# Protocol-based Conflict Resolution for Finite Information Horizon [1]

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## Abstract

This paper proposes a protocol-based multiple aircraft conflict resolution for a finite information horizon, in which the communication range of an aircraft is finite. A protocol for multiple aircraft conflict resolution for an infinite information horizon is presented and then this protocol is extended to a finite information horizon problem using graph theory. Communication topology among aircraft is important for the finite information horizon problem and is modeled by a graph called communication graph. We develop a protocol for multiple-aircraft conflict resolution based on the communication graph and show the safety of the protocol. Finally, we validate our protocol through simulations with a dynamic aircraft model.

## 1 Introduction

*Free flight* allows aircraft to follow their preferred trajectories which could economize flight time delay and fuel consumption while maintaining safe airborne separation[10, 13, 19]. Since conflict detection and resolution (CD&R) for safe airborne separation is critical for free flight, several recent studies have proposed methods for solving CD&R problems[12, 14, 17, 18, 19]. A comprehensive survey of CD&R methods is presented in [11]. We focus on conflict resolution in this paper. Previous research on conflict resolution has proposed analytic or numerical methods such as pairwise geometric optimization[3], randomized searches and convex optimization[7, 8], and a token allocation strategy combined with a genetic algorithm[2, 6].

Since an air traffic control system is a distributed system, information sharing between aircraft is important for safe conflict resolution. A proposed method for information sharing between aircraft is Automatic Dependent Surveillance - Broadcast (ADS-B)[15, 16]. A crucial point is that the finite communication range of ADS-B may affect the safety of conflict resolution.

This paper proposes a protocol-based multiple aircraft ($> 2$) conflict resolution algorithm for a finite information horizon (in which the communication range of an aircraft through an *augmented ADS-B*, which can provide information about the aircraft involved in a conflict, is finite). To derive a protocol for multiple-aircraft conflict resolution, we first review our previous work in solving an infinite information horizon problem; where we assume that each aircraft's position, heading, and velocity are available to all aircraft involved in the conflict[9]. We assume that all aircraft are on a horizontal plane and that conflict resolutions are achieved on the plane by heading change, velocity change, or a combination thereof. The protocol is easily understandable and implementable by all aircraft involved in the conflict, and provides guarantees of safety.

With this protocol, we then develop a multiple-aircraft collision avoidance algorithm for a finite information horizon. The communication topology among aircraft is important for a finite information horizon problem and is modeled by a graph called a *communication graph.* The multiple-aircraft conflict resolution for a finite information horizon is categorized into two cases: a *connected system*, in which the communication graph of the system is connected, i.e. there is a path between any pair of vertices in a graph, and a *disconnected system*, in which the communication graph is not connected.

In this paper, we focus on developing a conflict resolution protocol for a connected system: we show that the safety of the protocol for a finite information horizon is guaranteed. We also present simulation results with a dynamic aircraft model for various multiple-aircraft conflict scenarios.

This paper is organized as follows: Section 2 reviews the protocol-based conflict resolution for an infinite information horizon. In Section 3, a protocol for a finite information horizon is developed and then, we prove the safety of the protocol and also validate the protocol by simulation with a dynamic aircraft model. Conclusions are presented in Section 4.

## 2 Protocol-based conflict resolution for infinite information horizon [9]

The problem of conflict detection and resolution in a horizontal plane is considered in this paper, using only information about each aircraft's current position, ve-

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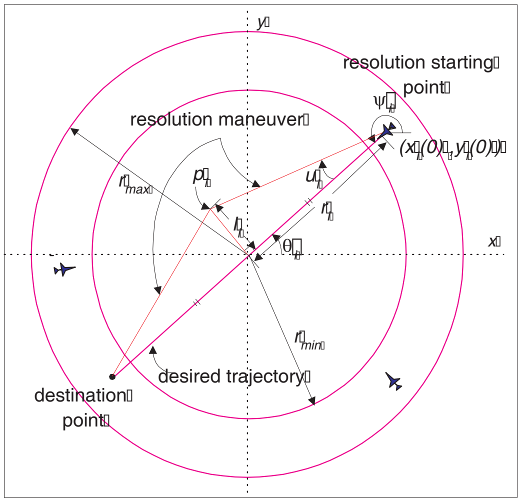

**Figure 1:** An illustration of the exact conflict resolution maneuver for one of $N$-aircraft and parameters involved.

locity, and heading. A conflict is defined as the event in which, at any time during a given time horizon, the distance between any pair of aircraft is less than $R$, which we assume to be $5nm$. We first solve a collision avoidance problem for the *exact conflict* in which all aircraft involved in a conflict come into conflict at a single point in space and time and then use these results to generalize to a conflict resolution protocol for the *inexact conflict*, in which at least one of the aircraft involved in a conflict comes into conflict at a different point in space and time than the others, as shown in Figure 2.

