3.2 Image Filtering (continued).

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Problem: suppress noise while preserving the useful image as much as possible.

Example 1. Linear space-invariant filtering

Averaging filter: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{8}$ better noise removal leads to more blurring

Example 2. Median filter - removes spikes; good for "impulsive" noise; not so good for white Gaussian noise.

Example 3 (Perona-Malik filter)

Linear filter of example 1:

$$g(m,n) = \left(1 - \frac{4}{8}\right) f(m,n) + \frac{1}{8} f(m-1,n) + \frac{1}{8} f(m+1,n) + \frac{1}{8} f(m,n-1) + \frac{1}{8} f(m,n+1)$$

$$= f(m,n) + \frac{1}{8}\Big\{\left[f(m-1,n) - f(m,n)\right] + \left[f(m+1,n) - f(m,n)\right] + \left[f(m,n-1) - f(m,n)\right] + \left[f(m,n+1) - f(m,n)\right]\Big\}$$

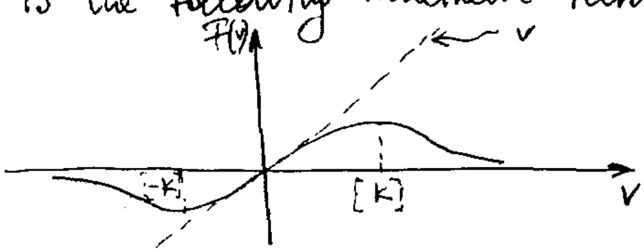
Instead, try the following nonlinear filter:

$$g(m,n) = f(m,n) + \alpha\Big\{\mathcal{F}\left[f(m-1,n) - f(m,n)\right] + \mathcal{F}\left[f(m+1,n) - f(m,n)\right]$$

not to be confused with F.T.

$$+ \mathcal{F}\left[f(m,n-1) - f(m,n)\right] + \mathcal{F}\left[f(m,n+1) - f(m,n)\right]\Big\},$$

where $\mathcal{F}$ is the following nonlinear function:



E.g., $\mathcal{F}(v) = \frac{v}{1 + \left(\frac{v}{K}\right)^2}$

What does this filter do?

not quite true {
- at an edge, Δf is large $\Rightarrow \mathcal{F}(\Delta f)$ is small
  $\Rightarrow g(m,n) \approx f(m,n)$

- for ~~small-amplitude~~ noise, $\mathcal{F}(v) \approx v$
  $\Rightarrow$ in uniform regions, this is approximately the linear low-pass filter of Example 1.

Edge-preserving noise removal:
- near edges, turn the filter off
- away from edges, smooth out the noise

(Note that this filter can be locally low-pass, high-pass, or all pass, depending on the local intensity of the image - i.e., we are doing different things to the same frequency, depending on where we are ~~in~~ the image - something that cannot be done with a linear space-invariant filter.)

If one filtering operation is insufficient, repeat this iteratively. Denote this filter by $\mathcal{A}$: $g = \mathcal{A}(f)$.

Iterate: $g^{(k)} = \underbrace{\mathcal{A}(\mathcal{A}(\mathcal{A}(\cdots\mathcal{A}}_{k \text{ times}}(f)\cdots)))$

$$g^{(0)} = f \longrightarrow \boxed{\mathcal{A}} \xrightarrow{g^{(1)}} \boxed{\mathcal{A}} \xrightarrow{g^{(2)}} \cdots \longrightarrow \boxed{\mathcal{A}} \xrightarrow{g^{(k)}}$$

$$g^{(i+1)}(m,n) = g^{(i)}(m,n) + \alpha \Big\{ \mathcal{F}\big[g^{(i)}(m-1,n) - g^{(i)}(m,n)\big] + \mathcal{F}\big[g^{(i)}(m+1,n) - g^{(i)}(m,n)\big] + \mathcal{F}\big[g^{(i)}(m,n-1) - g^{(i)}(m,n)\big] + \mathcal{F}\big[g^{(i)}(m,n+1) - g^{(i)}(m,n)\big] \Big\}$$

3.3 Multiscale Analysis.

Consider a slightly modified version of the homework problem on random walks.

Example 1 (random walk in 1-D).

A person wonders along a straight line. Every second, he either stays put, or moves 1 meter east, or moves 1 meter west, with probabilities:

$$\boxed{\begin{aligned} &P(\text{remain}) = 1-2\alpha \\ &P(\text{move 1 meter east}) = \alpha \\ &P(\text{move 1 meter west}) = \alpha, \qquad 0 \le \alpha \le \tfrac{1}{2} \end{aligned}}$$

His step is statistically independent of his past. ~~steps~~

Let

$g^{(i)}(n)$ = Prob. that, after i seconds, he is at the point n.

Then

$$g^{(i+1)}(n) = g^{(i)}(n-1)\cdot\alpha + g^{(i)}(n)\cdot(1-2\alpha) + g^{(i)}(n+1)\cdot\alpha$$

$$= g^{(i)}(n) + \alpha\left\{\left[g^{(i)}(n-1) - g^{(i)}(n)\right] + \left[g^{(i)}(n+1) - g^{(i)}(n)\right]\right\}$$

This is a 1-D averaging filter.

The pmf at time $i+1$ is a low-pass filtered version of the pmf at time i.

Example 1a. Suppose now he "moves" every Δt seconds, and either stays put, or moves by $\pm \Delta x$.

Let $u(t,x)$ = Prob. that, after t seconds, he is at the point x.

Then

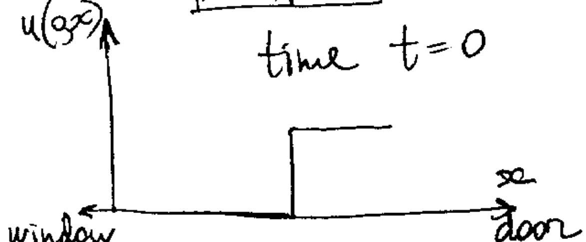
$$u(t+\Delta t, x) - u(t,x) = \alpha\left\{u(t,x+\Delta x) - 2u(t,x) + u(t,x-\Delta x)\right\}$$

$$\frac{u(t+\Delta t,x) - u(t,x)}{\Delta t} = \frac{\alpha(\Delta x)^2}{\Delta t}\left\{\frac{u(t,x+\Delta x) - 2u(t,x) + u(t,x-\Delta x)}{(\Delta x)^2}\right\}$$

Now let $\Delta t \to 0$, $\Delta x \to 0$, $\frac{\alpha(\Delta x)^2}{\Delta t} = 1$.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{- linear diffusion equation in 1-D.}$$

Concentration of particles in the room:



time $t=0$

Remove the partition, start the diffusion process: