Multiple-Target Tracking and Identity Management in Clutter, with Application to Aircraft Tracking

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Abstract— In this paper, the problem of tracking and managing the identity of multiple targets in a cluttered environment is discussed and applied to passive radar tracking of aircraft. The targets are assumed to be hybrid systems. We propose a filter based on joint probabilistic data association for target-measurement correlation combined with an identity management algorithm [1] and an algorithm that we have developed in earlier work [2] for hybrid state estimation. The Multiple-Target Tracking and Identity Management algorithm, also incorporates suitable local information, when available, in a manner that decreases the uncertainty, as measured by system entropy. In situations in which local information is not explicitly available, a version of local information incorporation based on multiple hypothesis testing is included to improve identity management. The algorithm allows us to track multiple targets, each capable of multiple modes of operation, in the presence of interference which could be both noise in the continuous processes as well as in the form of spurious measurements.

I. INTRODUCTION

The multiple-target tracking problem deals with correctly tracking several targets given noisy sensor measurements; the identity management problem tries to associate target identities with the state estimates available. Application of these problems includes tracking in sensor networks [1] and multiple-aircraft tracking [3]. As an example of the latter, the current Air Traffic surveillance system uses data from radar measurements to track aircraft. In spite of a substantial improvement in technology, the radar system is still vulnerable to several problems, such as extraneous measurements from clouds, birds and other objects, as well as "phantom" blips [4], [5]. Another issue that poses danger is the growing number of general aviation aircraft. These aircraft do not transmit their identities unless their transponders are switched on, and even then, the transponders may be problematic [6]. Since Air Traffic Controllers are instructed not to issue orders to aircraft unless they are certain of their identity [7], it becomes essential that they have access to reliable track data with identities, so that they can maintain safety.

Given a radar system (or a network of sensors), in addition to the continuous state measurements, local sensor information about identities is often available. In the case of radar systems, this information may be derived from either the physical attributes of an aircraft or from establishment of communication with one of the aircraft. In our previous work [8], we proposed the Multiple-Target Tracking and Identity Management (MTIM) algorithm, which is a combination of the Joint Probabilistic Data Association (JPDA) algorithm [4] in which a target's kinematic information (position and velocity) is used for associating measurements with targets, and the Identity Management (IM) algorithm for sensor networks [1]. MTIM utilizes target attribute information from local sensors to correctly maintain target identity. In [8], we assumed there were no extraneous measurements.

In reality, the MTIM problem could be complicated by several shortcomings in the quality of available information about the targets. The surveillance system may have measurement errors, and may even miss measurements entirely. In certain environments, the surveillance system may also measure extraneous signals, known as clutter. The behavior of the targets also adds complexity to the problem: many targets may be interacting in a small spatial region, and these interactions increase the uncertainty in what is being measured. These issues motivate the extension of the MTIM algorithm to cluttered environments.

Tracking multiple targets in clutter involves the problem of associating measurement data with targets [4], *i.e.*, computing the probability of a given measurement having originated from a given target. To compute these data association probabilities, we propose a modified version of the JPDA algorithm which works for the large number of measurement-target associations computationally efficiently. Assignment algorithms have been used to choose the correct measurement-target correlations among all possible ones [9], [10]. However, these assignment algorithms select measurements which are close to expected target positions without considering measurement-target correlation. Therefore, they lose the advantages of the JPDA algorithm which considers all possible correlations between measurements and targets. Thus, we propose a data association algorithm which considers measurement-target correlations and uses the *extended Munkres algorithm* [11], [12] in order

to maximize the overall data association probability. We also propose the use of a Multiple Hypothesis Testing (MHT) algorithm ([13], [14]) to correct the identities of the targets when targets are close and thus their identities are mixed. This can be interpreted as a method of generating local information in the system when such information is not explicitly available.

This paper is organized as follows: Section II presents the aircraft model for tracking and discusses the MTIM algorithm, including the modified approximate JPDA, the extended Munkres algorithm, and MHT for local information incorporation. Section III presents a multiple-aircraft scenario simulation as demonstration of the efficacy of the MTIM algorithm. Finally, our conclusions are presented in Section IV.

II. MULTIPLE-TARGET TRACKING AND IDENTITY MANAGEMENT (MTIM) ALGORITHM IN CLUTTER

In this section, we consider the problem of associating a time series of measurements to the tracking and managing of identity of one or more aircraft in the presence of clutter.