Each aircraft is assumed to have a desired trajectory which is a straight path of constant heading (though the treatment of segments of straight paths connected by waypoints is a simple extension, as we show in Figure 4). Simple projection of this nominal trajectory ahead by a fixed look-ahead time ($T$) is used for conflict detection; we assume $T$ is 20 minutes in this paper, though our method is general enough to work for any finite time horizon. During a resolution maneuver, the aircraft involved in the conflict are assumed to be flying at constant velocity. The velocities are bounded between known values $v_{min}$ and $v_{max}$.

We partition the airspace around the conflict point into two concentric circular discs of radii $r_{min} = v_{min}T$ and $r_{max} = v_{max}T$ as shown in Figure 1, which ensures that the aircraft lie in the annulus between the two radii at the initiation of the conflict resolution maneuver.

We assume aircraft can change heading instantaneously and thus, the control input is a piecewise constant heading change which we denote as $u_i$. Also, we assume synchronous maneuvers, in which all aircraft change their heading at the same time (though our method is robust to varying switches[9]). We use a kinematic model for each aircraft to design the protocol:

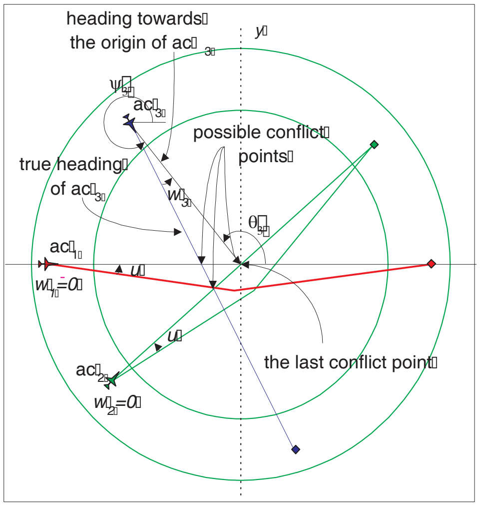

**Figure 2:** Inexact conflict general case; conflict points separated by distance greater than $R$.

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} v_i \cos\psi_i \\ v_i \sin\psi_i \end{bmatrix}, \quad \text{where } i = 1, 2, \cdots, N \tag{1}$$

All aircraft are assumed to share information about the other aircraft's position, velocity, and heading through ADS-B. In Section 3, we will analyze the effect on the protocol of differing information horizons.

For safe conflict resolution, the minimum distance between aircraft during the resolution maneuver should be greater than or equal to the predefined safety distance, $R$. If the heading changes of all aircraft are assumed to be the same and we define $u := u_i$ (for $i = 1, \cdots, n$), the condition for safe resolution is [9]:

$$\sin^2 u \geq \frac{R^2}{r_i^2 + r_j^2 - 2r_i r_j \cos(\theta_i - \theta_j)} \tag{2}$$

where $i, j = 1, 2, \cdots, N$, $i \neq j$.

Eq.(2) represents a *closed-form analytic solution* for the heading change required of all aircraft to resolve the conflict. The denominator in Eq.(2) is the distance between aircraft $i$ and aircraft $j$ at a resolution starting time. For a two-aircraft conflict case, Eq.(2) is the same as the result in [3]. Thus, it is not surprising that if Eq.(2) holds for all pairs in a multiple aircraft conflict, the multiple aircraft conflict resolution is safe. Here we show that Eq.(2) can be used to design a protocol for multiple aircraft conflict resolution, if additional constraints are imposed. The minimum heading change, $u_{min}$, can be obtained from Eq.(2) for a given initial configuration and known parameters. Since both $\pm u$ satisfy Eq.(2), we resolve this turn ambiguity for exact conflicts by restricting the control input to be such that $0^o < u < 90^o$.

