FIR Filter Design (Lab 5 Wk 2, 2-4)

Start with an even FIR filter:

$h_e(n) \neq 0$ for $n = -L, -L+1, \ldots, L$

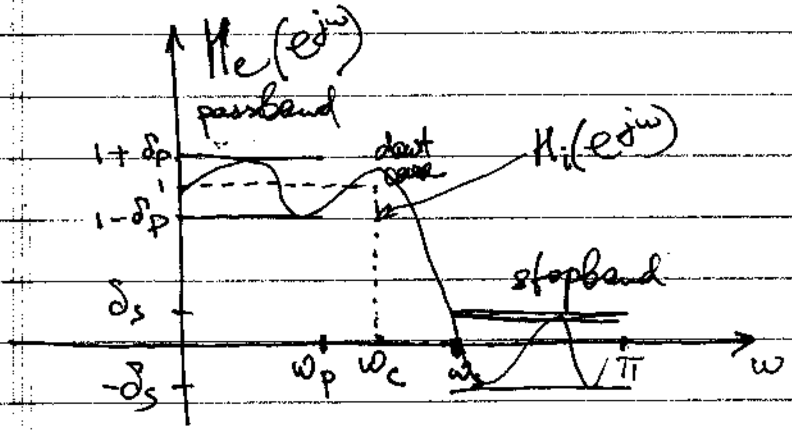
$h_e(n) = h_e(-n)$.

$$\boxed{H_e(e^{j\omega}) = \sum_{n=-L}^{L} h_e(n) e^{-j\omega n} = h_e(0) + \sum_{n=1}^{L} 2h_e(n)\cos(\omega n)}$$

Obtain a causal filter by delaying h_e by L samples:

$h(n) = h_e(n-L)$

$H(e^{j\omega}) = H_e(e^{j\omega}) e^{-j\omega L}$



Error: $E(\omega) = \underbrace{W(\omega)}_{\text{weight}} \left(\underbrace{H_i(e^{j\omega})}_{\text{ideal LPF}} - H_e(e^{j\omega}) \right)$

$$W(\omega) = \begin{cases} \frac{\delta_s}{\delta_p}, & 0 \le \omega \le \omega_p \\ 0, & \omega_p < \omega < \omega_s \\ 1, & \omega_s \le \omega \le \pi \end{cases}$$

Best approximation:

Pick the filter coefficients, ~~Fixed~~ $h_e(n)$, $n=0,\dots,L$ so as to minimize

$$\max_{0\le\omega\le\pi} |E(\omega)|$$

(max absolute error).