We model the dynamics of an aircraft as a stochastic linear hybrid system with discrete-time continuous-state dynamics:

$$
\begin{aligned}
x(k+1) &= A_j x(k) + w_j(k) \\
z(k) &= C_j x(k) + v_j(k)
\end{aligned}
\tag{1}
$$

and a Markov transition model of the discrete state (mode) given by:

$$
P[j(k+1)|i(k)] = H_{ij} \qquad i,j \in M = \{1, 2, \cdots, N\}
\tag{2}
$$

where $x \in \mathbb{R}^n$ and $z \in \mathbb{R}^p$ are the state and the output respectively. M is the set of discrete states, or modes. The terms w and v are respectively the mode-dependent, uncorrelated, white Gaussian process noise and measurement noise with zero means and covariances Q_j and R_j. H_{ij} is the Markov mode transition probability from mode i to mode j. This hybrid model is useful for tracking a maneuvering aircraft since the trajectory of an aircraft is composed of straight lines and circular arcs depending on the flight mode of the aircraft. For example, if a single linear (or nonlinear) continuous model is used for aircraft tracking, the process noise covariance in the model has to be large in order to account for model inaccuracy. This large process noise covariance leads to poor state estimates. Hybrid models with multiple modes that represent the flight regimes (flight modes) of an aircraft could represent the dynamics of the aircraft more accurately than one continuous model, and thus each continuous model could have a small process noise covariance that would give accurate state estimates. The flight mode changes of an aircraft depend on the pilots input which is usually unknown to the surveillance system. This unknown pilot's input makes the flight mode changes of an aircraft nondeterministic and can be modelled as a finite Markov process [15].

Given the above system parameters, hybrid estimation requires estimating both the continuous state and the discrete state at time k from the measurement sequence up to time $k-1$ ($k = 1, 2, \cdots$). The Residual-Mean Interacting Multiple Model algorithm (RMIMM) ([2], based on [13]) is a hybrid algorithm which computes the state estimate using a weighted sum of estimates from a bank of Kalman filters matched to different modes of the system, and uses information about the mean of the residual to improve estimation performance. A detailed explanation of RMIMM is provided in [2].

First, denote $\hat{z}(k+1|k)$ as the predicted measurement of a specific target at time $k+1$ using information up to time k. Assume that the true measurement at time $k+1$, $z(k+1)$, conditioned on the measurement sequence up to k (Z^k), is normally distributed. Then, the *validation gate* is defined as:

$$
\tilde{V}_{k+1}(\gamma) := \{z \mid r(k+1)^T S^{-1}(k+1) r(k+1) \leq \gamma^2\}
\tag{3}
$$

where $r(k+1) = z(k+1) - \hat{z}(k+1|k)$ is the residual, $S(k+1)$ is its covariance, and γ is a design parameter which determines the size of the validation gate. At each time $k+1$, all measurements that lie inside $\tilde{V}_{k+1}(\gamma)$ are considered valid possibilities. The problem of associating each validated measurement with an appropriate target or identifying it as clutter and discarding it is known as data association.

The MTIM algorithm approaches this problem using the three main blocks shown in Figure 1 at each time step. The first stage is *Data Association*, which consists of matching incoming measurements to the targets. Given state estimates of T targets from the previous time step and L measurements from the current time step, the *Data Association* block is used to generate an $L \times T$ matrix of association probabilities. Entries in this matrix represent the probability of a given measurement having originated from a given target. The *Tracking/Hybrid State Estimation* block of MTIM performs the tracking of T targets in parallel. At time k, the tracking algorithm for each target takes as input the hybrid state estimate from the previous time step $k-1$ and a single measurement from the current time k. The measurement input comes from the Data Association block. The hybrid state estimate comprises position and velocity estimates, their covariances, and a flight mode estimate. The output of the Tracking/Hybrid State Estimation block is the hybrid state estimate at time k. The *Identity Management* block takes as input the belief matrix from time $k-1$ whose entries represent the probability that a given target has given identity, and the $L \times T$ association probability matrix. This block maintains identity information over time given information about the interaction between T targets. This information is stored in a $T \times T$ *identity belief matrix* $B(k)$, where k is the current time step. The matrix is doubly stochastic; that is, $\sum_{i=1}^{T} B_{ij}(k) = 1$, for $j \in \{1, \ldots, T\}$ and $\sum_{j=1}^{T} B_{ij}(k) = 1$, for $i \in \{1, \ldots, T\}$. The evolution of this belief matrix is governed by a $T \times T$ *mixing matrix*

