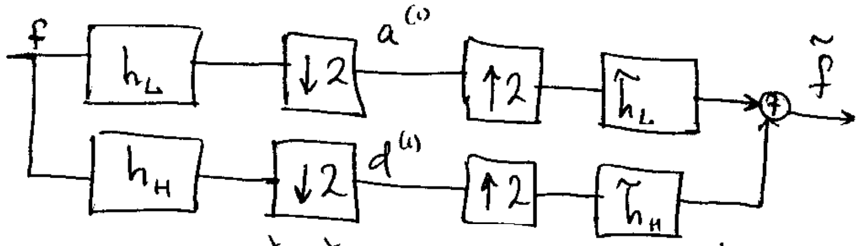
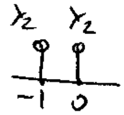
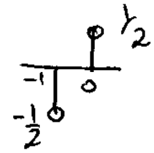
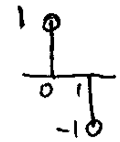
3.4 Perfect Reconstruction Filter Banks and Wavelets.

Lec 39 12/03/01



where $h_L =$  $\tilde{h}_L =$ 

$h_H =$  $\tilde{h}_H =$ 

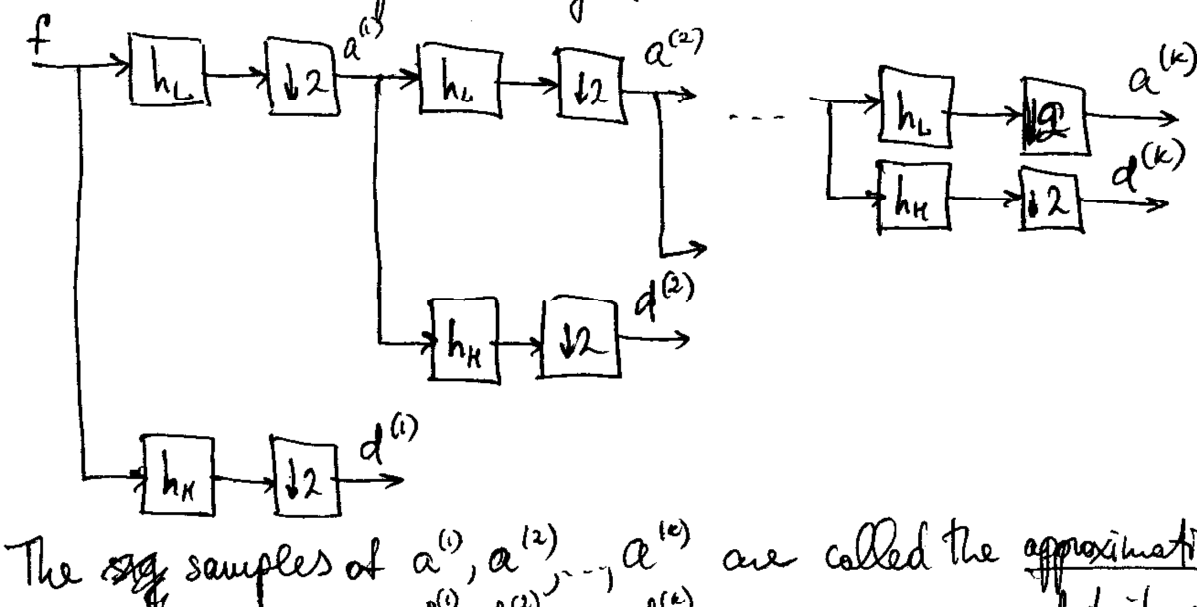
Perfect reconstruction filter bank.

In general, it can be shown that

$f = \tilde{f}$ for all inputs f iff

$$\left.\begin{aligned} H_L(\omega+\pi)\tilde{H}_L(\omega) + H_H(\omega+\pi)\tilde{H}_H(\omega) &= 0 \\ \text{and } H_L(\omega)\tilde{H}_L(\omega) + H_H(\omega)\tilde{H}_H(\omega) &= 2 \end{aligned}\right\} \text{perfect reconstruction conditions.}$$

Iterate the decomposition stage:



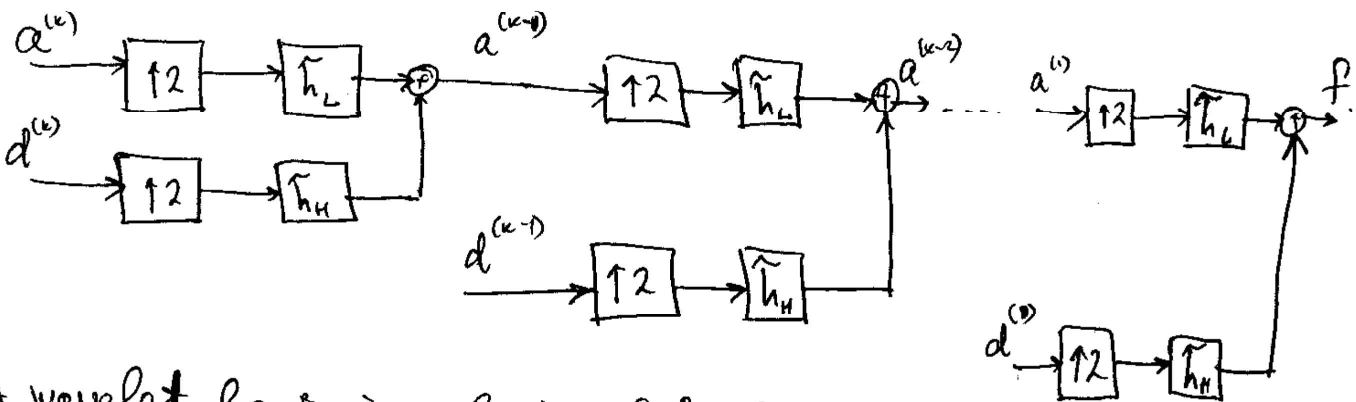
The samples of $a^{(1)}, a^{(2)}, \dots, a^{(k)}$ are called the approximation coeff.

