We saw last time that many problems ~~like this~~ should be thought of in the framework of the following picture:

```
 input x        ----------       output y = S[x]
  ------>      | system S |     ------>
                ----------
```

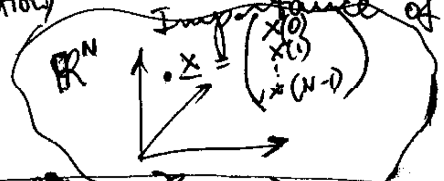
— too general

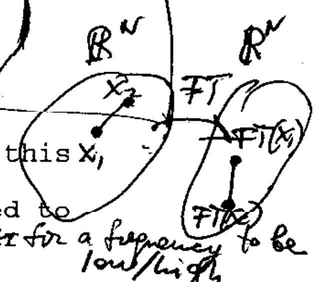
We will mostly analyze a certain class of systems, called linear time invariant systems.

1. Analysis of Discrete-Time (DT) Linear Time-Invariant (LTI) Systems.
   1.1. Signals. — HW1 Prob 1
   1.2. Systems. — HW2 Prob 3 (DT convolution)
   1.3. Fourier Series and Transforms.
   1.4. Sampling. — HW1 Prob 2
   1.5. Z-transform.
   1.6. FFT.
   1.7. Random sequences.

2. Speech
3. Image (enhancement, noise removal), Compression

Importance of geometry: Fourier





We will start by considering the basic ~~building~~ ingredients of this picture, namely, signals, and systems through which signals flow. We will then proceed to frequency analysis, and look at what it means ~~that this is lower~~ for a frequency to be low/high ~~frequency than this~~, AND how they can *appear* to be the same frequency if sampled incorrectly. More generally, the importance of this topic is due to the fact that, while most signals in the physical world are continuous-time or continuous-space, our most convenient and powerful signal processing tools deal with discrete-time and discrete-space signals. Well, sampling is how DT signals are obtained from CT signals.

Next, we will cover Z-transforms--a very useful technique for analyzing DT systems, and, in particular, for talking about system stability. Our next topic will be Fast Fourier Transform, which is not a new transform--it's just an algorithm to efficiently compute the Fourier transform.

Since many real-world signals are too complex to be modeled exactly, we will next consider random signals. Instead of asking what is the exact value of a signal at a point, we will be analyzing the average behavior of classes of signals. We will be interested in the probability that a signal from a certain class attains a particular value at a particular point.

1.1. Signals.

1.1.1 DEFINITIONS AND NOTATION.

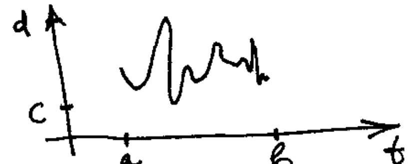
I'll come clean rightaway. A signal is a function.

"signal" and "function" are synonymous.

The two notions are the same, and we will be using them interchangeably. The historical reason for the existence of these two terms to denote the same thing is that "function" is the standard term from mathematics, whereas "signal" is an engineering term which originally was used to denote measurable physical ~~one-dimensional~~ quantities, like a voltage signal.

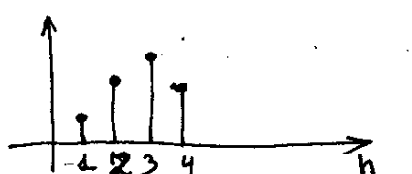
Continuous-time (CT) (or analog) signals are:
(i) defined for every value of time in an interval (possibly, an infinite interval), AND
(ii) take on values in an interval.

Graph of a continuous-time function:



Discrete-Time (DT) signals (or sequences) are defined only at integer values of time.

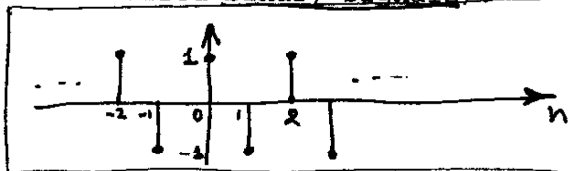
Graph of a discrete-time function:



To emphasize the difference between continuous-time and discrete-time, we will use n, instead of t, for discrete time.

Digital signal (or digital sequence) is a DT signal which can take on only integer values.