$M(k)$, which stores interaction information for a single time step. $M_{ij}(k)$ represents the probability that target i at time $k-1$ has become target j at time k. The belief matrix is updated according to the equation [1]:

$$B(k) = B(k-1)M(k) \qquad (4)$$

The Identity Management stage outputs the belief matrix at time k. The following sections discuss each structural block and the algorithms used to implement the stages in detail.
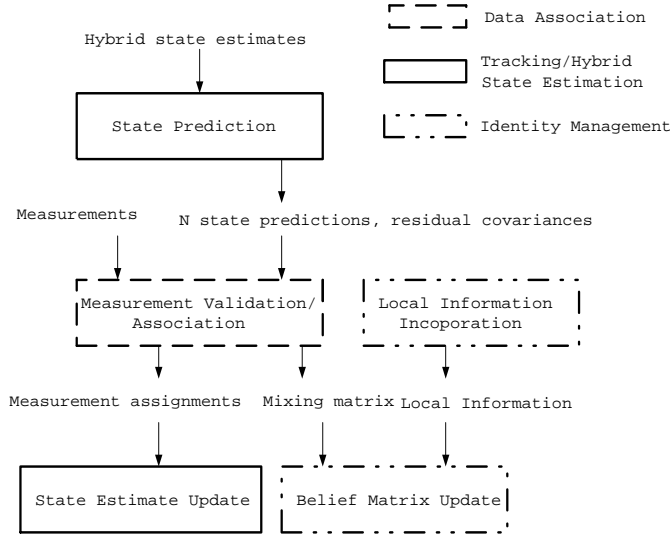

Fig. 1.   MTIM Block Diagram (single time step)

A. State Prediction

The State Prediction step, which generates an estimate of the state at time k based only on the outputs of MTIM at time $k-1$, is carried out for each of the T targets in parallel. This is done using the RMIMM algorithm. The details that follow refer to the procedure used for a single target. This stage takes as input the continuous state estimates $\hat{x}_i(k-1|k-1)$, covariances $P_i(k-1|k-1)$, and mode probabilities $\mu_i(k-1)$, which is a measure of how probable it is that the system is in mode i, where i refers to the mode of the target. The output of the block is a prediction of the state and its covariance at time k without information from time k. First, RMIMM combines the state estimates from the different modes, resulting in new initial states $\hat{x}_{0i}(k-1|k-1)$ and covariances $P_{0i}(k-1|k-1)$. These are input to a set of Kalman filters, one for each mode, without measurement inputs. The outputs of the Kalman filters are state predictions

$$\hat{x}_i(k|k-1) = A_i\hat{x}_{0i}(k-1|k-1), \qquad (5)$$

covariances

$$P_i(k|k-1) = A_iP_{0i}(k-1|k-1)A_i^T + Q_i, \qquad (6)$$

and residual covariances

$$S_i(k) = C_iP_i(k|k-1)C_i^T + R_i, \qquad (7)$$

The mode estimate $\hat{m}(k-1)$ from the previous time step is used to obtain a single continuous state prediction $\hat{x}(k|k-1)$ and a single residual covariance $S(k)$. Because the predicted state is assumed to have a Gaussian distribution, the state prediction is the mean (center) of the validation gate of the target, while the residual covariance is the covariance of the validation gate. $S(k)$ is also supposed to be used to determine the size of the validation gate, according to (3). However, when a target changes modes (starts a maneuver), the Kalman filter overestimates its confidence in its state estimate, which results in a smaller $S(k)$ than is appropriate. Often, the measurement of the maneuvering target does not fall inside its validation gate; as a result, the size of the validation gate must be increased. This increase is obtained by increasing the state covariance S with an additional term that compensates for the additional uncertainty about the maneuvering target. This additional term is related to the state velocity estimate $\hat{v}$ according to the expression

$$S_{extra} = \tau^2\hat{v}\hat{v}^T + \nu^2\hat{v}_\perp\hat{v_\perp}^T, \qquad (8)$$

where $\hat{v}_\perp$ is obtained by rotating $\hat{v}$ by 90° in the counterclockwise direction. The effective residual covariance S' is then equal to

$$S' = S + S_{extra}. \qquad (9)$$

Since S_{extra} is positive definite, the region covered by the validation gate created from S' is larger than that created by S, as shown in Figure 2. In this figure, the smaller ellipse is the validation gate as determined by S, while the larger ellipse is that determined by S'. The extended validation gate is longer in the cross-track direction to account for the likelihood of targets maneuvering to either side of their expected track. The constants τ and ν are chosen empirically to ensure that maneuvers are extremely unlikely to lead to measurements outside validation gates; the cross-track term ν is chosen to be larger than the along-track term τ. The additional term S_{extra} is related to velocity because errors in track due to a maneuver will be directly related to the velocity of the target. Thus, the outputs from the first block are state prediction $\hat{x}^t(k|k-1)$, residual covariance $S^t(k)$, and effective residual covariance $S^{t'}(k)$ for target t. There are T sets of outputs, one set for each target. The effective residual covariance S' is used for measurement validation only.

B. Measurement Validation/Association

Thus, the measurements are tested in validation gates defined in (3) with $S(k)$ replaced with the effective residual covariance $S'(k)$. The Joint Probabilistic Data Association (JPDA) algorithm can be used to choose T measurements and generate a $T \times T$ mixing matrix $M(k)$. However, in order to deal with many targets in clutter with good accuracy, in this section, we develop a Modified Approximate JPDA (MAJPDA) to generate the mixing matrix.

The Approximate JPDA (AJPDA) algorithm is a computationally abbreviated version of JPDA [16]. Denote
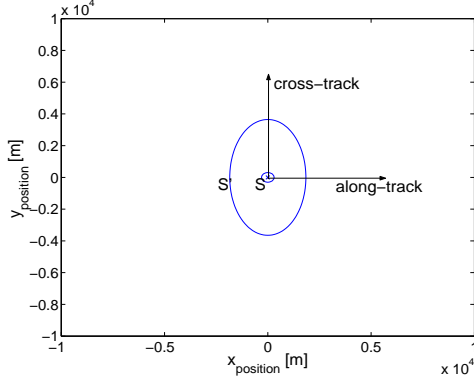
Fig. 2. Validation gates determined by the original residual covariance S and the effective residual covariance S' which accounts for the maneuvering uncertainty of a target.

the Gaussian probability density function of the residual $\mathcal{N}[z_j(k); \hat{z}^t(k|k-1), S^t(k)]$ as $G_{jt}(k)$ where $\hat{z}^t(k|k-1)$ denotes the predicted measurement for target t with an associated residual covariance S^t. Thus, $G_{jt}(k)$ is proportional to the Gaussian likelihood function that represents the closeness between target t and measurement j. We let

$$P_{st}(k) := \sum_j G_{jt}(k), \ P_{rj}(k) := \sum_t G_{jt}(k) \qquad (10)$$

Then the association probability is defined as [16]

$$\beta_{jt}(k) = \frac{G_{jt}(k)}{P_{st}(k) + P_{rj}(k) - G_{jt}(k) + \zeta}. \qquad (11)$$

Thus, (11) puts more weight on the target which does not fall into the validation gates of any other targets. ζ is a bias term set to 0 in most cases, including in this paper. The data association algorithm can be described as follows:

Algorithm 1: Data Association

- **Given**: validated measurements $z_j(k)$ $(j = \{1, \cdots, L\})$ and targets t $(t \in \{1, \cdots, T\})$ where $L \geq T$.

1) Modified Approximate JPDA (MAJPDA)

   a) Compute the $L \times T$ association probability matrix $\beta'(k) = [\beta'_{jt}(k)]$ using (11).
   b) Scaling: Find $\beta(k) = SI(\beta'(k))$ such that $\sum_{j=1}^{L} \beta_{jt} = 1$ and $\sum_{t=1}^{T} \beta_{jt} = 1$ where the operator SI represents the Sinkhorn scaling process.

2) Assignment (extended Munkres algorithm): Find a permutation Π such that

$$
\begin{aligned}
\max_{\Pi(t)} \quad & \textstyle\sum_{t=1}^{T} \beta_{\Pi(t)t} \\
\text{subject to} \quad & 1 \leq t \leq T, \ 1 \leq \Pi(t) \leq L \\
& i \neq j \Rightarrow \Pi(i) \neq \Pi(j)
\end{aligned}
$$

3) Mixing matrix: $M(k) = SI(\beta_{\Pi(t)t})$ for $t \in \{1, \cdots, T\}$.

