU time to reflect non-CPU costs such as paging due to high memory usage.

7.2 Tetris

Overview of Tetris The game **Tetris** is played in a rectangular board area, usually of size 10×20, that is initially empty. The program selects one of the seven shapes uniformly at random and the player rotates and drops the selected piece from the entry side of the board, which piles onto any remaining fragments of the pieces that were placed previously. In our implementation, whenever a full row of squares is occupied by fragments of pieces, that row is removed from the board and fragments on top of the removed row are moved down one row; a reward is also received when a row is removed. The process of selecting locations and rotations for randomly drawn pieces continues until the board is "full" and the new piece cannot be placed anywhere in the board. Tetris is stochastic since the next piece to place is always randomly drawn, but this is the only stochastic element

Trial #1												
# of features	0	1	2	3	11	11	12	17	17	18	18	18
Problem difficulty	5	5	5	5	5	6	6	6	7	7	8	9
Score	0.2	0.5	1.0	3.0	18	31	32	35	55	56	80	102
Accumulated time (Hr.)	0.0	2	4.2	5.2	20	21	24	39	42	46	50	57
Target size score	0.3	1.3	1.4	1.8	178	238	261	176	198	211	217	221

Trial #2											
# of features	0	1	2	3	8	8	12	12	14	14	14
Problem difficulty	5	5	5	5	5	6	6	7	7	8	9
Score	0.2	0.6	1.1	4.5	16	28	36	53	56	78	97
Accumulated time (Hr.)	0.0	2.4	3.9	4.9	15	15	27	29	39	44	49
Target size score	0.3	1.7	1.7	30	104	113	108	116	130	157	171

Figure 5: **Tetris** performance (averaged over 10,000 games). Score is shown in average rows erased, and problem difficulty is shown in the number of rows on the Tetris board. The number of columns is always 10. Difficulty increases when the average score is greater than 15+20*(n-5), where n is the number of rows in the Tetris board. Target problem size is 20 rows. Some columns are omitted as discussed on page 27.

in this game. Tetris is also used as an experimental domain in previous MDP and reinforcement learning research (Bertsekas & Tsitsiklis, 1996; Driessens et al., 2006). A set of human-selected features is described in (Bertsekas & Tsitsiklis, 1996) that yields very good performance when used in weighted linearly approximated value functions. We cannot fairly compare our performance in this domain to probabilistic planners requiring PPDDL input because we have found no natural PPDDL definition for Tetris.

Our performance metric for **Tetris** is the number of rows erased averaged over 10,000 trial games. The reward-scaling parameter r_{scale} is selected to be 1.

Tetris relational feature learning results We represent the **Tetris** grid using rows and columns as objects. We use three primitive predicates: **fill**(c, r), meaning that the square on column c, row r is occupied; **below**(r_1, r_2), meaning that row r_1 is directly below row r_2; and **beside**(c_1, c_2), meaning that column c_1 is directly to the left of column c_2. While our representation here uses only these primitive domain predicates, the RRL result we compare to uses human-specified Tetris-specific functions in the representation such as “number of holes” (Driessens et al., 2006). The quantifiers used in our relational Tetris hypothesis space are typed using the types “row” and “column”.

There are also state predicates representing the piece about to drop; however, for efficiency reasons our planner computes state value as a function only of the grid, not the next piece. This limitation in value-function expressiveness allows a significantly cheaper Bellman-backup computation. The one-step lookahead in greedy policy execution provides implicit reasoning about the piece being dropped, as that piece will be in the grid in all the next states.

We conduct our relational **Tetris** experiments on a 10-column, n-row board, with n initially set to 5 rows. Our threshold for increasing problem difficulty by adding one row is a score of at least $15 + 20(n - 5)$ rows erased. The target problem size for these experiments is 20 rows. The results for the relational Tetris experiments are given in Figures 5 and 6 and are discussed below.

Tetris propositional feature learning results For the propositional learner, we describe the **Tetris** state with 7 binary attributes that represent which of the 7 pieces is currently being dropped,

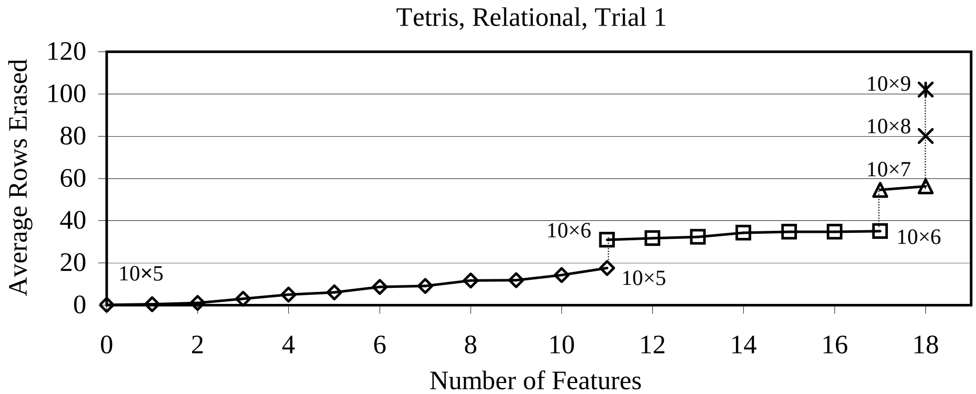


Figure 6: Plot of the average number of lines erased over 10,000 **Tetris** games after each run of AVI training during the learning of relational features (trial 1). Vertical lines indicate difficulty increases (in the number of rows), as labeled along the plot.

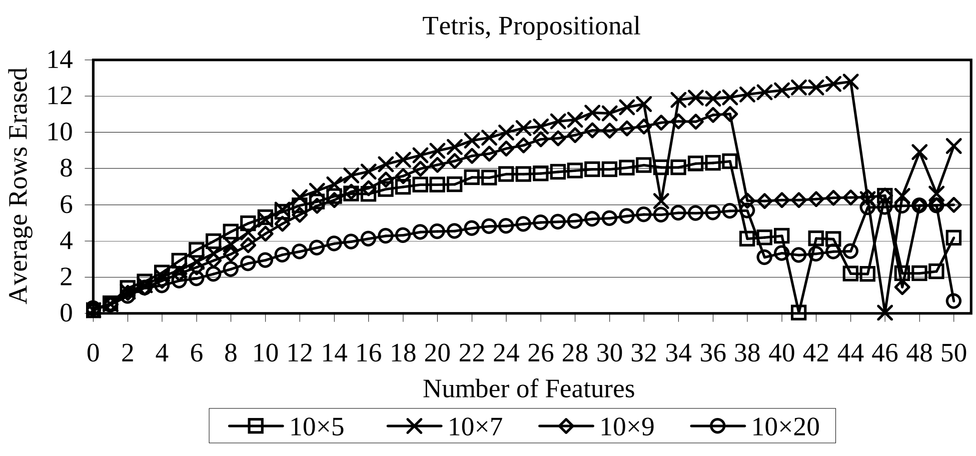


Figure 7: Plot of the average number of lines erased in 10,000 **Tetris** games after each iteration of AVI training during the learning of propositional features, averaged over two trials.

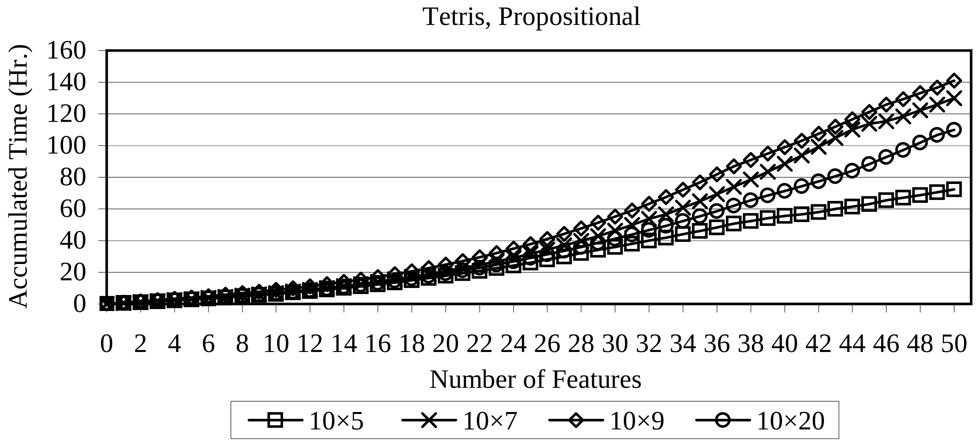


Figure 8: Plot of the accumulated time required to reach each point in Figure 7, averaged over two trials.

along with one additional binary attribute for each grid square representing whether that square is occupied. The adjacency relationships between the grid squares are represented only through the procedurally coded action dynamics. Note that the number of state attributes depends on the size of the Tetris grid, and learned features will only apply to problems of the same grid size. As a result, we show separate results for selected problem sizes.

We evaluate propositional feature learning in 10-column **Tetris** grids of four different sizes: 5 rows, 7 rows, 9 rows, and 20 rows. Results from these four trials are shown together in Figure 7 and the average accumulated time required to reach each point on Figure 7 is shown in Figure 8.

Evaluating random features in Tetris In **Tetris**, we compare the results of both our learning approaches against selecting features randomly to demonstrate that indeed using statewise Bellman error as the criterion to select features is essential to the success of our feature learning approaches. The only difference between constructing learned features and random features is we replace the target value in our feature training set with a random number from -1 to 1. We use the same approach to generate random features later in the planning domains. Again, in our propositional approach we only show results from using 200,000 states in random feature training sets.

In Figure 9 we show the results for random features in **Tetris**. For random features using our relational representation, we use the same schedule used for the learned relational features in Tetris by starting with the 10×5 problem size. However, the performance of random features is never good enough to increase the problem size. For the propositional approach we show the same problem sizes as in the learned propositional features.

Evaluating human features in Tetris In addition to evaluating our relational and propositional feature learning approach, we also evaluate how the human-selected features described in (Bertsekas & Tsitsiklis, 1996) perform in selected problem sizes. For each problem size, we start from all weights zero and use the AVI process described in Section 2.5 to train the weights for all 21 features until the performance appears to converge. We change the learning rate α from $\frac{3}{1+k/100}$ to $\frac{30}{1+k/100}$ as human features require a larger step-size to converge rapidly. The human features are normalized to a value between 0 and 1 here in our experiments. We run two independent trials for

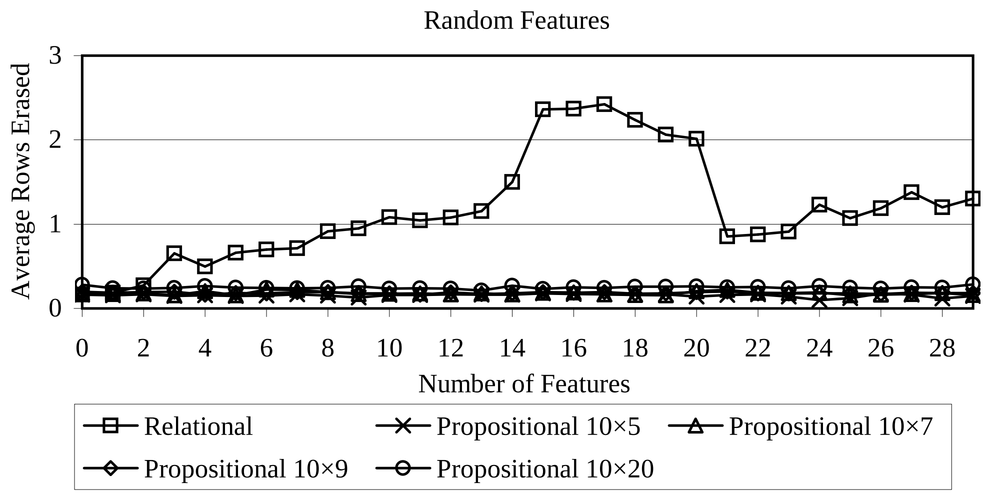


Figure 9: Plot of the average number of lines erased in 10,000 **Tetris** games using for randomly generated features. The relational trial starts with the 10×5 problem size and never achieves sufficient performance to increase that size.

each problem size and report the performance of the best-performing weight vector found in each trial, in Figure 10.

	10×5	10×7	10×9	10×20
Average rows erased, Trial 1	19	86	267	17,054
Average rows erased, Trial 2	19	86	266	18,125

Figure 10: The average number of lines erased in 10,000 **Tetris** games for the best weighted combination of human feature found in each of two trials of AVI and four problem sizes.

Performance comparison between different approaches to Tetris Several general trends emerge from the results on **Tetris**. First of all, the addition of new learned features is almost always increasing the performance of the resulting tuned policy (on the current size and on the target size), until a best performance point is reached. This suggests we are in fact selecting useful features. We also find clear evidence of the ability of the relational representation to usefully generalize between problem sizes: substantial performance is developed on the target problem size without ever training directly in that size.