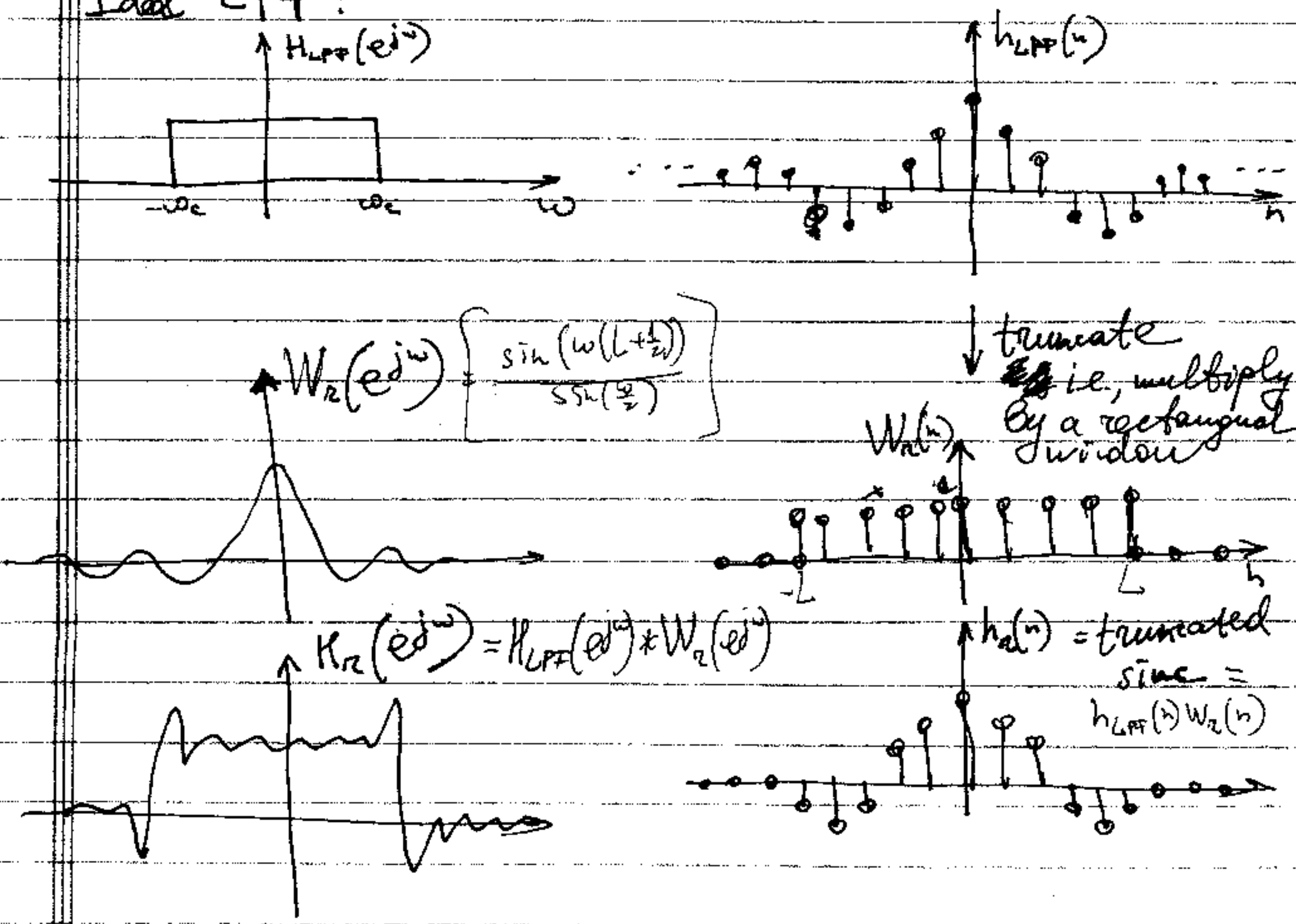
(Parks-McClellan Alg.) - Lab 5 wk2 Part 4

+ can also estimate a sufficient filter order, L.

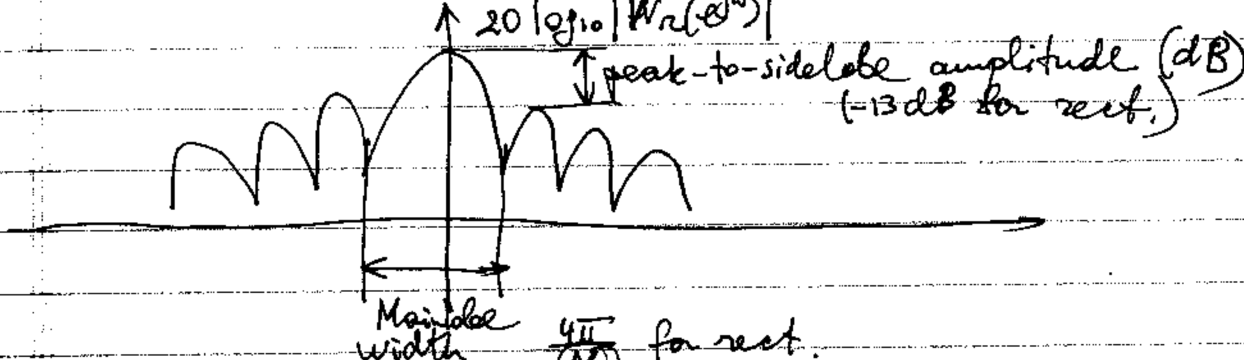
FIR Filter Design by Windowing.

- not optimal, but
- conceptually simpler than Parks-McClellan
- sometimes has computational advantages

Ideal LPF:



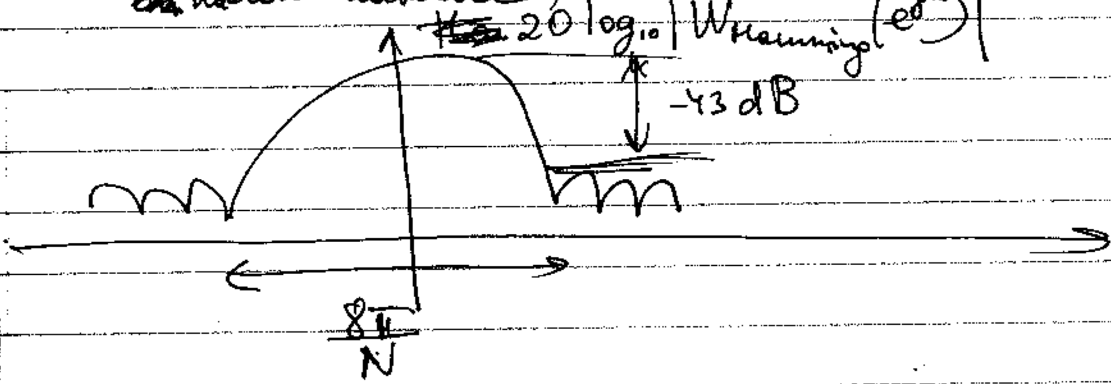
~~Use a different~~ $W_R(e^{j\omega})$ in dB:



$\frac{4\pi}{N}$ for rect.
N ← filter length

Want:
- small sidelobes
- ~~thi~~ narrow mainlobe

i.e., want this to be as close to 0 as possible



- small N

1.5.3 Properties of ROC, Poles, and Zeros (continued). ④

Recall: $a^n u(n) \leftrightarrow \frac{1}{1-az^{-1}}$, $|z|>|a|$

$-a^n u(-n-1) \leftrightarrow \frac{1}{1-az^{-1}}$, $|z|<|a|$

Example. Find all sequences whose z-transform is

$$X(z) = \frac{1-4z^{-1}}{1-3z^{-1}+2z^{-2}}.$$

Solution. $$X(z) = \frac{1-4z^{-1}}{(1-z^{-1})(1-2z^{-1})} = \frac{A_1}{1-z^{-1}} + \frac{A_2}{1-2z^{-1}}$$

How to find A_1 and A_2?

Method 1. $$X(z) = \frac{A_1(1-2z^{-1})+A_2(1-z^{-1})}{(1-z^{-1})(1-2z^{-1})} = \frac{A_1+A_2-(2A_1+A_2)z^{-1}}{(1-z^{-1})(1-2z^{-1})}$$

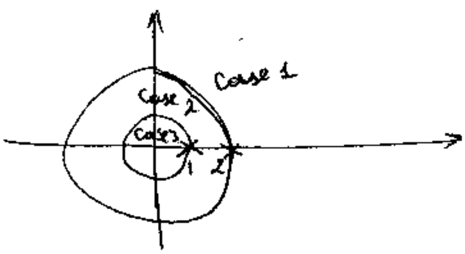
$$\begin{cases} A_1+A_2=1 \\ 2A_1+A_2=4 \end{cases} \Rightarrow \begin{cases} A_1=3 \\ A_2=-2 \end{cases}$$

Method 2. $$\left[X(z)(1-z^{-1})\right]_{z=1} = \left[A_1 + \frac{A_2(1-z^{-1})}{1-2z^{-1}}\right]_{z=1} = A_1$$

$$\left[\frac{1-4z^{-1}}{1-2z^{-1}}\right]_{z=1} = \frac{1-4}{1-2} = 3$$

$$\left[X(z)(1-2z^{-1})\right]_{z=2} = \left[\frac{1-4z^{-1}}{1-z^{-1}}\right]_{z=2} = \frac{1-\frac{4}{2}}{1-\frac{1}{2}} = -2 = A_2$$

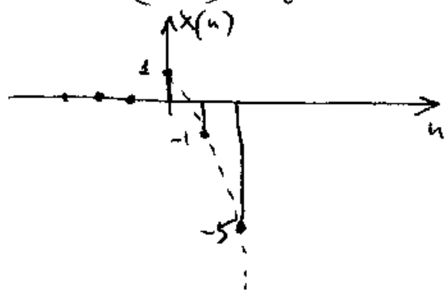
$$X(z) = \frac{3}{1-z^{-1}} - \frac{2}{1-2z^{-1}}$$



3 possible ROC's:

Case 1: ROC $|z|>2$

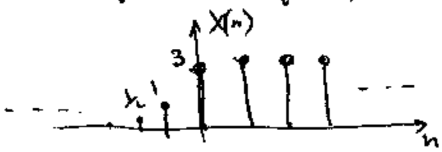
$$x(n) = 3\cdot 1^n u(n) - 2\cdot 2^n u(n) = (3-2^{n+1})\,u(n)$$



right sided

Case 2: ROC $1<|z|<2$

$$x(n) = 3\cdot 1^n u(n) + 2\cdot 2^n u(-n-1)$$



Case 3: ROC $|z|<1$.

$$x(n) = (-3+2^{n+1})\,u(-n-1)$$