Step 1-a computes association probabilities between the validated measurements and the targets. However, column sums of the association probability matrix computed by AJPDA might not be equal to 1, as in the case of JPDA. Thus, the accuracy of AJPDA might not be good enough for certain situations. To correct this and improve the performance of data association, we propose a Modified Approximate JPDA (MAJPDA) algorithm, which uses the Sinkhorn algorithm ([17], [8]) to make the association probability matrix $\beta(k)$ doubly stochastic (Step 1-b). Therefore, MAJPDA keeps the essential characteristics of JPDA, and thus outperforms AJPDA, with far less computational complexity than JPDA, for tracking many targets in clutter.

Because there can be more measurements than targets in a cluttered environment, there is a need to choose a subset of the full association probability matrix as the mixing matrix, which should be a square matrix [1]. The MAJPDA algorithm entails both the determination of the association probability matrix and the doubly stochastic, square mixing matrix. In the no-clutter case, the mixing matrix is nothing more than the doubly stochastic form of the association probability matrix. However, for a cluttered environment, if there are L measurements, then the association probability matrix has L rows and $T(\leq L)$ columns. The mixing matrix $M(k)$ must still have T rows and T columns. To choose T of the L rows, we use the extended Munkres algorithm [12], which is an assignment algorithm which chooses the set of T numbers with maximum sum from all sets of N numbers taken from a $T \times T$ matrix such that the numbers cover every row and every column [11]. Bourgeois and Lassalle extended this algorithm to rectangular matrices [12]. Thus, for a $L \times T$ matrix, with $L \geq T$, the extended Munkres algorithm picks T elements from the matrix with maximum sum so that these numbers cover T distinct rows and all of the T columns (Step 2). This extension lends itself to processing the data association probability matrix output by MAJPDA. The T numbers chosen by the extended Munkres algorithm constitute $z^t(k)$, which are the T measurements assigned to the T targets to maximize the sum of association probabilities. The assignment of measurements to targets is a one-to-one correspondence between measurement j and target t; that is, j is a function of t and vice-versa. The T rows of $\beta(k)$ representing these measurements form a $T \times T$ matrix. The doubly stochastic form of this matrix is the mixing matrix $M(k)$ (Step 3). The mixing matrix and measurement assignments are then passed to the Belief Matrix Update and State Estimate Update blocks respectively. The State Estimate Update block propagates the continuous state, its covariance, and mode probabilities to time k. The Belief Matrix Update blocks updates the belief matrix from time $k-1$ to time k using (4).

C. Local information incorporation

Local information is useful only if its use results in the uncertainty of the belief matrix being reduced, where uncertainty is measured as entropy (*Shannon Information*) [8]. Entropy of the belief matrix is defined as a measure of statistical uncertainty of the probability density of the iden-
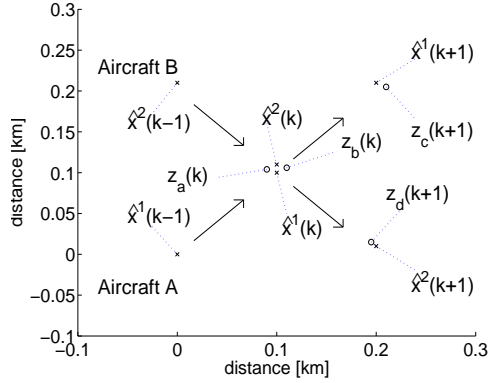
Fig. 3. State estimates (x) and measurements (o) for a two-aircraft example. The solid arrows denote the direction of movement of the targets. The dotted lines do not denote distances, they give the association between the labels and the points.

tity of the T targets. From our earlier work [8], it has been shown that local information that identifies one or more targets with absolute certainty can always be incorporated, since such information will never increase entropy. In this paper, an additional source of possible local information is presented. This set is automatically generated whenever targets interact and the entropy increases significantly.
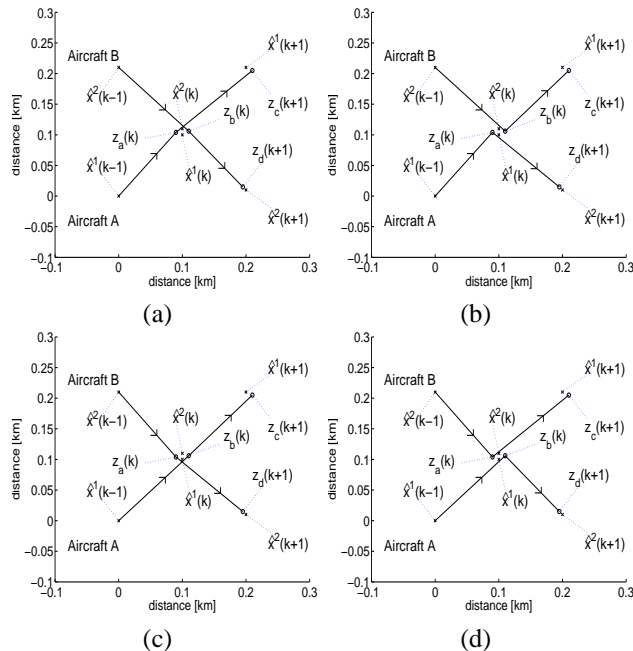