In our two relational learning trials the best target size performances were 261 and 171 rows erased, respectively. For comparison, the best RRL result reported in (Driessens et al., 2006) was around 55 rows erased (as we estimate from reading the published plot). Our relational approach thus produces performance far superior to the policy learned by RRL, even though RRL is using human-engineered Tetris-specific features much more sophisticated than those in the primitive domain description. The setting for the RRL work is substantially different in access to the world model from our setting in that RRL leverages only the ability to execute actions, whereas our learner requires the ability to compute Bellman backups. It is a suitable topic for future research to design an approximate Q-learning version of our methods that would require only action execution for

	Relational	Prop. 10×5	Prop.10×7	Prop.10×9	Prop.10×20
Average feature learning time (Min.)	112	44	52	60	44

Figure 11: Table for the average feature learning time for relational and propositional approaches.

training. What we can conclude here is that our learner is able to exploit this simple additional model access to much more than compensate for the lack of any human input regarding feature selection.

We find that the best performance of learned propositional features is much lower than that of learned relational features in all problem sizes shown here, even though a larger feature training set size and many more learned features are used for the propositional approach. This suggests that the rich relational representation indeed is able to better capture the dynamics in Tetris than the propositional representation.

We find that the performance of using random features in **Tetris** is significantly worse than that of using learned features, demonstrating the our performance improvements in feature learning are due to useful feature selection (using Bellman error), not simply due to increasing the number of features.

Our learned relational feature performance in 10×20 **Tetris** is far worse than that obtained by using the human-selected features with AVI here. However, in 10×5 Tetris the relational feature performance is close to that of the human features. The human features are engineered to perform well in the 10×20 Tetris hence some concepts that are useful in performing well in smaller problem sizes may not exist in these features.

Our feature learner has definitely not replaced the value of human engineering in selecting features in this domain, though it does produce the best machine-learned policy known to date that is found without exploiting human engineering of the feature set. We suggest that humans use sophisticated reasoning about the domain model as well as a richer feature representation in order to develop more useful features than our technique can.

Time to learn each feature In Figure 11 we show the average time required to learn a relational feature or a propositional feature in **Tetris**.

The time required to learn a relational feature is significantly longer than that required to learn a propositional feature, even though for the propositional approach a larger feature training set size is being used.

Key Factors to Finding Human Tetris Features Automatically Here we discuss key factors that may need to be addressed to be able to automatically construct features similar to the human-selected ones in **Tetris**. Of course, the rich knowledge representation used by human engineers is a critical factor. The human feature set we evaluate in this paper contains features defined with flexible usage of defined concepts such as “column height,” “difference in height of adjacent columns,” and “maximum height of all columns.” One can imagine searching a feature hypothesis language automatically containing such richer constructs, but control of such richness to avoid unacceptable runtime cost could possibly become a major issue. Generally, we suggest careful enrichment of the knowledge representation in the direction of capturing distinguished quantified concepts concisely as defined concepts using few or no explicit quantifiers as a fruitful direction for future research.

The human feature set contains many features defined once for each column. This suggest considering type-based feature discovery so that a new feature would be added for each object of the same type. Nothing like this is done by our current methods.

The human feature set also contains a feature counting the number of covered up “holes” in the board. This feature is likely derived by reasoning about the rules of the game and realizing that such holes are difficult to fill. Bellman error evaluation could play a role in such reasoning. The state of the art in planning, learning, and reasoning is far short of finding such a feature via reasoning without also doing an unmanageable amount of other, useless reasoning. Nonetheless, using some form of targeted reasoning from the rules to define an enriched feature-description space is a feasible direction for future research.

7.3 Probabilistic Planning Competition Domains

Throughout the evaluations of our learners in planning domains, we use a lower plan-length cutoff of 1000 steps when evaluating success ratio during the iterative learning of features, to speed learning. We use a longer cutoff of 2000 steps for the final evaluation of policies for comparison with other planners and for all evaluations on the target problem size. The reward-scaling parameter r_{scale} is selected to be 1 throughout the planning domains.

For domains with multi-dimensional problem sizes, it remains an open research problem on how to change problem size in different dimensions automatically to increase difficulty during learning. Here, in **Conjunctive-Boxworld** and **Zenotravel**, we hand-design the sequence of increasing problem sizes.

Blocksworld In the probabilistic, non-reward version of **Blocksworld** from the first IPPC, the actions **pickup** and **putdown** have a small probability of placing the handled block on the table object instead of on the selected destination.

For our relational learner, we start with 3 blocks problems. We increase from n blocks to $n+1$ blocks whenever the success ratio exceeds 0.9 and the average successful plan length is less than $30(n-2)$. The target problem size is 20 blocks. Results are shown in Figures 12 and 13.

For our propositional learner, results for problem sizes of 3, 4, 5, and 10 blocks are shown in Figure 14, with accumulated time taken shown in Figure 15.

Our relational learner consistently finds value functions with perfect or near-perfect success ratio up to 15 blocks. This performance compares very favorably to the recent RRL (Driessens et al., 2006) results in the deterministic blocksworld, where goals are severely restricted to, for instance, single **ON** atoms, and the success ratio performance of around 0.9 for three to ten blocks (for the single **ON** goal) is still lower than that achieved here. Our results in blocksworld show the average plan length is far from optimal. We have observed large plateaus in the induced value function: state regions where all states are given the same value so that the greedy policy wanders. This is a problem that merits further study to understand why feature-induction does not break such plateaus. Separately, we have studied the ability of local search to break out of such plateaus (Wu, Kalyanam, & Givan, 2008).

The performance on the target size clearly demonstrates successful generalization between sizes for the relational representation.

The propositional results demonstrate the limitations of the propositional learner regarding lack of generalization between sizes. While very good value functions can be induced for the small problem sizes (3 and 4 blocks), slightly larger sizes of 5 or 10 blocks render the method ineffective.

Trial #1								
# of features	0	1	2	2	3	3	3	3
Problem difficulty	3	3	3	4	4	5	10	15
Success ratio	1.00	1	1	0.95	1	1	1	0.97
Plan length	89	45	20	133	19	33	173	395
Accumulated time (Hr.)	0.5	1.0	1.5	2.2	3.3	3.9	10	36
Target size SR	0	0	0	0	0.98	0.96	0.98	0.97
Target size Slen.	–	–	–	–	761	724	754	745

Trial #2								
# of features	0	1	2	2	3	3	3	3
Problem difficulty	3	3	3	4	4	5	10	15
Success ratio	1	1	1	0.94	1	1	1	0.96
Plan length	80	48	19	125	17	34	167	386
Accumulated time (Hr.)	0.5	1.0	1.4	2.0	3.3	3.8	9.4	33
Target size SR	0	0	0	0	0.97	0.98	0.98	0.98
Target size Slen.	–	–	–	–	768	750	770	741

Figure 12: **Blocksworld** performance (averaged over 600 problems) for relational learner. We add one feature per column until success ratio exceeds 0.9 and average successful plan length is less than $30(n-2)$, for n blocks, and then increase problem difficulty for the next column. Plan lengths shown are successful trials only. Problem difficulties are measured in number of blocks, with a target problem size of 20 blocks. Some columns are omitted as discussed on page 27.

In 10 block problems, the initial random greedy policy cannot be improved because it never finds the goal. In addition, these results demonstrate that learning additional features once a good policy is found can degrade performance, possibly because AVI performs worse in the higher dimensional weight space that results.

Conjunctive-Boxworld The probabilistic, non-reward version of **Boxworld** from the first IPPC is similar to the more familiar **Logistics** domain used in deterministic planning competitions, except that an explicit connectivity graph for the cities is defined. In Logistics, airports and aircraft play an important role since it is not possible to move trucks from one airport (and the locations adjacent to it) to another airport (and the locations adjacent to it). In Boxworld, it is possible to move all the boxes without using the aircraft since the cities may all be connected with truck routes. The stochastic element introduced into this domain is that when a truck is being moved from one city to another, there is a small chance of ending up in an unintended city. As described in Section 6.1, we use **Conjunctive-Boxworld**, a modified version of Boxworld, in our experiments.

We start with 1-box problems in our relational learner and increase from n boxes to $n+1$ boxes whenever the success ratio exceeds 0.9 and the average successful plan length is better than $30n$. All feature-learning problem difficulties use 5 cities. We use two target problem sizes: 15 boxes and 5 cities, and 10 boxes and 10 cities. Relational learning results are shown in Figures 16 and 17, and results for the propositional learner on 5 cities with 1, 2, or 3 boxes are shown in Figures 18 and 19.

In interpreting the **Conjunctive-Boxworld** results, it is important to focus on the average successful plan-length metric. In Conjunctive-Boxworld problems, random walk is able to solve the

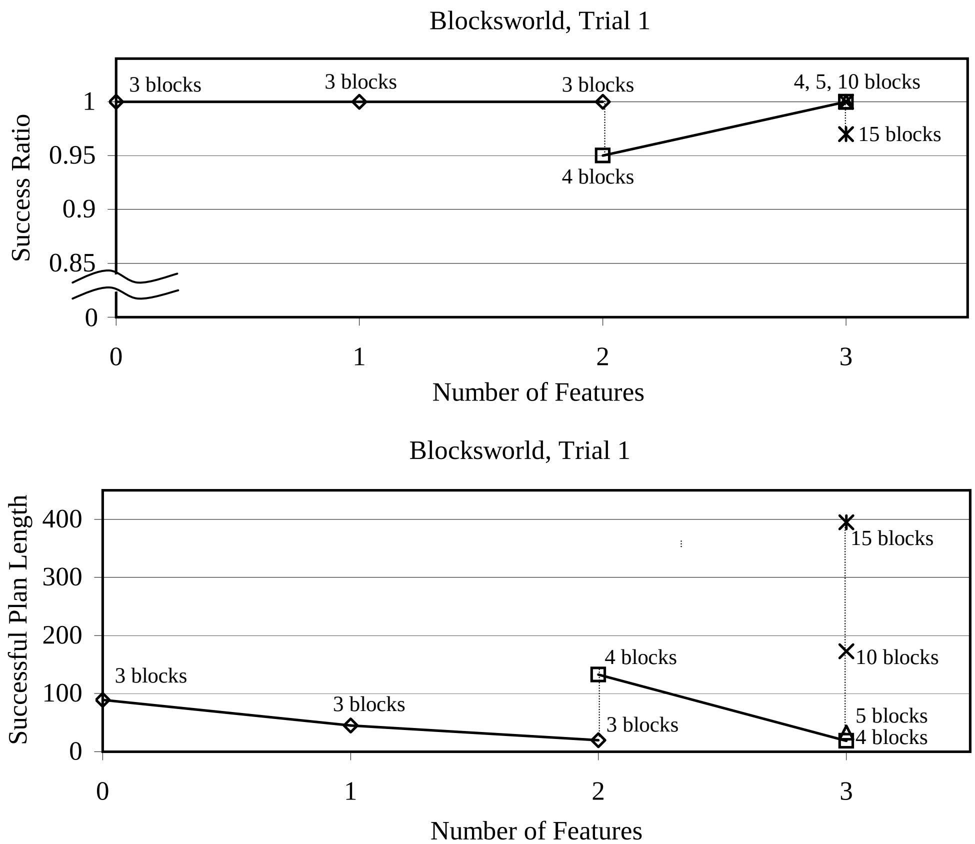


Figure 13: **Blocksworld** success ratio and average successful plan length (averaged over 600 problems) for the first trial from Figure 12 using our relational learner.

problem nearly always, but often with very long plans[8]. The learned features enable more direct solutions as reflected in the average plan-length metric.

Only two relational features are required for significantly improved performance in the problems we have tested. Unlike the other domains we evaluate, for the **Conjunctive-Boxworld** domain the learned features are straightforwardly describable in English. The first feature counts how many boxes are correctly at their target city. The second feature counts how many boxes are on trucks.

We note the lack of any features rewarding trucks for being in the "right" place (resulting in longer plan lengths due to wandering on value-function plateaus). Such features can easily be written in our knowledge representation (e.g. count the trucks located at cities that are the destinations for some package on the truck), but require quantification over both cities and packages. The severe limitation on quantification currently in our method for efficiency reasons prevents consideration of these features at this point. It is also worth noting that regression-based feature discovery, as studied in (Sanner & Boutilier, 2006; Gretton & Thiébaux, 2004), can be expected to identify such features

8. We note that, oddly, the IPPC competition domain used here has action preconditions prohibiting moving a box away from its destination. These preconditions bias the random walk automatically towards the goal. For consistency with the competition results, we retain these odd preconditions, although these preconditions are not necessary for good performance for our algorithm.

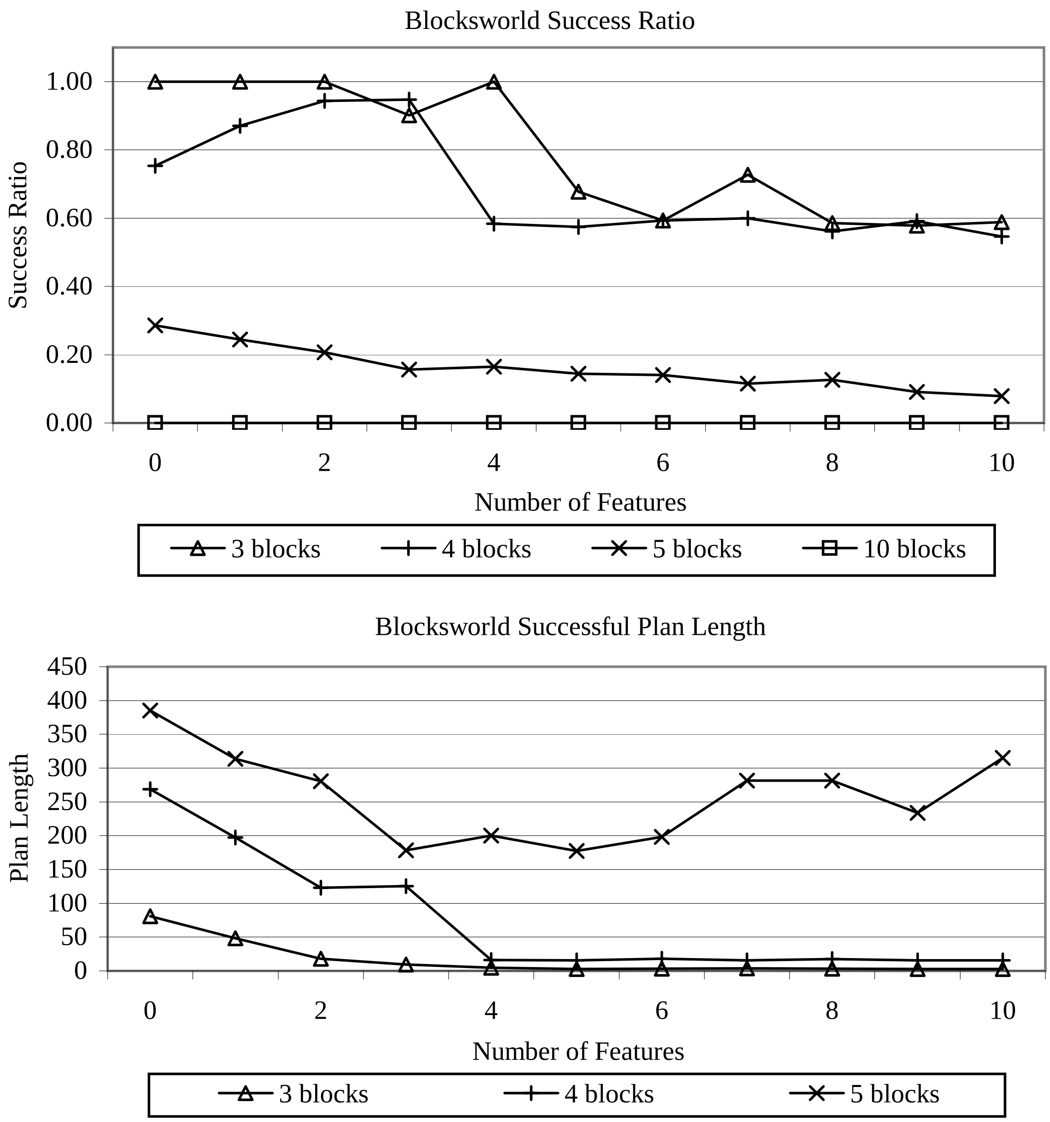


Figure 14: **Blocksworld** performance success ratio and average successful plan length (averaged over 600 problems) for our propositional learner, averaged over two trials.

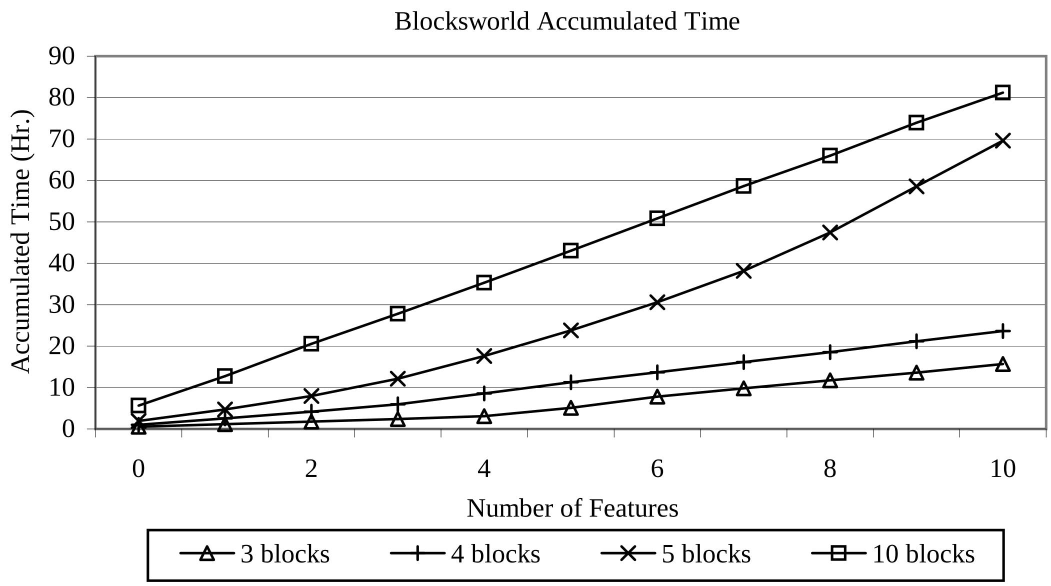


Figure 15: Accumulated run-time in **Blocksworld** for our propositional learner, averaged over two trials.

regarding trucks by regressing the goal through the action of unloading a package at the destination. Combining our Bellman-error-based method with regression-based methods is a promising future direction.

Nevertheless, our relational learner discovers two concise and useful features that dramatically reduce plan length relative to the initial policy of random walk. This is a significant success for automated domain-independent induction of problem features.

One trial of the relational feature learner in **Conjunctive-Boxworld** takes several days, even though we have fixed the number of cities for the training problems at five cities. New techniques are required for improving the efficiency of feature learning before we can provide results for training in larger numbers of cities. Our results here demonstrate that the current representation and learning methods adequately manage small city graphs even with larger and larger numbers of boxes to deliver, and that the resulting value functions successfully generalize to 10-city problems.

In this domain, a well known weakness of AVI is apparent. While AVI often works in practice, there is no theoretical guarantee on the quality of the weight vector found by AVI training. (Alternatively, an approximate linear programming step could replace AVI training to provide a more expensive but perhaps more robust weight selection.) In the **Conjunctive-Boxworld** results, AVI training goes astray when selecting weights in the 12 box domain size in Trial 1. As a result, the selected weights overemphasize the first feature, neglecting the second feature. This is revealed in the data shown because the plan-length performance degrades significantly for that one column of data. When AVI is repeated at the next problem size (13 boxes), good performance is restored. A similar one-column degradation of plan length occurs in trial 2 at the 10-box and 12-box sizes.

For our propositional experiments in the **Conjunctive-Boxworld**, we note that, generally, adding learned propositional features degrades the success-rate performance relative to the initial random walk policy by introducing ineffective loops into the greedy policy. The resulting greedy policies find the goal in fewer steps than random walk, but generally pay an unacceptable drop in

Trial #1											
# of features	0	1	2	2	2	2	2	2	2	2	2
Problem difficulty	1	1	1	2	3	5	10	11	12	13	15
Success ratio	0.97	1	1	1	1	1	1	1	1	1	1
Plan length	226	84	23	37	44	54	77	80	313	87	92
Accumulated time (Hr.)	7.2	10	13	14	16	21	42	49	57	65	84
Target size #1 SR	0.98	1	1	1	1	1	1	1	1	1	1
Target size #1 Slen.	1056	359	93	91	90	92	90	92	355	90	91
Target size #2 SR	0.16	0.90	0.97	0.97	0.96	0.98	0.96	0.98	0.90	0.98	0.96
Target size #2 Slen.	1583	996	238	230	233	244	240	238	1024	240	239

Trial #2												
# of features	0	1	2	2	2	2	2	2	2	2	2	2
Problem difficulty	1	1	1	2	3	5	9	10	11	12	13	15
Success ratio	0.97	1	1	1	1	1	1	1	1	1.00	1	1
Plan length	235	85	24	34	43	54	72	299	80	310	84	91
Accumulated time (Hr.)	7.3	11	14	16	18	23	39	45	51	60	68	86
Target size #1 SR	0.96	1	1	1	1	1	1	1	1	1	1	1
Target size #1 Slen.	1019	365	90	91	91	92	89	359	89	363	90	90
Target size #2 SR	0.19	0.9	0.97	0.97	0.98	0.98	0.97	0.92	0.98	0.91	0.97	0.96
Target size #2 Slen.	1574	982	226	230	233	233	242	1006	231	1026	240	233

Figure 16: **Conjunctive-Boxworld** performance (averaged over 600 problems). We add one feature per column until success ratio is greater than > 0.9 and average successful plan length is less than $30n$, for n boxes, and then increase problem difficulty for the next column. Problem difficulty is shown in number of boxes. Throughout the learning process the number of cities is 5. Plan lengths shown are successful trials only. Two target problem sizes are used. Target problem size #1 has 15 boxes and 5 cities. Target problem size #2 has 10 boxes and 10 cities. Some columns are omitted as discussed on page 27.

success ratio to do so. The one exception is the policy found for one-box problems using just two propositional features, which significantly reduces plan length while preserving success ratio. Still, this result is much weaker than that for our relational feature language.

These problems get more severe as problem size increases, with 3-box problems suffering severe degradation in success rate with only modest gains in successful plan length. Also please note that accumulated runtime for these experiments is very large, especially for 3-box problems. AVI training is very expensive for policies that do not find the goal. Computing the greedy policy at each state in a long trajectory requires considering each action, and the number of available actions can be quite large in this domain. For these reasons, the propositional technique is not evaluate at sizes larger than three boxes.

Tireworld We use the **Tireworld** domain from the second IPPC. The agent needs to drive a vehicle through a graph from the start node to the goal node. When moving from one node to an adjacent node, the vehicle has a certain chance of suffering a flat tire (while still arriving at the adjacent node). The flat tire can be replaced by a spare tire, but only if there is such a spare tire present in the node containing the vehicle, or if the vehicle is carrying a spare tire. The vehicle can pick up a spare tire if it is not already carrying one and there is one present in the node containing the vehicle. The default setting for the second-IPPC problem generator for this domain defines a problem distribution that includes problems for which there is no policy achieving the goal with probability one. Such problems create a tradeoff between goal-achievement probability and expected number

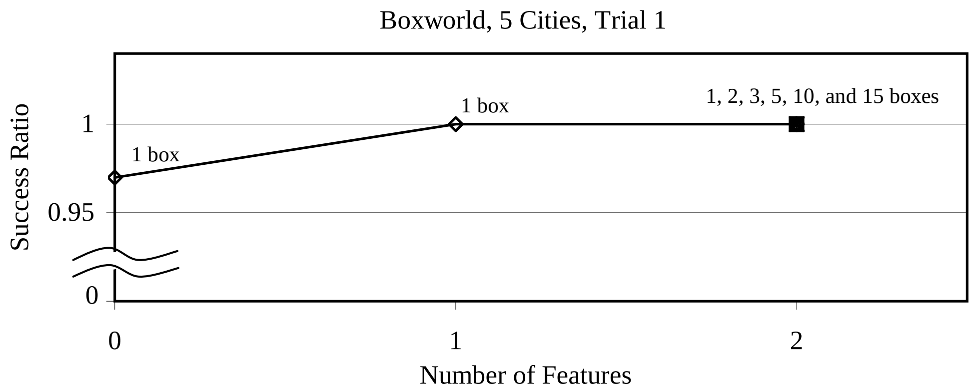


Figure 17: **Conjunctive-Boxworld** success ratio and average successful plan length (averaged over 600 problems) for the first trial using our relational learner.

of steps to the goal. How strongly our planner favors goal achievement versus short trajectories to the goal is determined by the choice of the discount factor made in Section 6.1.

We start with 4-node problems in our relational learner and increase from n nodes to $n + 1$ nodes whenever the success ratio exceeds 0.85 and the average successful plan length is better than $4n$ steps. The target problem size is 30 nodes. The results are shown in Figures 20 and 21.