We then construct finite partitions of the airspace in the angular direction around the conflict to find a finite solution set to infinitely many conflict geometries, and we derive a protocol for resolving all possible conflicts within each partition using our analytic solution in Eq.(2). If we use six partitions, the protocol for an exact conflict is:

- $90^o \leq \delta\theta_{min} \leq 120^o \quad \Rightarrow \quad u \geq 0.03 = 1.56^o$
- $60^o \leq \delta\theta_{min} < 90^o \quad \Rightarrow \quad u \geq 0.04 = 2.21^o$
- $45^o \leq \delta\theta_{min} < 60^o \quad \Rightarrow \quad u \geq 0.051 = 2.89^o$
- $30^o \leq \delta\theta_{min} < 45^o \quad \Rightarrow \quad u \geq 0.075 = 4.27^o$
- $10^o \leq \delta\theta_{min} < 30^o \quad \Rightarrow \quad u \geq 0.23 = 12.75^o$
- $0^o \leq \delta\theta_{min} < 10^o \quad \Rightarrow \quad u \geq 0.34 = 19.48^o$

where the minimum relative angle between aircraft is defined: $\delta\theta_{min} = \min\{|\theta_i - \theta_j|\,|i,j = 1,2,\cdots,N, \text{and}\, i \neq j\}$.

The exact conflict case is unrealistic and rarely occurs, yet we use the results to generalize to the inexact conflict case, the topology of which is illustrated in Figure 2.

In order to solve this general conflict, we first transform the problem to that of an exact conflict, and then use the solution for an exact conflict to resolve the transformed problem. We then map the resolution maneuver back to the coordinates of the general conflict, and the general resolution results. We introduce new variables $w_i$, defined as the heading difference between the true heading of aircraft $i$ and the origin of our reference frame, as shown in Figure 2 for aircraft 3 ($ac_3$). We define the counter-clockwise direction as positive. Then, the protocol for a general inexact conflict is as follows:

**Algorithm 1 (Multiple-aircraft protocol in the case of inexact conflict (general case))**
*For $i = 1, 2, \cdots, N$:*

1. *Select the last conflict point among possible conflict points as the center of conflict resolution.*
2. *If aircraft $i$ is not involved in the conflict at the origin, adjust its velocity such that $v_i = \frac{r_i}{T}$ ($\leftarrow$ new velocity).*
3. *Aircraft $i$ computes a protocol $u$ with $u_{exact}$ from the protocol for an exact conflict and $w_{max} = \max_i |w_i| sign(w_i)$:*

$$u = \begin{cases} u_{exact}\, sign(w_{max}) & \text{if } u_{exact} > |w_{max}| \\ w_{max} & \text{if } u_{exact} \leq |w_{max}| \end{cases}$$

4. *Aircraft $i$ changes its heading by $u - w_i$.* ■

Then, Algorithm 1 guarantees the safety for the multiple-aircraft inexact conflict (general case)[9].

Though a kinematic aircraft model is used to design the protocol, a dynamic model for a B747 with Mach 8 at $40,000ft$[4], is used for validation. The simulation result with a look-ahead time $T = 6$ minutes in Figure 3 shows the result of our protocol. This solution is robust to uncertainties in the aircraft's position, heading, and velocity, as well as to path smoothing, and asynchronous maneuvers[9].