Fig. 4. (a)-(d): Possible joint events in MHT. The solid arrows denote the direction of movement of the targets. The dotted lines do *not* denote distances, they give the association between the labels and the points.

Without using extra sensors to get attribute information about the targets to correct target identities, we propose to use the Multiple Hypothesis Testing (MHT) algorithm to get local (attribute) information about interacting targets. The reason for using MHT is that it covers all possible target identity hypotheses. MHT is used only when the minimum

diagonal element of the mixing matrix is below a threshold, which we treat as a design parameter. The local information comes from applying the MHT algorithm on track estimates for two time steps. This information is useful in the situation portrayed in Figure 3. In this figure, two aircraft cross perpendicularly at time k. Their estimated positions are marked with x's, while the radar measurements are marked with o's. The expression $\hat{x}^t(l)$ denotes the state estimate for target t at time l. The measurements $z_a(k)$, $z_b(k)$, $z_c(k+1)$, and $z_d(k+1)$ are indexed by letters to reflect that of possibly many choices, two measurements were chosen by MAJPDA at each time step to correspond to the two targets. Aircraft A starts at the bottom left at time $k-1$ and moves to the top right at time $k+1$, while Aircraft B starts at the top left and moves to the bottom right. The assumption is that aircraft A(or B) is Target 1(respectively, 2) with absolute certainty at time $k-1$. That is, the belief matrix $B(k-1)$ is the identity $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. At time k, the two targets are close together and almost equally likely to be associated with each of two measurements. Assume the mixing matrix $M(k)$, and thus the belief matrix $B(k) = B(k-1)M(k)$, is $\begin{bmatrix} 0.51 & 0.49 \\ 0.49 & 0.51 \end{bmatrix}$ at time k. At time $k+1$, the aircraft have diverged, and the validation gates of the two aircraft no longer intersect. Thus, the mixing matrix $M(k+1)$ from MAJPDA is the identity, and the belief matrix $B(k+1) = B(k)M(k+1)$ remains at $\begin{bmatrix} 0.51 & 0.49 \\ 0.49 & 0.51 \end{bmatrix}$. The MAJPDA algorithm cannot differentiate between the two measurements at time k; as a result, uncertainty in the belief matrix is essentially maximum. This uncertainty remains even after the aircraft separate. However, from the dynamics of the two aircraft, it is clear that neither aircraft can execute a 90° turn in one time step. Thus, the only possible outcome is that aircraft A(B) remains associated with Target 1(2) and this yields a belief matrix equal to the identity matrix, which has minimum entropy.

The MHT algorithm is utilized to obtain a lower entropy belief matrix than MAJPDA and standard Belief Matrix Updates can achieve. This algorithm is discussed in detail in [4], [14]. Given initial conditions $\hat{x}^1(k-1)$ and $\hat{x}^2(k-1)$, as well as measurements $z_a(k)$, $z_b(k)$, $z_c(k+1)$, and $z_d(k+1)$, there are four possible target-measurement matchings that can occur; these are illustrated in Figure 4. Figure 4(a) refers to the outcome chosen by MAJPDA, since Target 1 is assumed to have gone through measurements $z_a(k)$ and $z_c(k+1)$. Each plot in Figure 4 is a joint event made up of four events represented by the line segments in the plot. The likelihood of the joint event that each target chooses its pair of measurements is the product of these individual events. The result is four likelihoods for the four joint events portrayed in the plots of Figure 4.

