1.7.8. Sampling from a Distribution

(Lec 24) Mon 10/22/01 ①

Suppose you've hypothesized or built a probabilistic model — i.e., you've ~~estimated~~ assumed or estimated a probability distribution of your observation (e.g., using techniques from the previous section). It is often very important to be able to synthesize samples from this probability distribution.

E.g., texture synthesis is needed in rendering, printing, and other computer vision and imaging applications:

- model texture as a 2-D random process
- synthesize an image or a texture patch by sampling from the prob. model

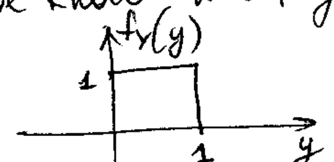
) one possible approach to texture synthesis

[see, e.g., http://www.ai.mit.edu/~jsd/jsd.doit/Research/TextureSynthesis]

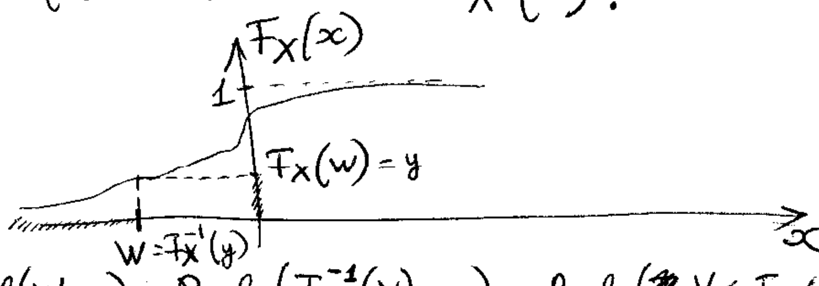
(Note: here, the word "sampling" has a completely different meaning.)

Simplest example: suppose we have a cdf $F_X(x)$. How to generate samples of a random variable with this cdf?

Last part of HW9: Suppose we know how to generate samples of a uniform variable Y,



What is the cdf of $W = F_X^{-1}(Y)$?



$$F_W(w) = \text{Prob}(W \le w) = \text{Prob}\left(F_X^{-1}(Y) \le w\right) \underset{\uparrow}{=} \text{Prob}\left(Y \le F_X(w)\right) = F_X(w)$$

because F_X is monotone

So, the cdf of W is F_X!
Use F_X^{-1} to transform the uniform distribution into F_X.

1.7.9 Filtering a random process.

What happens to a random process when it's put through a linear system? In general, this is a very difficult question; however, we **can** say what will happen to its 1-st and 2-nd order statistics.

Def. If $X(n)$, $Y(n)$ are real-valued random sequences, the autocorrelation function of $X(n)$ is

$$r_{XX}(m,n) = E\{X(m)X(n)\}$$

(How strongly are the points of X at m and n are correlated?)

The cross-correlation function of $X(m)$ and $Y(n)$ is

$$C_{XY}(m,n) = E\{X(m)Y(n)\}$$

(How strongly are X and Y correlated?)

[A NOTE on complex-valued random variables and processes:

$$Cov(X,Y) = E((X-m_x)(Y-m_y)^*)$$

correlation is $E(XY^*)$

$$r_{XX}(m,n) = E\{X(m)X^*(n)\}$$

$$C_{XY}(m,n) = E\{X(m)Y^*(n)\}$$
]

Def. A random sequence $X(n)$ is "wide-sense stationary" (WSS) if

(1) $E\{X(n)\} = \text{const}$ for all n

(2) $r_{XX}(m,n) = r_{XX}(0,n-m)$ for any n,m

$$= r_{XX}(n-m).$$

abuse of notation

If both X and Y are WSS, then

$$C_{XY}(k,n) = C_{XY}(n-k).$$