" " $d^{(1)}, d^{(2)}, \dots, d^{(k)}$ " " " detail or wavelet coefficients.

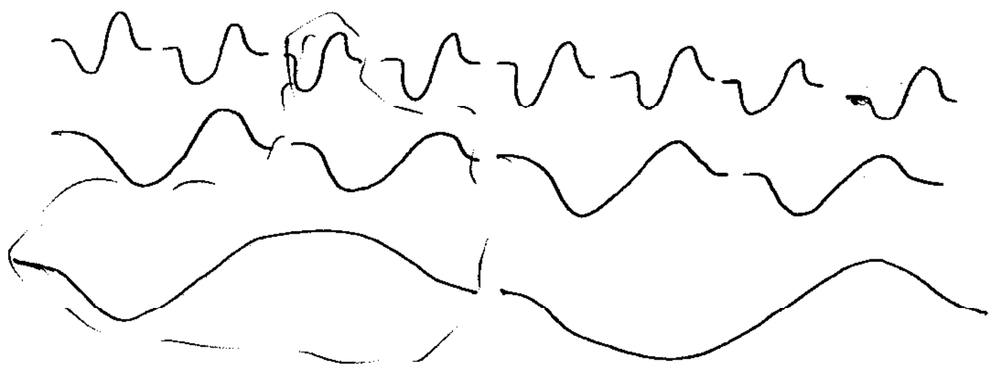
If there are $\tilde{h}_L, \tilde{h}_H$ such that $h_L, h_H, \tilde{h}_L, \tilde{h}_H$ satisfy the perfect reconstruction conditions, then we can reconstruct f

from the coarsest-scale approximation coefficients and the detail coefficients, ②

$$\{d^{(1)}, d^{(2)}, \dots, d^{(k)}, a^{(k)}\}:$$



A <u>wavelet basis</u> is a basis of localized functions which are scaled and translated versions of one function, e.g.



E.g., if

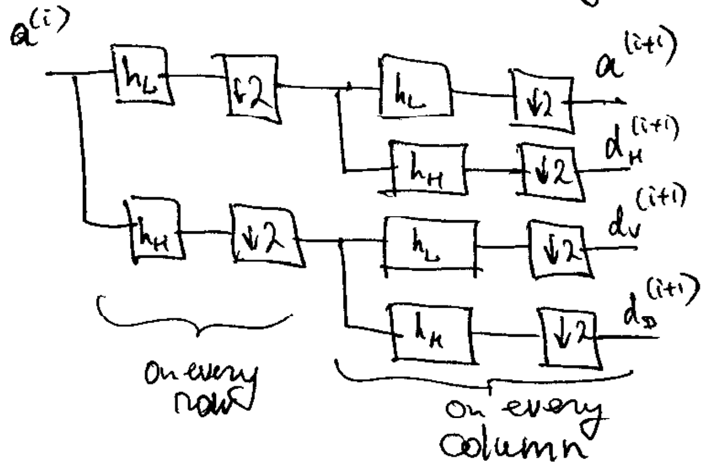
$f =$ 

<u>all</u> Fourier coefficients are large (because of the sharp transition), but only a few wavelet coefficients are large. In the representation

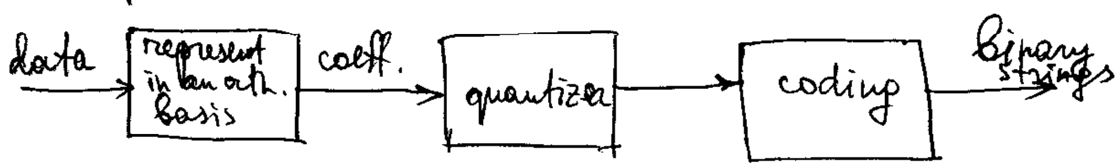
$$f = \sum \alpha_k g_k,$$

- if g_k's are sinusoids, then <u>all</u> g_k's are "responsible" for the discontinuity
- if g_k's are wavelets, then only a few g_k's in the vicinity of the discontinuity are "responsible" for it.

For images, filter every row, and then every column:



3.4.1 Compression:



Advantages of wavelets:

- for piecewise smooth signals and images, many coefficients will be small and therefore quantized to zero.
- possibility of embedded coding due to the multiresolution nature of the Basis

3.4.2 Noise Removal via Thresholding (HW 15)