In **Tireworld**, our relational learner again is able to find features that generalize well to large problems. Our learner achieves a success ratio of about 0.9 on 30 node problems. It is unknown whether any policy can exceed this success ratio on this problem distribution; however, neither comparison planner, FOALP nor FF-Replan, finds a higher success-rate policy.

We note that some improvements in success rate in this domain will necessarily be associated with increases in plan length because success-rate improvements may be due to path deviations to acquire spare tires.

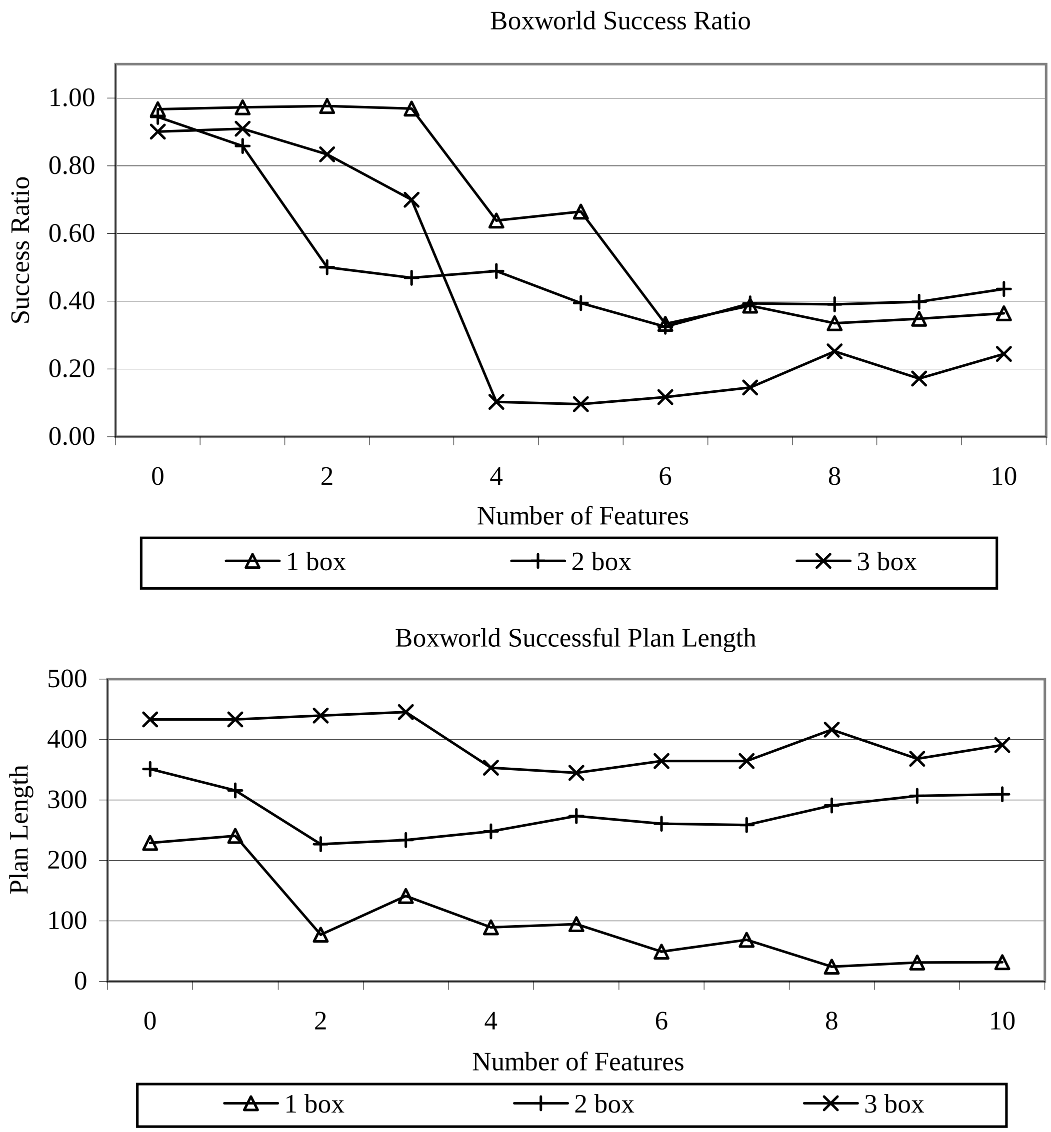


Figure 18: **Conjunctive-Boxworld** performance (averaged over 600 problems) for propositional learner, averaged over two trials. Throughout the learning process the number of cities is 5.

Zenotravel We use the **Zenotravel** domain from the second IPPC. The goal of this domain is to fly all travelers from their original location to their destination. Planes have (finite-range, discrete) fuel levels, and need to be re-fuelled when the fuel level reaches zero to continue flying. Each available activity (boarding, debarking, flying, zooming, or refueling) is divided into two stages, so that an activity X is modelled as two actions start-X and finish-X. Each finish-X activity has a (high) probability of doing nothing. Once a "start" action is taken, the corresponding "finish" action must be taken (repeatedly) until it succeeds before any conflicting action can be started. This structure allows the failure rates on the "finish" actions to simulate action costs (which were not used explicitly in the problem representation for the competition). A plane can be moved between

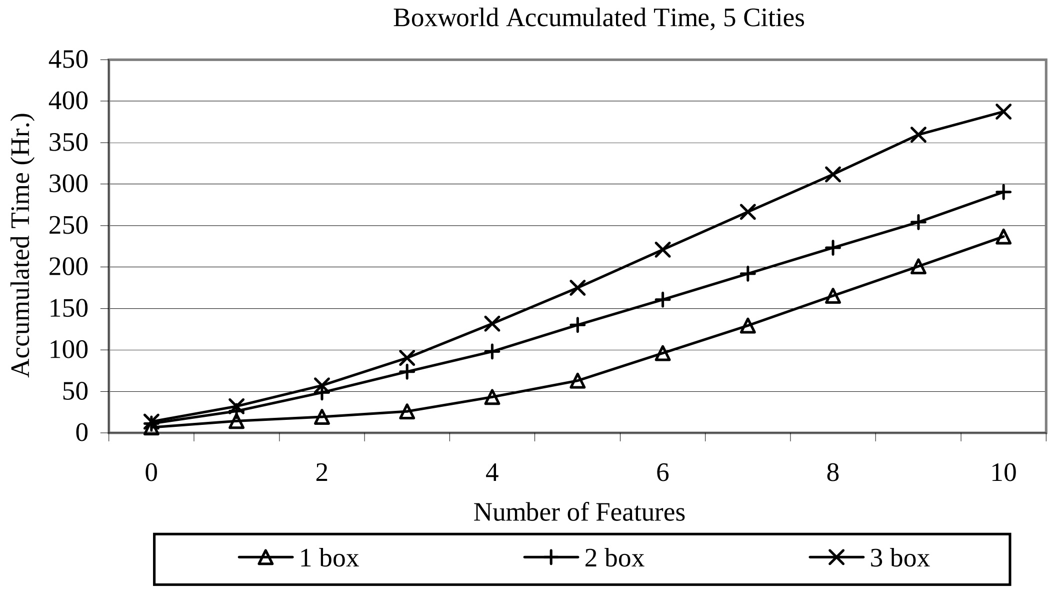


Figure 19: **Conjunctive-Boxworld** accumulated time (averaged over 600 problems) for propositional learner, averaged over two trials.

Trial #1												
# of features	0	1	2	3	3	3	4	4	5	5	5	5
Problem difficulty	4	4	4	4	5	6	6	9	9	10	20	30
Success ratio	0.52	0.81	0.84	0.86	0.86	0.84	0.88	0.85	0.86	0.86	0.91	0.91
Plan length	4	3	4	2	2	2	3	3	4	4	5	5
Accumulated time (Hr.)	0.3	3.1	12	17	18	18	19	21	22	23	29	36
Target size SR	0.17	0.53	0.81	0.83	0.83	0.82	0.90	0.91	0.91	0.91	0.92	0.92
Target size Slen.	5	4	9	5	4	4	6	6	6	6	5	6

Trial #2										
# of features	0	1	2	3	3	3	4	4	4	4
Problem difficulty	4	4	4	4	5	6	6	10	20	30
Success ratio	0.52	0.81	0.85	0.86	0.93	0.81	0.89	0.85	0.86	0.88
Plan length	4	3	3	2	3	2	3	4	4	5
Accumulated time (Hr.)	0.5	3.7	6.9	10	11	11	12	14	18	24
Target size SR	0.19	0.49	0.80	0.82	0.91	0.62	0.92	0.91	0.90	0.88
Target size Slen.	7	3	9	4	5	2	5	5	6	6

Figure 20: **Tireworld** performance (averaged over 600 problems) for relational learner. We add one feature per column until success ratio exceeds 0.85 and average successful plan length is less than $4n$, for n nodes, and then increase problem difficulty for the next column. Plan lengths shown are successful trials only. Problem difficulties are measured in number of nodes, with a target problem size of 30 nodes. Some columns are omitted as discussed on page 27.

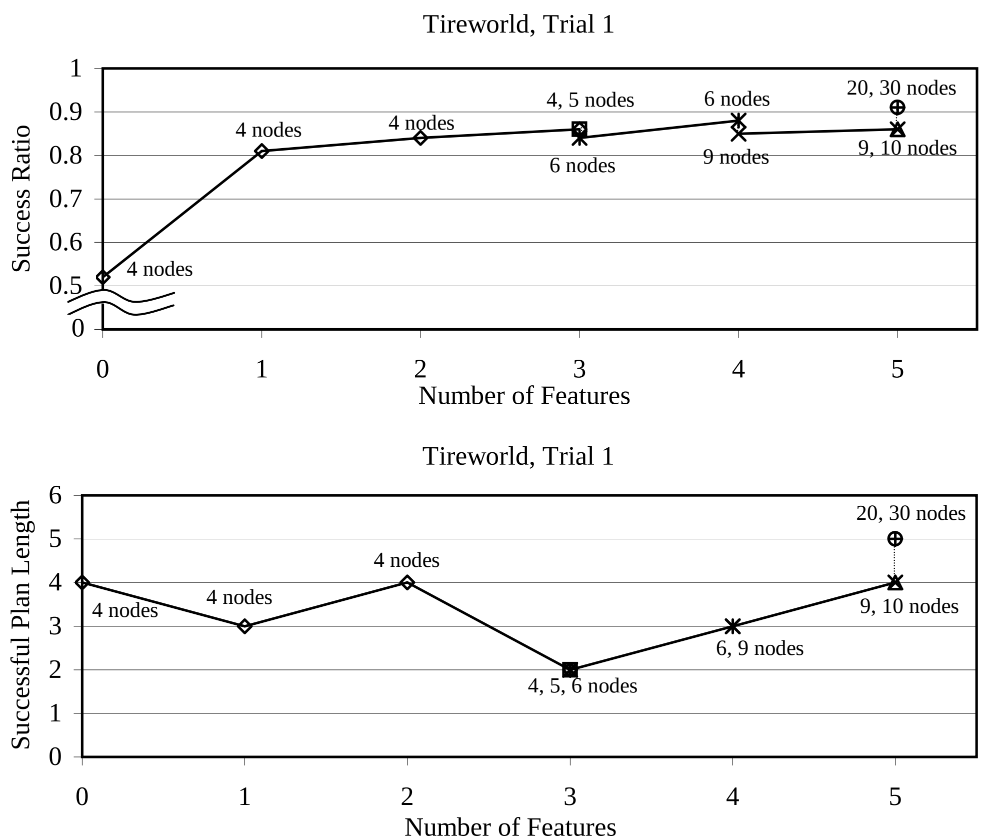


Figure 21: **Tireworld** success ratio and average successful plan length (averaged over 600 problems) for the first trial using our relational learner.

Trial #1											
# of features	0	1	1	2	3	4	5	6	7	8	9
Problem difficulty	3,1,1	3,1,1	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2
Success ratio	0.79	0.8	0.59	0.52	0.54	0.55	0.54	0.52	0.56	0.53	0.55
Plan length	253	255	413	440	437	450	411	440	426	428	451
Accumulated time (Hr.)	0.75	1.7	3.4	7.1	11	15	19	25	30	36	41
Target size SR	0.06	0.08	0.09	0.09	0.12	0.11	0.10	0.08	0.11	0.08	0.12
Target size Slen.	916	1024	1064	1114	1050	1125	1111	1115	1061	1174	1195

Trial #2											
# of features	0	1	2	2	3	4	5	6	7	8	9
Problem difficulty	3,1,1	3,1,1	3,1,1	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2	3,2,2
Success ratio	0.77	0.79	0.80	0.55	0.55	0.50	0.53	0.12	0.12	0.12	0.10
Plan length	262	254	233	391	425	415	422	0	0	0	0
Accumulated time (Hr.)	1.3	2.3	3.3	5.3	8.9	13	17	22	29	36	43
Target size SR	0.05	0.10	0.10	0.09	0.09	0.08	0.10	0.02	0.02	0.02	0.01
Target size Slen.	814	1008	1007	1067	1088	1014	1078	0	0	0	0

Figure 22: **Zenotravel** performance (averaged over 600 problems) for relational learner. The problem difficulty shown in this table lists the numbers of cities, travelers, and aircraft, with a target problem size of 10 cities, 2 travelers, and 2 aircraft. We add one feature per column until success ratio exceeds 0.8, and then increase problem difficulty for the next column. Plan lengths shown are successful trials only.

locations by flying or zooming. Zooming uses more fuel than flying, but has a higher success probability.