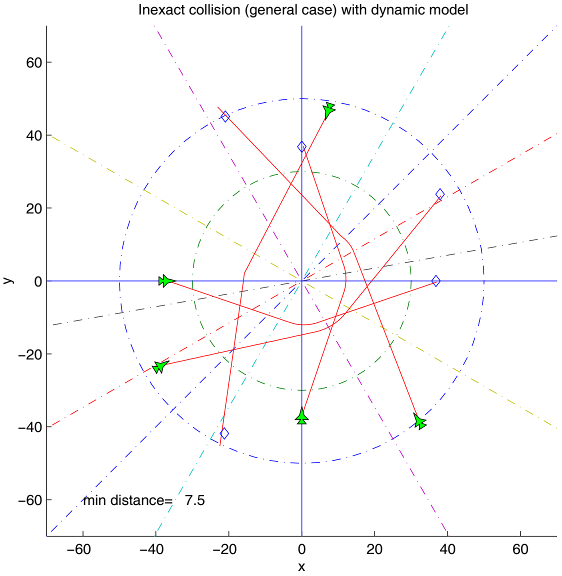

**Figure 3:** Inexact conflict (general case) for five aircraft with $r_{min} = 30nm$ and $r_{max} = 50nm$.

## 3 Algorithm for finite information horizon

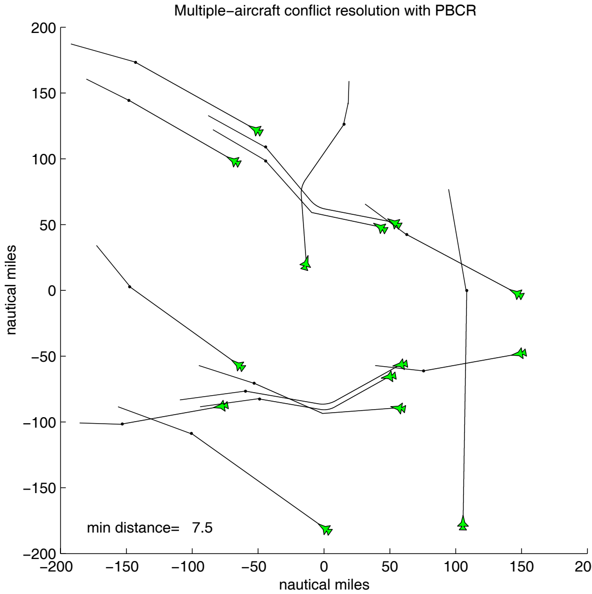

**Figure 4:** Multiple-aircraft conflict resolution with protocol-based conflict resolution algorithm.

Even though Algorithm 1 may applied to airborne separation assurance, the assumption that an aircraft's information is available to all aircraft through ADS-B is unrealistic. For example, conflicts in Figure 4 cannot be resolved safely by Algorithm 1 if an information horizon is less than $2r_{max}$. Thus, in this section, we consider a *finite information horizon* problem such that an aircraft can share information about the aircraft within its communication range, which is assumed to be a circle with radius $D_I$ as shown in Figure 5. If $D_I \geq 2r_{max}$,

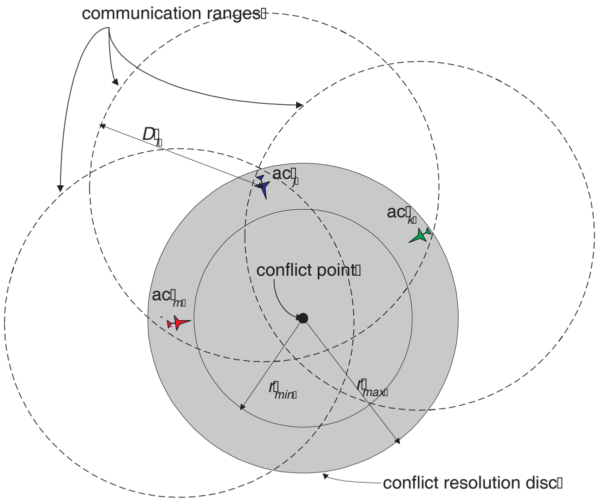

**Figure 5:** An illustration of a three-aircraft conflict with a communication range $D_I$.

Algorithm 1 can be applied safely for conflict resolution because all aircraft within the conflict resolution disc can share information with each other. Therefore, we consider the case in which $D_I < 2r_{max}$ in this section. Communication between aircraft is assumed to be achieved through an augmented ADS-B which can provide information about the aircraft involved in a conflict.

### 3.1 Topology of communication

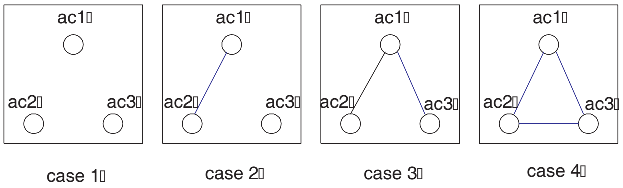

**Figure 6:** Topology of communication.

For a finite information horizon problem, the communication topology is important because sharing information among all aircraft within a conflict resolution disc as shown in Figure 5 is necessary for safe conflict resolution using Algorithm 1. Communication topology can be modeled by a graph.

A *graph* is pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ that consists of a set of vertices $\mathcal{V}$ and a set of edges $\mathcal{E}$ where $e \in \mathcal{E} = (v, w)$ and $v, w \in \mathcal{V}$. A *path* on $\mathcal{G}$ from $v_0$ to $v_N$ ia an ordered set of distinct vertices $\{v_0, \cdots, v_N\}$ such that $(v_{i-1}, v_i) \in \mathcal{E}\ \forall i \in \{1, \cdots, N\}$. $v_j$ is a *neighbor* of $v_i$ if there is an edge connecting them. A graph is *connected* if there is a path connecting every pair of vertices and a *complete* graph is one in which every possible edge exists[5].