To determine belief, one is only interested in whether Target 1 reaches the expected position of Aircraft A at time $k+1$ or not. Thus, the sum of the likelihoods from

Figure 4(a) and (c) is the likelihood that Target 1(2) remains identified as Aircraft A(B); let this quantity be denoted L_1. The sum of the likelihoods from Figure 4(b) and (d) is the likelihood that the targets swap identities; let this quantity be denoted L_{-1}. Because a 90° turn in one time step is not allowed in the dynamic models of the aircraft, $L_{-1} = 0$.

The doubly stochastic version of the matrix $\begin{bmatrix} L_1 & L_{-1} \\ L_{-1} & L_1 \end{bmatrix}$ represents the mixing matrix for the two aircraft between time steps $k-1$ and $k+1$. This matrix, a two-step mixing matrix, is denoted as $\Gamma(k+1)$. For the example presented, $\Gamma(k+1)$ is the identity. Thus, the belief matrix determined by MHT at time $k+1$ is $B'(k+1) = B(k-1)\Gamma(k+1) = \begin{bmatrix} L_1 & L_{-1} \\ L_{-1} & L_1 \end{bmatrix}$. The resulting belief matrix $B'(k+1)$ is the identity, which has lower entropy than the $B(k+1)$ from the standard MTIM model. The local information can thus be incorporated through the Belief Matrix Update block of MTIM.

Because there is no guarantee that automated MHT local information will improve the entropy of the belief matrix, it is only incorporated if this local information decreases entropy. This automated local information and identity local information are both handled by the Incorporate Local Information block of the MTIM algorithm.

III. APPLICATION OF MTIM TO MULTIPLE AIRCRAFT FLYING IN CLUTTERED ENVIRONMENT

One of many scenarios where multiple aircraft are interacting in a cluttered environment is presented below to demonstrate the efficacy of the MTIM algorithm in clutter. Measurement points are assumed to be made available every 5 seconds. Measurement covariance R is $\begin{bmatrix} (100)^2 & 0 \\ 0 & (100)^2 \end{bmatrix}$, which means the standard deviation of position error is 100m in both dimensions. Process noise is set to be $\begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}$ for straight flight and $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ for turning mode. The above constants are realistic values for aircraft in clutter and are taken from [18]. Clutter is uniformly distributed in space and Poisson distributed in number; the density of clutter points is $0.5 * 10^{-6}$ clutter points per square meter, or 0.5 points per square kilometer. The validation gate parameter γ is set to 9.2, which would correspond to a 3σ confidence level if residual covariance S were used. The effective residual covariance S' that is actually used is determined with system constants τ and ν set to 3 and 6, respectively. The threshold for initiating MHT is set to 0.75. In the example, the target 1 is initially identified as Aircraft A, 2 as B, and so on.

The example shown in Figure 5 is an extreme (acrobatic, not realistic for air traffic, but nonetheless interesting) scenario where four aircraft fly at each other directly and maneuver; this example is useful in understanding the capabilities of MTIM. Figure 5 (top) shows a shot of the radar screen including the entire flight data, but without the trajectories explicitly indicated. This gives us an idea of the clutter density, as well as how unclear the system is, especially when the aircraft come close to each other. Figure 5(center) displays the actual and estimated positions of four aircraft following symmetric paths that first converge, then maneuver around a common point, and finally diverge. The dashed lines with dots as markers are the noisy measurements from the targets. The solid lines with markers as shown in the legend are the estimated positions found by MTIM. The fainter dots interspersed throughout the plot are clutter points. Aircraft A, B, C, and D fly with constant velocity of 200 m/s. All turns are executed at $3^o/\text{sec}$. Target tracking is accurate except for overshoot when aircraft start turning. Indeed, the dashed lines depicting the noisy measurements are not clearly visible because the solid lines depicting estimated target positions match them almost exactly. Figure 5(bottom) displays the evolution of the belief matrix in graphical form. The plots, from top to bottom, show the probability that any aircraft is identified with targets one through four, respectively. From this figure, it is clear that the belief matrix is unchanged while the aircraft are distant from and not interacting with each other. When paths cross, the belief matrix is changed significantly only if the measurements for both targets happen to nearly coincide. For example, at time 30, targets 1 and 2 nearly coincide, leading to the belief that both targets 1 and 2 are nearly 0.5 Aircraft A and 0.5 Aircraft B. However, the automated MHT local information generated by this interaction restores the belief matrix to nearly identity at the following time step. At time 30, targets 3 and 4 also interact with equally drastic loss of identity between Aircraft C and D. Again, the local information restores the belief matrix at the following time step. At time 32, targets 1 and 3 interact, with similar jump in belief matrix entropy followed by belief matrix restoration from local information. Targets 2 and 4 also interact in the same fashion at time 32. The scenario depicted in Figure 5 establishes the efficacy of the MTIM algorithm in clutter.