We start with a problem difficulty of 3 cities, 1 traveler, and 1 aircraft using our relational feature learner. Whenever the success ratio exceeds 0.8, we increase the number n of travelers and aircraft by 1 if the number of cities is no less than $5n-2$, and increase the number of cities by one otherwise. The target problem size is 10 cities, 2 travelers, and 2 aircraft. **Zenotravel** results for the relational learner are shown in Figures 22 and 23.

The relational learner is unable to find features that enable AVI to achieve the threshold success rate (0.8) for 3 cities, 2 travelers, and 2 aircraft, although 9 relational features are learned. The trials were stopped because no improvement in performance was achieved for several iterations of feature addition. Using a broader search ($W = 160$, $q = 3$, and $d = 3$) we are able to find better features and extend the solvable size to several cities with success rate 0.9 (not shown here as all results in this paper use the same search parameters, but reported in (Wu & Givan, 2007)), but the runtime also increases dramatically, to weeks. We believe the speed and effectiveness of the relational learning needs to be improved to excel in this domain, and a likely major factor is improved knowledge representation for features so that key concepts for **Zenotravel** are easily represented.

Trial two in Figure 22 shows a striking event where adding a single new feature to a useful value function results in a value function for which the greedy policy cannot find the goal at all, so that the success ratio degrades dramatically immediately. Note that in this small problem size, about ten percent of the problems are trivial, in that the initial state satisfies the goal. After the addition of the sixth feature in trial two, these are the only problems the policy can solve. This reflects the unreliability of the AVI weight-selection technique more than any aspect of our feature discovery: after all, AVI is free to assign a zero weight to this new feature, but does not. Additional study of

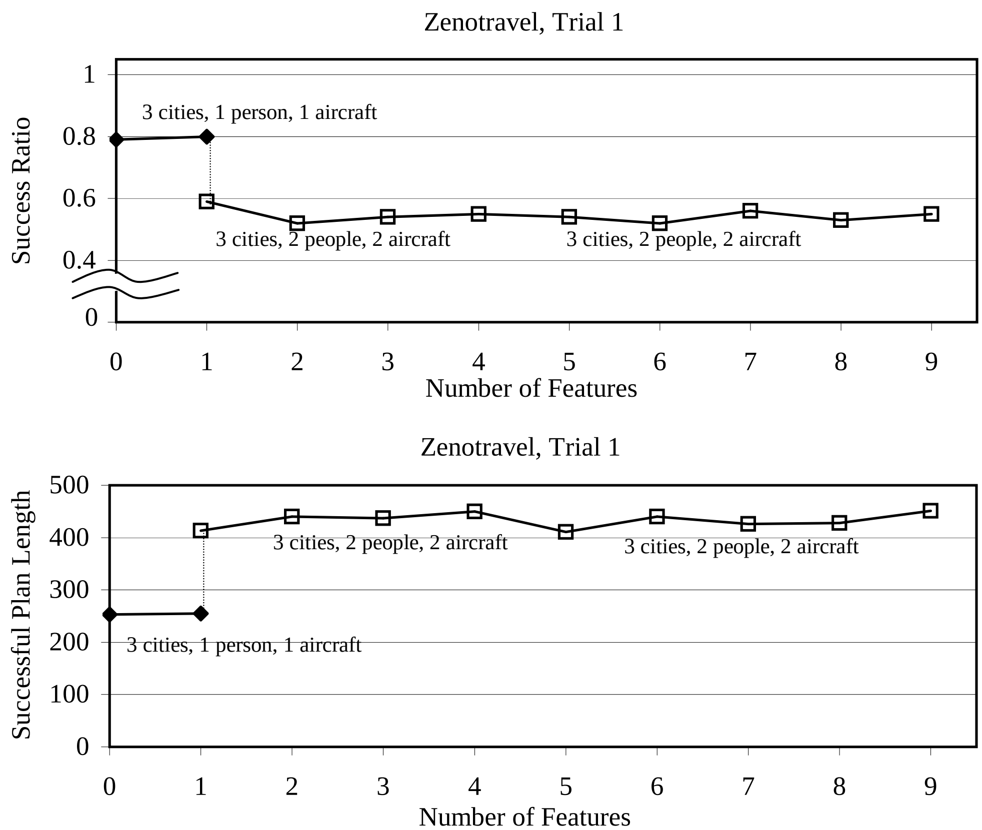


Figure 23: **Zenotravel** success ratio and average successful plan length (averaged over 600 problems) for the first trial using our relational learner.

Trial #1											
# of features	0	1	2	3	4	5	6	7	7	8	9
Problem difficulty	3	3	3	3	3	3	3	3	4	4	4
Success ratio	0.56	0.58	0.56	0.63	0.56	0.68	0.62	0.71	0.4	0.45	0.43
Plan length	1	2	1	2	1	1	2	2	4	5	4
Accumulated time (Hr.)	0.6	1.4	2.2	3.1	4.2	5.9	8.7	11	12	20	28
Target size #1 SR	0.12	0.12	0.14	0.22	0.20	0.31	0.16	0.34	0.33	0.31	0.31
Target size #1 Slen.	3	3	3	5	4	6	9	6	6	5	5
Target size #2 SR	0	0	0	0.00	0.00	0.03	0	0.02	0.03	0.02	0.02
Target size #2 Slen.	–	–	–	10	4	24	–	19	26	23	22

Trial #2											
# of features	0	1	2	3	4	5	5	6	7	8	9
Problem difficulty	3	3	3	3	3	3	4	4	4	4	4
Success ratio	0.56	0.56	0.55	0.63	0.55	0.75	0.45	0.45	0.43	0.42	0.36
Plan length	1	2	1	2	1	2	4	5	5	4	4
Accumulated time (Hr.)	0.6	1.3	2.1	2.9	3.7	4.6	5.3	14	22	31	39
Target size #1 SR	0.14	0.15	0.12	0.18	0.17	0.33	0.31	0.32	0.31	0.28	0.30
Target size #1 Slen.	4	3	4	6	4	6	6	6	6	5	5
Target size #2 SR	0	0	0	0.01	0.00	0.02	0.01	0.01	0.02	0.01	0.01
Target size #2 Slen.	–	–	–	19	18	26	27	15	21	15	18

Figure 24: **Exploding Blocksworld** performance (averaged over 600 problems) for relational learner. Problem difficulties are measured in number of blocks. We add one feature per column until success ratio exceeds 0.7, and then increase problem difficulty for the next column. Plan lengths shown are successful trials only. Target problem size #1 has 5 blocks, and target problem size #2 has 10 blocks.

the control of AVI and/or replacement of AVI by linear programming methods is indicated by this phenomenon; however, this is a rare event in our extensive experiments.

Exploding Blocksworld We also use **Exploding Blocksworld** from the second IPPC to evaluate our relational planner. This domain differs from the normal Blocksworld largely due to the blocks having certain probability of being "detonated" when they are being put down, destroying objects beneath (but not the detonating block). Blocks that are already detonated once will not be detonated again. The goal state in this domain is described in tower fragments, where the fragments are not generally required to be on the table. Destroyed objects cannot be picked up, and blocks cannot be put down on destroyed objects (but a destroyed object can still be part of the goal if the necessary relationships were established before or just as it was destroyed).

We start with 3-block problems using our relational learner and increase from n blocks to $n+1$ blocks whenever the success ratio exceeds 0.7. The target problem sizes are 5 and 10 blocks. **Exploding Blocksworld** results for the relational learner are shown in Figures 24 and 25. The results in Exploding Blocksworld are not good enough for the planner to increase the difficulty beyond 4-block problems, and while the results show limited generalization to 5-block problems, there is very little generalization to 10-block problems.

Our performance in this domain is quite weak. We believe this is due to the presence of many dead-end states that are reachable with high probability. These are the states where either the table or one of the blocks needed in the goal has been destroyed, before the object in question achieved the required properties. Our planner can find meaningful and relevant features: the planner discovers

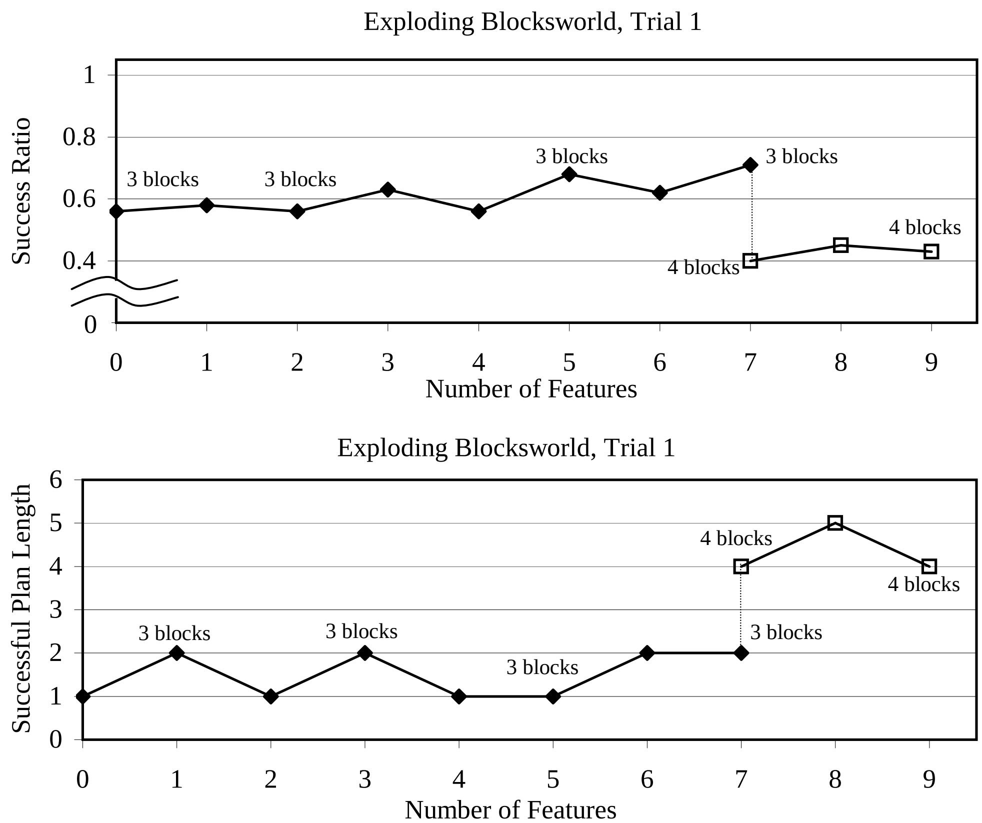


Figure 25: **Exploding Blocksworld** success ratio and average successful plan length (averaged over 600 problems) for the first trial using our relational learner.

that it is undesirable to destroy the table, for instance. However, the resulting partial understanding of the domain cannot be augmented by random walk (as it is in some other domains such as Blocksworld and Conjunctive-Boxworld) to enable steady improvement in value, leading to the goal; random walk in this domain invariably lands the agent in a dead end. Very short successful plan length, low probability of reaching the goal, and (not shown here) very high unsuccessful plan length (caused by wandering in a dead end region) suggest the need for new techniques aimed at handling dead-end regions to handle this domain. These results demonstrate that our technique relies on random walk (or some other form of search) so that the learned features need not completely describe the desired policy.

Towers of Hanoi We use the domain **Towers of Hanoi** from the first IPPC. In this probabilistic version of the well-known problem, the agent can move one or two discs simultaneously, but there is a small probability of going to a dead-end state on each move, and this probability depends on whether the largest disc has been moved and which type of disc move (one or two at a time) is being used. We note that there is only one planning problem in each problem size here.

It is important to note that 100% success rate is generally unachievable in this domain due to the unavoidable dead-end states.