A digital signal which takes on only two different values (sometimes such signals are called binary signals): — temporarily DNE



Instead of using all these words and pictures, I could have simply written: $\mathbb{R}$->$\mathbb{R}$, $\mathbb{Z}$->$\mathbb{R}$, and $\mathbb{Z}$->$\mathbb{Z}$. Let's talk a little about this notation, since it will prove very useful.

$\mathbb{R}$ = the set of all real numbers, i.e. the real line.

$\mathbb{Z}$ = the set of all integers, {..., -2, -1, 0, 1, 2, ... }

In order to completely understand this notation, it is important to realize that a function is not a number and is not a set of numbers.

11.2

WHAT'S A FUNCTION?

A function is NOT a number;
it is NOT even a set of numbers;
it is a RULE for producing a number in its range, given a number from its domain.

```
         _______                       _______
        /       \                     /       \
       /  Domain \        f          /  Range  \
      |   of f,   |    ------->     |   of f,   |
       \    D    /                   \    R    /
        \_______/                     \_______/
```

Notation: $f: \mathcal{D} \rightarrow \mathcal{R}$,

Again, it's useful to think of a function in terms of a block diagram:

```
        a single number                       a single number
        n, taken from the    ______________   f(n) in the range of
        domain of f, D      |              |  f, R
                    ------->| function f   |------>
                            |______________|
```

Example.

```
   argument n, an integer  | function            |   value f(n)=n/3, a real number
                   ------->| f = "divide by 3"   |------->
                           |_____________________|
```

The programming metaphor of the day is quite straightforward: think of a signal as a program that takes a single number as its input and produces another number as the output.

```
float divide_by_3(n)
      int n;
{
    float x;

    x = n/3.0;
    return(x);
}
```

The function is this box, this module of code, a rule, an algorithm. Then you can call it from elsewhere, and evaluate it for a particular argument.

```
main()
```

```
main()
{
...
x = divide_by_3(5);
...
}
```

When you evaluate the function, you'll be assigning to x a particular number, in this case, 5/3, or approximately 5/3.

So, a function is a box which takes in one number and produces another number.

This notation means that continuous-time functions take in a real number and produce another real number: $\mathbb{R} \to \mathbb{R}$

This means that discrete-time functions can only take in an integer number, but can produce a real number. $\mathbb{Z} \to \mathbb{R}$

This means that digital functions take in an integer and produce an integer. $\mathbb{Z} \to \mathbb{Z}$

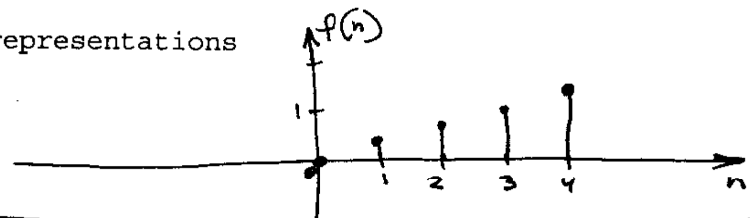
Now, how you represent a function on paper is a different matter.

1.1.3

REPRESENTATIONS OF A SIGNAL.

(a) formula,
e.g. $f(n) = n/3$ for $n=0,1,2,3,4$.

(b) graphical representations



(note that, for 2-D functions, surface plots and intensity images can be very useful--as you are finding out from the last exercise of Lab 1).

(c) a list of all values for all arguments:

n	0	1	2	3	4
f(n)	0	1/3	2/3	1	4/3

(d) A vector in an N-dimensional space
we will use this for:
- N-point signals
- periodic signals with period N.

Different types of functions require different processing tools. It will be important for us to know: is a function periodic or not? Is it finite duration? Is it bounded? Is its energy finite?

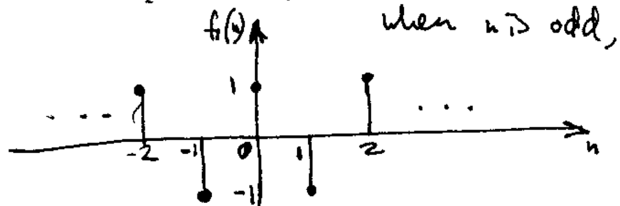
1.1.4

PROPERTIES OF SIGNALS.

a) Periodicity.
   If f(n) = f(n+T) for some fixed T and all n, we say that f is periodic with period T.