If we consider a three-aircraft case, there are four possible communication topologies as shown in Figure 6, called *communication graphs* in this paper. In a communication graph, a vertex represents an aircraft and an edge connecting two vertices represents that the two aircraft can communicate with each other. Each aircraft $i$ ($i \in \{1, 2, 3\}$) has a communication group $C_i$ ($i \in \{1, 2, 3\}$) such that $C_i$ is a set whose elements are neighbors of aircraft $i$. In a communication graph, aircraft $i$ can communicate with only the aircraft in $C_i$. Case 4 is a complete graph and Case 3 is a connected graph. Cases 1 and 2 are not connected graphs. Since Case 4 is a complete graph, all three aircraft share information with each other and thus, Algorithm 1 can be applied safely. However, since Case 1 and 2 are not connected graphs, they cannot share information with each other and so, Algorithm 1 cannot guarantee safe conflict resolution. Case 3 is in between the above two cases. Algorithm 1 can or cannot resolve a conflict safely depending on conflict geometry. In this paper, we focus on developing a protocol-based conflict resolution algorithm for a *connected system*, that is a distributed system whose communication graph is connected.

### 3.2 Connected system

We here analyze only a connected system whose communication graph is not complete, such as Case 3 in Figure 6 because Algorithm 1 can resolve conflicts in a connected system whose communication graph is complete. Before detailed analysis of a connected system, we introduce a new term:

**Definition 1** *A distributed system is* **observable** *when a local disturbance can propagate through the whole system[1].*

From the above definition, we relate connectivity in graph theory to observability in control theory.

**Proposition 1** *If a distributed system is observable, a communication graph of the distributed system is connected.*

**Proof:** Consider a communication graph with $N$ vertices. Since a distributed system is observable, any local information can be shared with any other elements in the system. To share information between any pair of elements, they must communicate each other. This means there should be a path between any pair of vertices in a corresponding communication graph. Thus, the communication graph should be connected. ■

For safe application of Algorithm 1, a given conflict resolution problem must be observable within a conflict resolution disc. Even for a connected system, all aircraft within a conflict resolution disc may not necessarily share information with each other through ADS-B since an ADS-B message of an aircraft does not contain information about other aircraft[15, 16]. To make all aircraft within a conflict resolution disc share information about the conflict, ADS-B must be modified to carry information about the aircraft involved in the

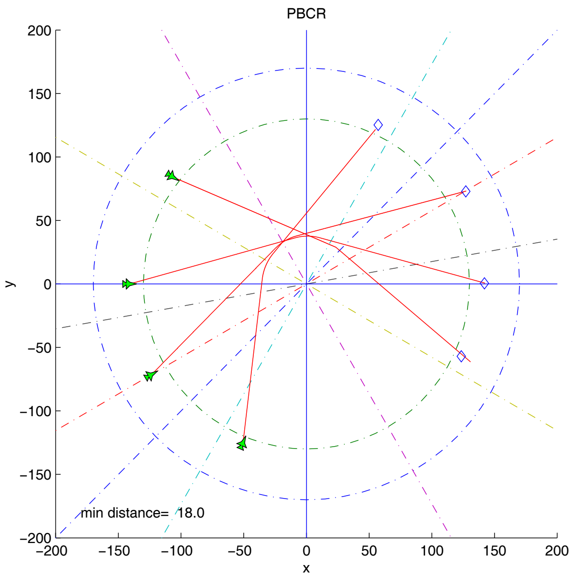

**Figure 7:** Protocol-based conflict resolution for a finite horizon connected system with augmented ADS-B ($nm$): 4 aircraft.

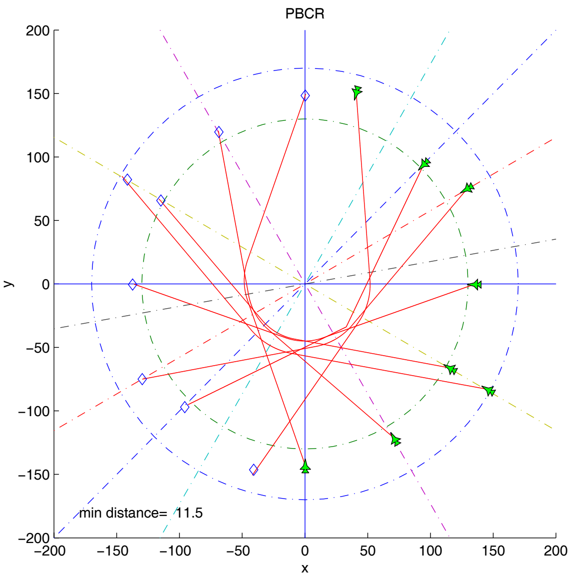

**Figure 8:** Protocol-based conflict resolution for a finite horizon connected system with augmented ADS-B ($nm$): 8 aircraft.