IV. CONCLUSIONS

We have developed a Multiple-Target Tracking and Identity Management algorithm in a cluttered environment, which can track and manage identities of multiple maneuvering targets simultaneously. This algorithm is composed of three different blocks: Data Association, Tracking/Hybrid State Estimation, and Identity Management. For data association, we have proposed a Modified Approximate Joint Probability Density (MAJPDA) algorithm which can handle many targets with low computational complexity, yet possessing the main advantages of the standard JPDA algorithm. For tracking multiple-maneuvering targets, we used the Residual-Mean Interacting Multiple Model (RMIMM) algorithm which gives better estimation performance than IMM by using the mean of the residual computed by individual Kalman filter. For identity management, we extended

the algorithm developed by the authors earlier work in [8]
to account for clutter.

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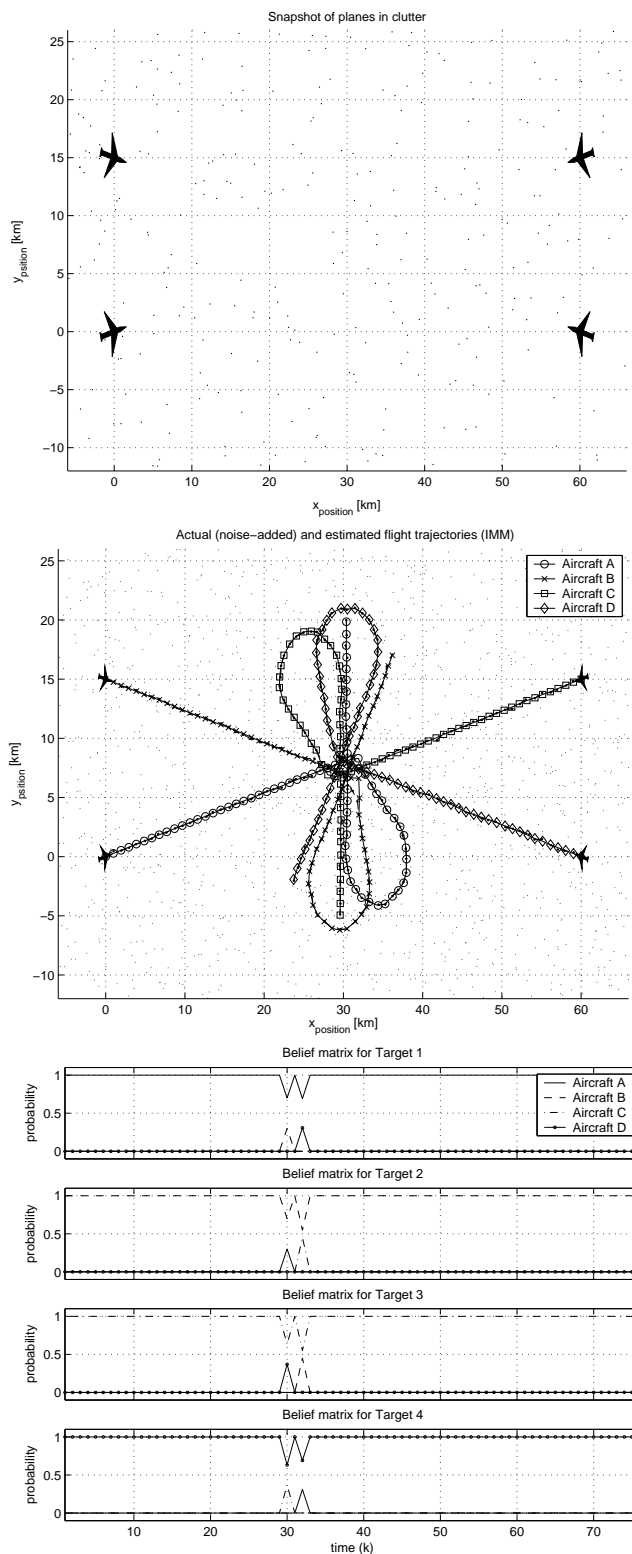
Fig. 5. Measurement points with clutter (top), aircraft trajectories (center) and accompanying belief matrix plot (bottom).