Trial #1														
# of features	0	1	1	2	3	3	4	5	6	7	8	8	20	38
Problem difficulty	2	2	3	3	3	4	4	4	4	4	4	5	5	5
Success ratio	0.70	0.75	0.11	0.44	0.73	0	0	0	0	0	0.51	0	0	0
Plan length	4	2	43	26	4	–	–	–	–	–	4	–	–	–
Accumulated time (Hr.)	0.0	0.0	0.1	0.2	0.3	0.4	0.5	1.1	1.2	2.1	2.2	2.3	18	53
Target size #1 SR	0.07	0.15	0.01	0.08	0.03	0	0	0	0	0	0.52	0.53	0	0.43
Target size #1 Slen.	13	9	90	95	37	–	–	–	–	–	4	4	–	4
Target size #2 SR	0.00	0	0	0	0.00	0	0	0	0	0	0	0	0	0
Target size #2 Slen.	11	–	–	–	107	–	–	–	–	–	–	–	–	–

Trial #2														
# of features	0	0	1	2	3	3	4	5	6	7	8	8	20	38
Problem difficulty	2	3	3	3	3	4	4	4	4	4	4	5	5	5
Success ratio	0.71	0.23	0.14	0.42	0.75	0	0	0	0	0	0.53	0	0	0
Plan length	4	12	37	25	4	–	–	–	–	–	4	–	–	–
Accumulated time (Hr.)	0.0	0.0	0.2	0.3	0.3	0.4	0.5	1.1	1.9	2.3	2.6	2.7	6	16
Target size #1 SR	0.1	0.09	0.0	0.09	0.03	0	0	0	0	0	0.49	0	0	0
Target size #1 Slen.	14	11	105	95	41	–	–	–	–	–	4	–	–	–
Target size #2 SR	0.00	0.1	0	0	0.00	0	0	0	0	0	0	0	0	0
Target size #2 Slen.	16	29	–	–	107	–	–	–	–	–	–	–	–	–

Figure 26: **Towers of Hanoi** performance (averaged over 600 problems) for relational learner. We add one feature per column until success ratio exceeds 0.7^{n-1} for n discs, and then increase problem difficulty for the next column. Plan lengths shown are successful trials only. Problem difficulties are measured in number of discs, with a target problem size #1 of 4 discs and size #2 of 5 discs.

We start with the 2-disc problem in our relational learner and increase the problem difficulty from n discs to $n+1$ discs whenever the success ratio exceeds 0.7^{n-1}. The target problem sizes are 4 and 5 discs. **Towers of Hanoi** results for the relational learner are shown in Figures 26 and 27.

The learner is clearly able to adapt to three- and four-disc problems, achieving around 50% success rate on the four disc problem in both trials. The optimal solution for the four disc problem has success rate 75%. This policy uses single disc moves until the large disc is moved and then uses double disc moves. Policies that use only single disc moves or only double disc moves can achieve success rates of 64% and 58%, respectively, on the four disc problem. The learned solution occasionally moves a disc in a way that doesn't get closer to the goal, reducing its success.

Unfortunately, the trials show that an increasing number of new features are needed to adapt to each larger problem size, and in our trials even 38 total features are not enough to adapt to the five-disc problem. Thus, we do not know if this approach can extend even to five discs. Moreover, the results indicate poor generalization between problem sizes.

We believe it is difficult for our learner (and for humans) to represent a good value function across problem sizes. Humans deal with this domain by formulating a good recursive policy, not by establishing any direct idea of the value of a state. Finding such a recursive policy automatically is an interesting open research question outside the scope of this paper.

Lifted-Fileworld3 As described in Section 6.1, we use the domain **Lifted-Fileworld3**, which is a straightforwardly lifted form of **Fileworld** from the first IPPC, restricted to three folders. To reach the goal of filing all files, an action needs to be taken for each file to randomly determine which

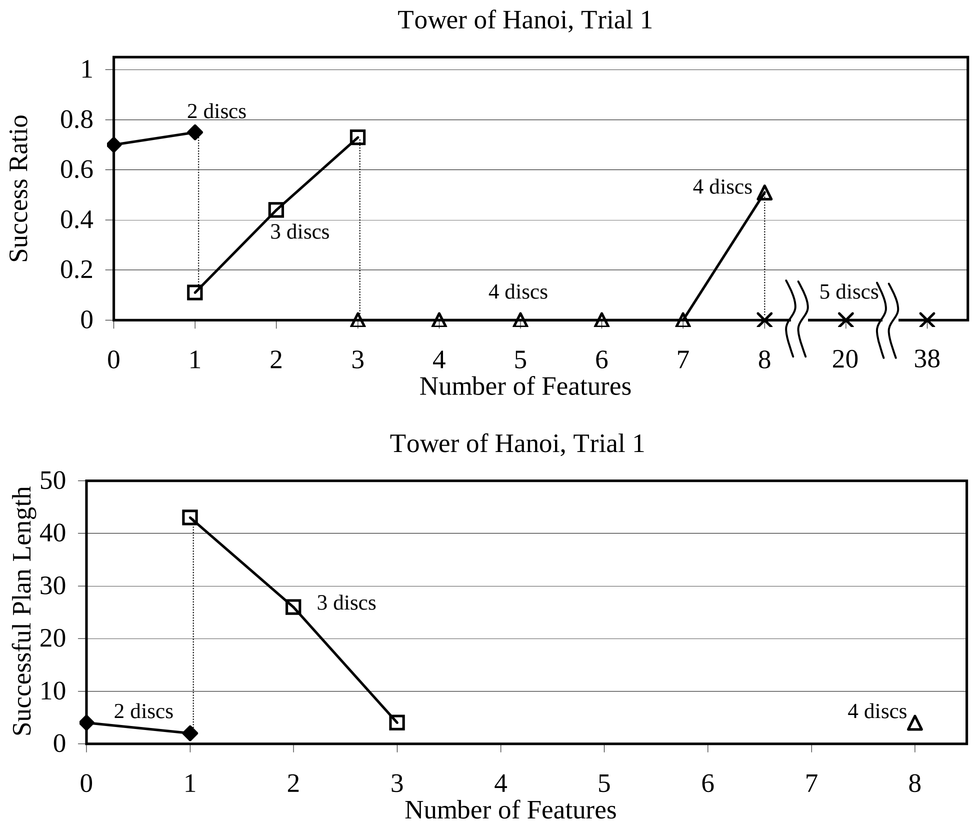


Figure 27: **Towers of Hanoi** success ratio and average successful plan length (averaged over 600 problems) for the first trial using our relational learner.

folder that file should go into. There are actions for taking out a folder, putting a file in that folder, and returning the folder to the cabinet. The goal is reached when all files are correctly filed in the targeted folders.

We note that both **Fileworld** and **Lifted-Fileworld3** are very benign domains. There are no reachable dead ends and very few non-optimal actions, each of which is directly reversible. Random walk solves this domain with success rate one even for thirty files. The technical challenge posed then is to minimize unnecessary steps so as to minimize plan length. The optimal policy solves the n-file problem with between $2n+1$ and $2n+5$ steps, depending on the random file types generated.

Rather than preset a plan-length threshold for increasing difficulty (as a function of n), here we adopt a policy of increasing difficulty whenever the method fails to improve plan length by adding features. Specifically, if the success ratio exceeds 0.9 and one feature is added without improving plan length, we remove that feature and increase problem difficulty instead.[9]

9. It is possible to specify a plan-length threshold function for triggering increase in difficulty in this domain, as we have done in other domains. We find that this domain is quite sensitive to the choice of that function, and in the end it must be chosen to trigger difficulty increase only when further feature addition is fruitless at the current difficulty. So, we have directly implemented that automatic method for triggering difficulty increase.

Trial #1																						
# of features	0	1	2	3	3	4	4	4	4	4	4	4	4	4	5	5	5	6	7	7	7	7
Problem difficulty	1	1	1	1	2	2	3	4	8	10	11	12	13	14	14	15	16	16	16	18	19	20
Success ratio	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Plan length	14	8	4	3	7	6	9	11	21	25	30	29	31	49	37	35	55	37	37	41	43	45
Accumulated time (Hr.)	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	2.4	3.8	4.8	5.9	7.3	8.9	10	13	15	17	19	37	49	62
Target size SR	1	1	1	0	0	0	0	1.00	1.00	1	1	1	1	1	1	1	1	1	1	1	1	1
Target size Slen.	251	134	87	–	–	–	–	87	82	91	88	93	65	90	91	65	91	65	65	65	111	65

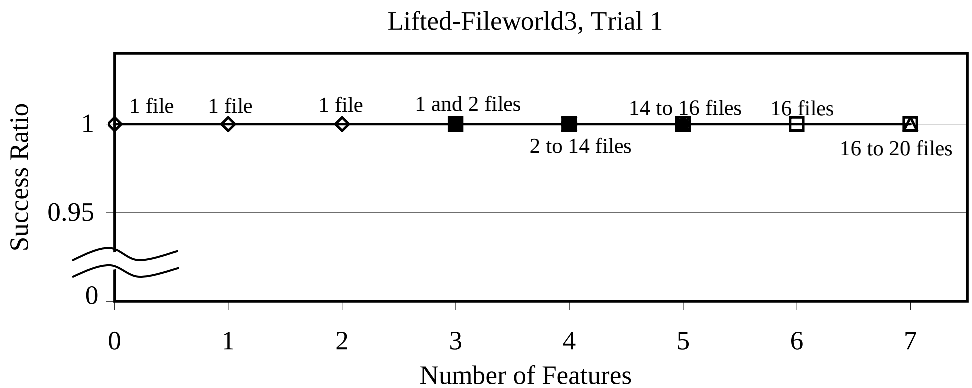
Trial #2																						
# of features	0	1	2	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
Problem difficulty	1	1	1	1	2	2	3	4	5	8	9	10	14	15	16	17	18	19	20	23	24	25
Success ratio	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Plan length	14	8	4	3	7	6	9	12	14	21	23	25	33	35	62	65	41	43	49	91	53	55
Accumulated time (Hr.)	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.6	2.5	3.1	3.9	9.0	11	13	19	27	30	34	50	66	74
Target size SR	1	1	1	0	0	0	0	0.96	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Target size Slen.	251	135	88	–	–	–	–	85	88	82	82	91	96	87	91	93	97	65	65	107	82	65

Figure 28: **Lifted-Fileworld3** performance (averaged over 600 problems) for relational learner. We add one feature per column until success ratio exceeds 0.9 and adding one extra feature does not improve plan length, and then increase problem difficulty for the next column (after removing the extra feature). Plan lengths shown are successful trials only. Problem difficulties are measured in number of files, with a target problem size of 30 files. Some columns are omitted as discussed on page 27.

We start with 1 file problems in our relational learner and increase from n files to $n+1$ files whenever the performance does not improve upon feature addition. The target problem size is 30 files. **Lifted-Fileworld3** results for the relational learner are shown in Figures 28 and 29.

The results show that our planner acquires an optimal policy for the 30-file target size problem after learning four features, in each of the two trials. The results in this domain again reveal the weakness of the AVI weight-selection method. Although four features are enough to define an optimal policy, as problem difficulty increases, AVI often fails to find the weight assignment producing such a policy. When this happens, further feature addition can be triggered, as in trial 1. In this domain, the results show that such extra features do not prevent AVI from finding good weights on subsequent iterations, as the optimal policy is recovered again with the larger feature set. Nonetheless, here is another indication that improved performance may be available via work on alternative weight-selection approaches, orthogonal to the topic of feature selection.

Random Features In order to show that our performance is not simply due to the number of features, but to the feature-selection criterion, we generate two greedy policies in each domain using random feature selection within our relational representation, alternating AVI training, difficulty increase, and feature generation as in the experiments reported above. For each domain, we select the best performing policy generated in this manner, running the algorithm until there are nine random features selected or until the target problem difficulty is reached. We evaluate each greedy policy acquired in this manner, measuring the average target-problem-size performance in each domain using the target problem sizes shown for each domain above. The results are shown in Figure 30. In no domain does random-feature generation perform comparably to our relational feature learner, with the exception of three domain/size combinations where both learners perform very poorly (Zenotravel, 10-block Exploding Blocksworld, and 5-disc Towers of Hanoi).



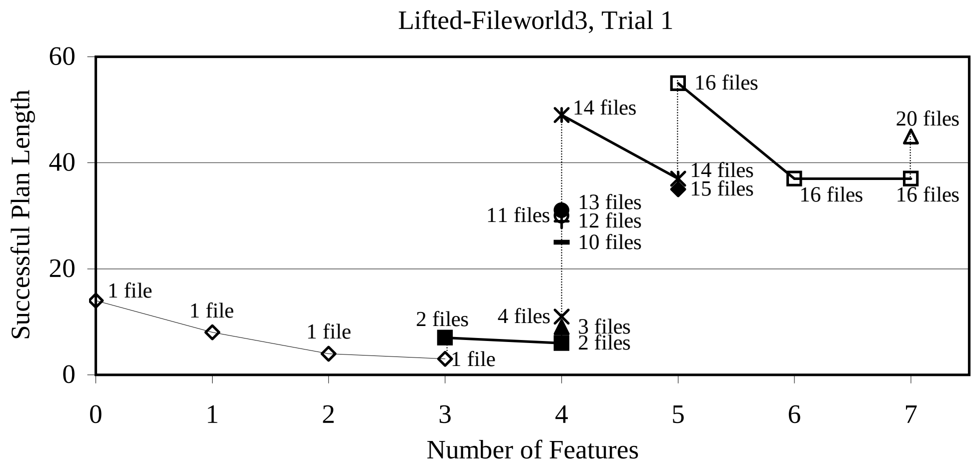


Figure 29: **Lifted-Fileworld3** success ratio and average successful plan length (averaged over 600 problems) for the first trial using our relational learner.

Domain Size	BW 20	Box (15,5)	Box (10,10)	Tire 30	Zeno (10,2,2)	EX-BW 5	EX-BW 10	TOH 4	TOH 5	File 30
Random features SR	0	0.99	0.21	0.67	0.05	0.26	0.01	0.24	0.03	1
Random features SLen.	–	946	1582	6	910	7	12	13	26	215
Learned features SR	0.98	1	0.98	0.92	0.11	0.34	0.03	0.51	0.00	1
Learned features SLen.	748	90	235	5	1137	6	23	4	14	65
Random walk SR	0	0.97	0.18	0.18	0.06	0.13	0	0.09	0.00	1
Random walk SLen.	–	1038	1579	6	865	4	–	14	14	251

Figure 30: Target-problem-size performance (averaged over 600 problems) of random relational features, learned relational features, and random walk, averaged over the best results of two independent trials for each target problem size.

Comparison to FF-Replan and FOALP We compare the performance of our learned policies to FF-Replan and FOALP on each of the PPDDL evaluation domains used above. We use the problem generators provided by the planning competitions to generate 30 problems for each tested problem size except for **Towers of Hanoi** and **Lifted Fileworld3**, where there is one fixed problem for

	15 blocks BW	20 blocks BW	25 blocks BW	30 blocks BW	
RFAVI #1	1 (483)	1 (584)	0.85 (1098)	0.75 (1243)	
RFAVI #2	1.00 (463)	1.00 (578)	0.85 (1099)	0.77 (1227)	
FF-Replan	0.93 (52)	0.91 (71)	0.7 (96)	0.23 (118)	
FOALP	1 (56)	0.73 (73)	0.2 (96)	0.07 (119)	

	(10BX,5CI)Box	(10BX,10CI)Box	(10BX,15CI)Box	(15BX,5CI)Box	(20BX,20CI)Box
RFAVI #1	1 (76)	0.97 (225)	0.93 (459)	1 (90)	0.82 (959)
RFAVI #2	1 (75)	0.97 (223)	0.93 (454)	1 (90)	0.82 (989)
FF-Replan	1 (70)	0.98 (256)	0.93 (507)	1 (88)	0.35 (1069)
FOALP	1 (35)	0.70 (257)	0.28 (395)	0.99 (56)	0.0 (711)

	20 nodes Tire	30 nodes Tire	40 nodes Tire	(10CI,2PR,2AT)Zeno
RFAVI #1	0.87 (5)	0.85 (7)	0.98 (6)	0.06 (1240)
RFAVI #2	0.85 (4)	0.84 (7)	0.97 (6)	0.07 (1252)
FF-Replan	0.76 (2)	0.73 (3)	0.83 (3)	1 (99)
FOALP	0.92 (4)	0.90 (5)	0.91 (5)	N/A

	5 blocks EX-BW	10 blocks EX-BW	4 discs TOH	5 discs TOH	30 files Lifted-File
RFAVI #1	0.25 (8)	0.02 (30)	0.43 (4)	0 (–)	1 (65)
RFAVI #2	0.25 (8)	0.01 (35)	0.47 (4)	0 (–)	1 (65)
FF-Replan	0.91 (7)	0.45 (20)	0.57 (3)	0.37 (7)	1 (66)
FOALP	N/A	N/A	N/A	N/A	N/A

Figure 31: Comparison of our planner (RFAVI) against FF-Replan and FOALP. Success ratio for a total of 900 attempts (30 attempts for **Towers of Hanoi** and **Lifted Fileworld3**) for each problem size is reported, followed by the average successful plan length in parentheses. The two rows for RFAVI map to two learning trials shown in the paper.

	30 BW	(20,20) BX	40 Tire	(10,2,2) Zeno	10 EX-BW	5 TOH	30 Files
RFAVI #1	106s	83s	1s	51s	2s	–	1s
RFAVI #2	105s	86s	0s	51s	3s	–	1s
FF-Replan	872s	739s	0s	1s	8s	3s	10s
FOALP	16s	173s	24s	N/A	N/A	N/A	N/A

Figure 32: Average runtime of the successful attempts, from the results shown in Figure 31, on the largest problem size for each domain.

each problem size. We evaluate the performance of each planner 30 times for each problem, and report in Fig. 31 the success ratio of each planner in each problem size (averaged over all attempts). Our policies, learned from the two independent trials shown above, are indicated as RFAVI #1 and RFAVI #2. Each planner has a 30-minute time limit for each attempt. The average time required to finish a successful attempt for the largest problem size in each domain is reported in Figure 32.

For each of the two trials of our learner in each domain, we evaluate here the policy that performed the best in the trial on the (first) target problem size. (Here, a "policy" is a set of features and a corresponding weight vector learned by AVI during the trial.) Performance is measured by success rate, with ties broken by plan length. Any remaining ties are broken by taking the later policy in the trial from those that are tied. In each case, we consider that policy to be the "policy learned from the trial."

The results show that our planner's performance is incomparable with that of FF-Replan (winning in some domains, losing in others) and generally dominates that of FOALP.

RFAVI performs the best of the planners in larger **Blocksworld**, **Conjunctive-Boxworld**, and **Tireworld** problems. RFAVI is essentially tied with FF-Replan in performance in **Lifted-Fileworld3**. RFAVI loses to FF-Replan in the remaining three domains, **Exploding Blocksworld**, **Zenotravel**, and **Towers of Hanoi**. Reasons for the difficulties in the last three domains are discussed above in the sections presenting results for those domains. We note that FOALP does not have a learned policy in Zenotravel, Exploding Blocksworld, Towers of Hanoi, and Lifted-Fileworld3.

RFAVI relies on random walk to explore plateaus of states not differentiated by the selected features. This reliance frequently results in long plan lengths and at times results in failure. We have recently reported elsewhere on early results from ongoing work remedying this problem by using search in place of random walk (Wu et al., 2008).

The RFAVI learning approach is very different from the non-learning online replanning used by FF-Replan, where the problem is determinized, dropping all probability parameters. It is an important topic for future research to try to combine the benefits obtained by these very different planners across all domains.

The dominance of RFAVI over FOALP in these results implies that RFAVI is at the state of the art among first-order techniques — those that work with the problem in lifted form and use lifted generalization. Although FOALP uses first-order structure in feature representation, the learned features are aimed at satisfying goal predicates individually, not as a whole. We believe that the goal-decomposition technique can sometimes work well in small problems but does not scale well to large problems.

In these comparisons, it should also be noted that FOALP does not read PPDDL domain descriptions directly, but requires human-written domain axioms for its learning, unlike our completely automatic technique (requiring only a few numeric parameters characterizing the domain). This requirement for human-written domain axioms is one of the reasons why FOALP did not compete in some of the competition domains and does not have a learned policy for some of the domains tested here.

In **Conjunctive-Boxworld**[10], we note that FF-Replan uses an "all outcomes" problem determinization that does not discriminate between likely and unlikely outcomes of truck-movement actions. As a result, plans are frequently selected that rely on unlikely outcomes (perhaps choosing to move a truck to an undesired location, relying on the unlikely outcome of "accidentally" moving to the desired location). These plans will usually fail, resulting in repeated replanning until FF luckily selects the high-likelihood outcome or plan execution happens to get the desired low-likelihood outcome. This behavior is in effect similar to the behavior our learned value function exhibits because, as discussed on page 35, our learner failed to find any feature rewarding appropriate truck moves. Both planners result in long plan lengths due to many unhelpful truck moves. However, our learned policy conducts the random walk of trucks much more efficiently (and thus more successfully) than the online replanning of FF-Replan, especially in the larger problem sizes. We believe even more dramatic improvements will be available with improved knowledge representation for features.

10. We hand-convert the nested universal quantifiers and conditional effects in the original boxworld domain definition to an equivalent form without universal quantifiers and conditional effects to allow FF-Replan to read the domain.

7.4 SysAdmin

A full description of the **SysAdmin** domain is provided in (Guestrin, Koller, & Parr, 2001). Here, we summarize that description. In the SysAdmin domain, machines are connected in different topologies. Each machine might fail at each step, and the failure probability depends on the number of failed machines connected to it. The agent works toward minimizing the number of failed machines by rebooting machines, with one machine rebooted at each time step. For a problem with n machines and a fixed topology, the dynamic state space can be sufficiently described by n propositional variables, each representing the on/off status of a certain machine.

We test this domain for the purpose of direct comparison of the performance of our propositional techniques to the published results in (Patrascu et al., 2002). We test exactly the topologies evaluated there and measure the performance measure reported there, sup-norm Bellman error.

We evaluate our method on the exact same problems (same MDPs) used for evaluation in (Patrascu et al., 2002) for testing this domain. Two different kinds of topologies are tested: 3-legs and cycle. The "3-legs" topology has three three-node legs (each a linear sequence of three connected nodes) each connected to a single central node at one end. The "cycle" topology arranges the ten nodes in one large cycle. There are 10 nodes in each topology. The target of learning in this domain is to keep as many machines operational as possible, so the number of operating machines directly determines the reward for each step. Since there are only 10 nodes and the basic features are just the on/off statuses of the nodes, there are a total of 1024 states. The reward-scaling parameter r_{scale} is selected to be 10.

(Patrascu et al., 2002) uses $L_{\inf}$ (sup norm) Bellman error as the performance measurement in **SysAdmin**. Our technique, as described above, seeks to reduce mean Bellman error more directly than $L_{\inf}$ Bellman error. In particular, because we allow duplicated states in our AVI training sets, our weight selection prefers weights that have low mean Bellman error, even if the largest Bellman error encountered is larger for such weights. For this reason, we here evaluate two versions of our technique: that described and evaluated above, and a variation which is identical except that duplicate states are removed from AVI training sets. We report the $L_{\inf}$ Bellman error, averaged over two trials, on both versions in Figure 33.

Also included in Figure 33 are the results shown in (Patrascu et al., 2002). We select the best result shown there (from various algorithmic approaches) from the 3-legs and cycle topologies shown in their paper. These correspond to the "d-o-s" setting for the cycle topology and the "d-x-n setting" for the 3-legs topology, in the terminology of that paper.

Both topologies show that both variants of our algorithm reduces the $L_{\inf}$ Bellman error more effectively per feature as well as more effectively overall than the experiments previously reported in (Patrascu et al., 2002). Both topologies also show that leaving duplicate states in the AVI training sets eventually encourages value functions that with high $L_{\inf}$ Bellman error after an initial substantial success in reducing that Bellman error. The method allowing duplicate states can still achieve low Bellman error by remembering and restoring the best-performing weighted feature set once weakened performance is detected.