conflict. Thus, we propose a new ADS-B protocol, *augmented ADS-B* which has following properties:

1. ADS-B information is augmented with a center of a conflict resolution maneuver and position, velocity, and heading of all aircraft involved in the conflict.
2. Only when an aircraft is involved in a conflict, does it broadcast augmented ADS-B information.
3. If an aircraft receiving augmented ADS-B information has a neighbor within the conflict resolution disc, it broadcasts the augmented ADS-B message.

We show the following to be true:

**Proposition 2** *For a connected system whose communication graph may not necessarily be complete, if at least one communication group among the N communication groups within a conflict resolution disc has a conflict, then Algorithm 1 with augmented ADS-B can be applied safely for multiple-aircraft conflict resolution.*

**Proof:** Assume there are $N$ aircraft within a conflict resolution disc and $ac_j(j \in \mathcal{I}$ where $\mathcal{I} = \{1, 2, \cdots, N\}$ which is an index set) has a conflict with its neighbor $ac_k(k \in C_j \subset \mathcal{I})$ at the center of the conflict resolution disc as shown in Figure 5. The two aircraft compute a conflict resolution protocol using Algorithm 1 and broadcast augmented ADS-B information. Since the communication graph within the conflict resolution disc is connected and from the properties of augmented ADS-B, all $N$ aircraft within the conflict resolution disc share information with each other. If there are other conflicts among the aircraft within the conflict resolution disc, which are not detected due to finite information horizon, they can be detected using information obtained through augmented ADS-B. All aircraft within the conflict resolution disc compute their own conflict resolution protocol using Algorithm 1 and broadcast it. Since they can share this information through augmented ADS-B, Algorithm 1 for multiple-aircraft conflict resolution can resolve the conflicts safely[9]. ■

Thus, Algorithm 1 with augmented ADS-B can resolve conflicts of a connected system which satisfy the conditions in Proposition 2.

For simulation, the same dynamic aircraft model as in Section 2 is used. Simulation results with different conflict scenarios are shown in Figure 7 and 8. We assume communication range $D_I = 100nm$[15]. The motivating example in Figure 4, in which all aircraft fly through their waypoints, can be safely resolved by Algorithm 1 with augmented ADS-B and the simulation result is the same as that of the infinite horizon problem in Figure 4.

However, the condition that there is at least one conflict within a communication group is restrictive. To relax this condition, we should consider conflict resolution for a disconnected system. It is clear, however, that whatever information is augmented to ADS-B, Algorithm 1 cannot solve a general finite information horizon problem in which the communication graph within a conflict resolution disc may not be connected. Therefore, to resolve conflicts in a disconnected system, we must make the disconnected system connected in some way. Even if the communication graph is connected, if there is a conflict between aircraft within different communication groups but no conflict in the same communication group, there may be an unsafe conflict situation which Algorithm 1 with augmented ADS-B cannot resolve. This issue is a topic in our current research.

## 4 Conclusion

This paper has proposed a protocol-based multiple-aircraft conflict resolution for a finite information horizon. In the free flight concept, aircraft can fly user-preferred trajectories; in such a situation, automatic airborne conflict detection and resolution is crucial.

We have first reviewed conflict resolution protocols for an infinite information horizon problem based on the assumptions that aircraft have nominal trajectories which are at constant altitude and heading, with possibly varying velocities, and that all resolutions take place in the horizontal plane by heading change, velocity change, or both. We compute an analytic solution to the minimum heading change for all aircraft involved a conflict for safe resolution. The conflict resolution protocols are developed with this simple analytic solution.

For a finite information horizon, communication topology is important and is modeled by a graph. Using graph theory, we categorize a finite information horizon problem into two cases: a connected and a disconnected system. We have developed a protocol for multiple-aircraft conflict resolution for a connected system and shown the safety of the protocol. This protocol-based method has a couple of advantages over other proposed methods: 1) it is for multiple-aircraft conflicts, 2) it can be used in real-time as it is based on an simple analytic solution, and 3) it is simple to understand for pilots and to design embedded software implementations for autopilots. Finally, we validate our protocol through simulations using a dynamic aircraft model.

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