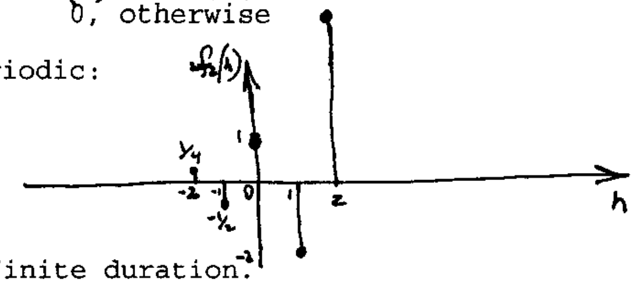
   For example, the function given by the formula
   f1(n) = (-1)^n

is periodic with period 2: when n is even, it's 1
when n is odd, it's -1



But $f2(n) = \begin{cases} (-2)^{n}, & |n| \leq 2 \\ 0, & \text{otherwise} \end{cases}$

is not periodic:



b) Finite/infinite duration.

If $f(n) = 0$ outside of a finite interval,
f is finite duration;
otherwise, f is infinite duration.
For example, f1(n) is infinite duration; f2(n) is finite duration.

c) The energy of a signal f is $\sum_{n=-\infty}^{\infty} f(n)$.

For example, the energy of f1 is $\sum_{n=-\infty}^{\infty} 1$, which is infinite.
The energy of f2 is $\left(\frac{1}{4}\right)^2 + \left(-\frac{1}{2}\right)^2 + 1^2 + (-2)^2 + 4^2 = 21\frac{5}{16}$

An important remark here is that, since we will often be dealing with sums of the type:

$$1 + q + q^2 + \cdots$$

it is useful to remember the formulas for summing the geometric series:

$$1 + q + q^2 + \cdots + q^{N-1} = \frac{1-q^N}{1-q}$$

and, if $0<|q|<1$,

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

d) The magnitude of a signal f is the maximum of its absolute value:

$$\max_{-\infty<n<\infty} |f(n)|.$$

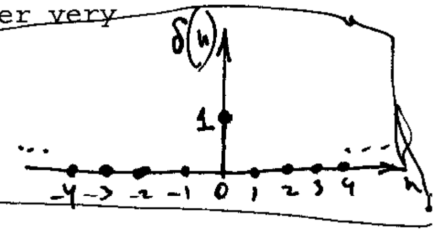
The magnitude of f1 is 1; the magnitude of f2 is 4.
If a signal has a finite magnitude, we say, naturally, that it is bounded; otherwise, it's unbounded.

There are several special signals which we will encounter very often.

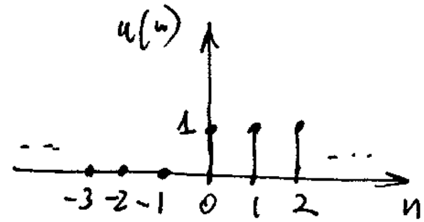
1.1.5

SPECIAL SIGNALS.

a) Unit sample (or unit impulse) $\delta(n) = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$



b) Unit step. $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



c) Sinusoid.

---

start Lec 3 F01

1.1.6 PECULIAR PROPERTIES OF DT SINUSOIDS.

(a) the highest frequency is pi:

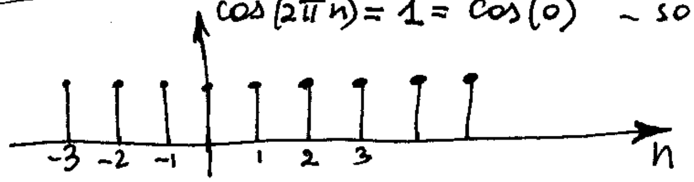
(b) adding 2*pi to the frequency does not change the signal

(c) DT sinusoids are not necessarily periodic!

Transformations:
- reflection
- shift
- scaling (downsampling and upsampling)
- their compositions

The ~~first~~ one looks strange. What happens if I write $\cos(2\pi n)$? Won't the frequency of the signal be ~~[illegible]~~ larger than π? But remember that n is integer. Therefore, cosine of $2\pi n$ is 1:

$\cos(2\pi n) = 1 = \cos(0)$ ~ so, the DT frequency of 2π is the same as DT frequency 0!



Let's do another example very carefully, to see what's happening here. Let's plot the continuous-time signal $\cos \pi t$ end Lec 2 F01