8. Discussion and future research

We have presented a general framework for automatically learning state-value functions by feature-discovery and gradient-based weight training. In this framework, we greedily select features from

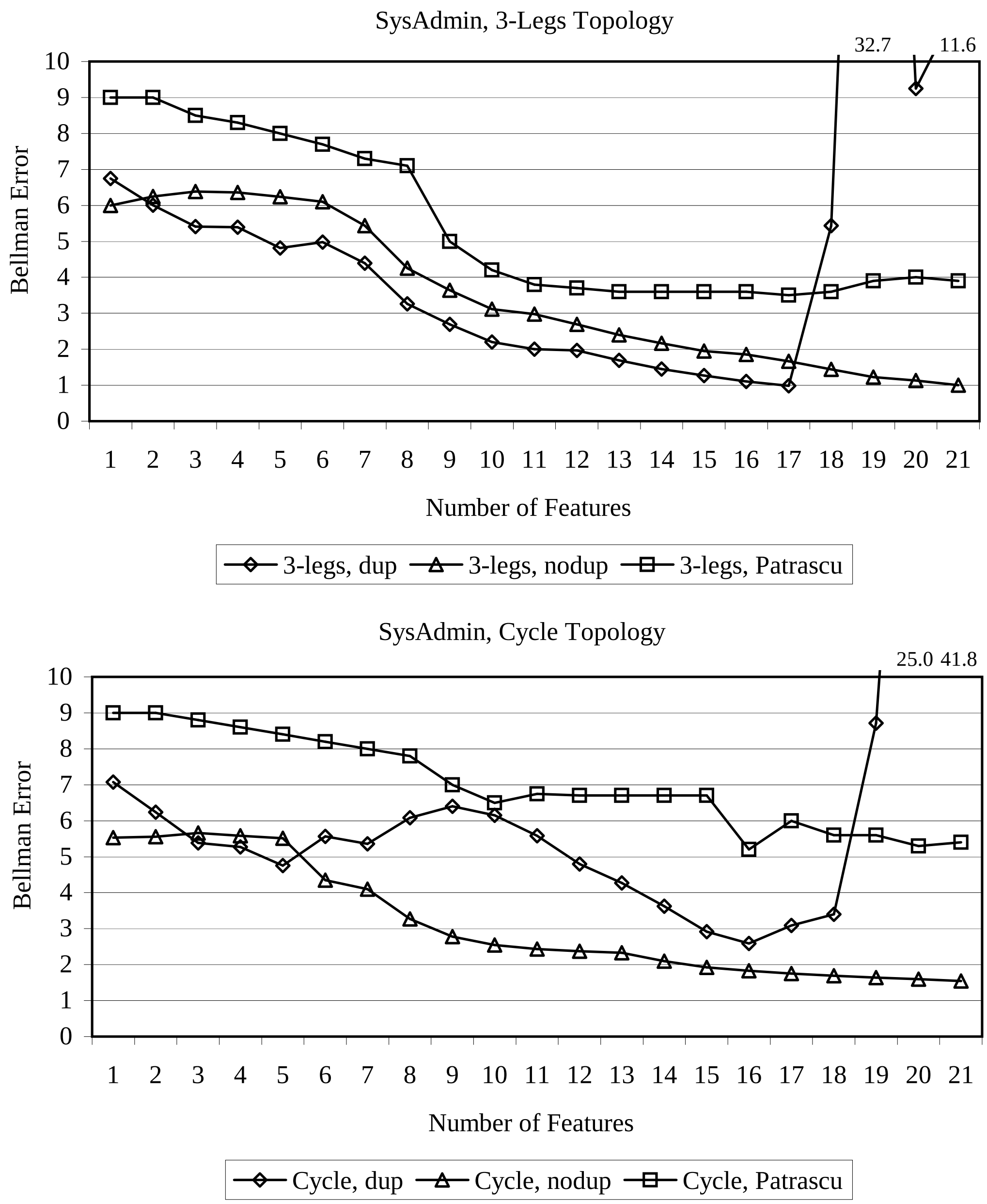


Figure 33: L_{inf} Bellman error for the **SysAdmin** domain (10 nodes) for two topologies. Values for the results from (Patrascu et al., 2002) are taken from Figure 2 and 3 of (Patrascu et al., 2002). The label **nodup** represents trials of a variant of our method for which the AVI weight training sets have duplicate states removed.

a provided hypothesis space (which is a parameter of the method) to best correlate with Bellman error features, and use AVI to find weights to associate with these features.

We have proposed two different candidate hypothesis spaces for features. One of these two spaces is a relational one where features are first-order formulas with one free-variable, and a beam-search process is used to greedily select a hypothesis. The other hypothesis space we have considered is a propositional feature representation where features are decision trees. For this hypothesis space, we use a standard classification algorithm C4.5 (Quinlan, 1993) to build a feature that best correlates with the sign of the statewise Bellman error, instead of using both the sign and magnitude.

The performance of our feature-learning planners is evaluated using both reward-oriented and goal-oriented planning domains. We have demonstrated that our relational planner represents the state-of-the-art for feature-discovering probabilistic planning techniques. Our propositional planner does not perform as well as our relational planner, and cannot generalize between problem instances, suggesting that knowledge representation is indeed critical to the success of feature-discovering planners.

Bellman-error reduction is of course just one source of guidance that might be followed in feature discovery. During our experiments in the IPPC planning domains, we find that in many domains the successful plan length achieved is much longer than optimal, as we discussed above on page 52. A possible remedy other then our work (deploying search) in (Wu et al., 2008) is to learn features targeting the dynamics inside the plateaus, and use these features in decision-making when plateaus are encountered.

References

Bacchus, F., & Kabanza, F. (2000). Using temporal logics to express search control knowledge for planning. *Artificial Intelligence*, *116*, 123–191.

Bertsekas, D. P. (1995). *Dynamic programming and optimal control*. Athena Scientific.

Bertsekas, D. P., & Tsitsiklis, J. N. (1996). *Neuro-Dynamic Programming*. Athena Scientific.

Bonet, B., & Givan, R. (2006). Non-deterministic planning track of the 2006 international planning competition. Website. http://www.ldc.usb.ve/ bonet/ipc5/.

Darken, C., & Moody, J. (1992). Towards faster stochastic gradient search. In *Proceedings of the 4th Conference on Advances in Neural Information Processing*.

Davis, R., & Lenat, D. (1982). *Knowledge-Based Systems in Artificial Intelligence*. McGraw-Hill, New York.

Driessens, K., & Džeroski, S. (2004). Integrating guidance into relational reinforcement learning. *Machine Learning*, *57*, 271–304.

Driessens, K., Ramon, J., & Gärtner, T. (2006). Graph kernels and gaussian processes for relational reinforcement learning. *Machine Learning*, *64*, 91–119.

Džeroski, S., DeRaedt, L., & Driessens, K. (2001). Relational reinforcement learning. *Machine Learning*, *43*, 7–52.

Estlin, T. A., & Mooney, R. J. (1997). Learning to improve both efficiency and quality of planning. In *Proceedings of the 15th International Joint Conference on Artificial Intelligence*, pp. 1227–1232.

Fawcett, T. (1996). Knowledge-based feature discovery for evaluation functions. *Computational Intelligence*, *12*(1), 42–64.

Fern, A., Yoon, S., & Givan, R. (2006). Approximate policy iteration with a policy language bias: Solving relational markov decision processes. *Journal of Artificial Intelligence Research*, *25*, 75–118.

Fox, M., & Long, D. (1998). The automatic inference of state invariants in TIM. *Journal of Artificial Intelligence Research*, *9*, 367–421.

Gerevini, A., & Schubert, L. (1998). Inferring state constraints for domain-independent planning. In *Proceedings of the 15th National Conference on Artificial Intelligence*, pp. 905–912.

Gretton, C., & Thiébaux, S. (2004). Exploiting first-order regression in inductive policy selection. In *20th Conference on Uncertainty in Artificial Intelligence (UAI-04)*.

Guestrin, C., Koller, D., & Parr, R. (2001). Max-norm projections for factored MDPs. In *Proceedings of the 17th International Joint Conference on Artificial Intelligence*, pp. 673–680.

Harris, R., Chabries, D., & Bishop, F. (1986). A variable step (vs) adaptive filter algorithm. *IEEE transactions on acoustics, speech, and signal processing*, *34*(2), 309– 316.

Jacobs, R. (1988). Increased rates of convergence through learning rate adaptation. *Neural Networks*, *1*, 295–307.

Kambhampati, S., Katukam, S., & Qu, Y. (1996). Failure driven dynamic search control for partial order planners: an explanation based approach. *Artificial Intelligence*, *88*(1-2), 253–315.

Keller, P., Mannor, S., & Precup, D. (2006). Automatic basis function construction for approximate dynamic programming and reinforcement learning. In *Proceedings of the 23th International Conference on Machine Learning*.

Khardon, R. (1999). Learning action strategies for planning domains. *Artificial Intelligence*, *113*(1-2), 125–148.

Kwong, R., & Johnston, E. (1992). A variable step size lms algorithm. *IEEE Transactions on Signal Processing*, *40*(7), 1633–1642.

Martin, M., & Geffner, H. (2004). Learning generalized policies from planning examples using concept languages. *Applied Intelligence*, *20*, 9–19.

Mathews, V., & Xie, Z. (1993). A stochastic gradient adaptive filter with gradient adaptive step size. *IEEE Transactions on Signal Processing*, *41*(6), 2075–2087.

Mitchell, T. M. (1997). *Machine Learning*. McGraw-Hill.

Parr, R., Painter-Wakefield, C., Li, L., & Littman, M. (2007). Analyzing feature generation for value-function approximation. In *Proceedings of the 24th International Conference on Machine Learning*.

Patrascu, R., Poupart, P., Schuurmans, D., Boutilier, C., & Guestrin, C. (2002). Greedy linear value-approximation for factored markov decision processes. In *In Proceedings of the 18th National Conference on Artificial Intelligence*, pp. 285–291.

Quinlan, J. R. (1993). *C4.5: Programs for Machine Learning*. Morgan Kaufmann.

Sanner, S., & Boutilier, C. (2006). Practical linear value-approximation techniques for first-order mdps. In *Proceedings of the 22nd Conference on Uncertainty in Artificial Intelligence*.

Sutton, R. S. (1988). Learning to predict by the methods of temporal differences. *Machine Learning*, *3*, 9–44.

Sutton, R. S., & Barto, A. G. (1998). *Reinforcement Learning: An Introduction*. MIT Press.

Tesauro, G. (1995). Temporal difference learning and td-gammon. *Communications of the ACM*, *38*(3), 58–68.

Tsitsiklis, J., & Roy, B. V. (1997). An analysis of temporal-difference learning with function approximation. *IEEE Transactions on Automatic Control*, *42*(5), 674–690.

Veloso, M., Carbonell, J., Perez, A., Borrajo, D., Fink, E., & Blythe, J. (1995). Integrating planning and learning: The PRODIGY architecture. *Journal of Experimental and Theoretical AI*, *7*(1).

Widrow, B., & Hoff, Jr, M. E. (1960). Adaptive switching circuits. *IRE WESCON Convention Record*, 96–104.

Williams, R. J., & Baird, L. C. (1993). Tight performance bounds on greedy policies based on imperfect value functions. Tech. rep., Northeastern University.

Wu, J., & Givan, R. (2007). Discovering relational domain features for probabilistic planning. In *Proceedings of the Seventeenth International Conference on Automated Planning and Scheduling*, pp. 344–351.

Wu, J., Kalyanam, R., & Givan, R. (2008). Stochastic enforced hill-climbing. In *Proceedings of the Eighteenth International Conference on Automated Planning and Scheduling*, pp. 396–403.

Wu, J., & Givan, R. (2005). Feature-discovering approximate value iteration methods. In *Proceedings of the Symposium on Abstraction, Reformulation, and Approximation*, pp. 321–331.

Yoon, S., Fern, A., & Givan, R. (2002). Inductive policy selection for first-order MDPs. In *Proceedings of the 18th Conference on Uncertainty in Artificial Intelligence*.

Yoon, S., Fern, A., & Givan, R. (2007). FF-Replan: A baseline for probabilistic planning. In *Proceedings of the Seventeenth International Conference on Automated Planning and Scheduling*, pp. 352–358.

Younes, H., Littman, M., Weissman, D., & Asmuth, J. (2005). The first probabilistic track of the international planning competition. *Journal of Artificial Intelligence Research*, *24*, 851–887.

APPENDIX

A-1. PPDDL Source for Lifted-Fileworld3

The PPDDL source for **Lifted-Fileworld3** with a problem size of 10 files.

```
(define (domain file-world)
        (:requirements  :typing
                        :disjunctive-preconditions
                        :negative-preconditions
                        :conditional-effects
                        :probabilistic-effects
                        :universal-preconditions)
        (:types  file folder)
```

```
        (:predicates  (has-type ?p - file)
                      (goes-in ?p - file ?f - folder)
                      (filed ?p - file)
                      (have ?f - folder))
        (:constants  F0 F1 F2 - folder )

(:action get-type
        :parameters  (?p - file)
        :precondition  (and (not (has-type ?p)))
        :effect  (and (has-type ?p)
                      (probabilistic
                         0.333   (goes-in ?p F0)
                         0.333   (goes-in ?p F1)
                         0.334   (goes-in ?p F2))))

(:action get-folder
        :parameters  (?f - folder)
        :precondition  (and (forall (?x -folder) (not (have ?x))))
        :effect  (have ?f))

(:action file-F
        :parameters  (?p - file ?f - folder)
        :precondition  (and (have ?f) (has-type ?p)
                            (goes-in ?p ?f))
        :effect  (filed ?p))

(:action return-folder
        :parameters  (?f - folder)
        :precondition  (have ?f)
        :effect  (not (have ?f)))
)

(define (problem file-prob)
        (:domain file-world)
        (:objects p0 p1 p2 p3 p4 p5 p6 p7 p8 p9 )
        (:goal (and (filed p0) (filed p1) (filed p2) (filed p3)
                    (filed p4) (filed p5) (filed p6) (filed p7)
                    (filed p8) (filed p9)))
)
```