ECE 438 Homework 12, due in class Friday, 12/03/2004.

Problem 1. Short-time Fourier transform.
Compute the short-time Fourier transform

$$X(m, e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)w(n-m)e^{-j\omega n}$$

for the signal

$$x(n) = a^n u(n)$$

and an N-point rectangular window:

$$w(n) = u(n) - u(n-N).$$

Here, a is a fixed positive real number, N is a fixed positive integer, and $u(n)$ is the discrete-time unit step function.

Problem 2. Random variables as vectors.
Let X, Y, Z be zero-mean, unit-variance, real-valued random variables which satisfy

$$Var(X + Y + Z) = 0.$$

Find the covariance matrix of X, Y, Z, i.e., find the matrix

$$\begin{pmatrix} E[X^2] & E[XY] & E[XZ] \\ E[YX] & E[Y^2] & E[YZ] \\ E[ZX] & E[ZY] & E[Z^2] \end{pmatrix}.$$

Problem 3. AR models and linear prediction.
Consider the following causal system:

$$S(n) = 0.5S(n-1) + X(n),$$

where the input signal $X(n)$ is a zero-mean white noise sequence with variance σ^2. Suppose we would like to design a third-order linear predictor for the output process $S(n)$:

$$\hat{S}(n) = a_1 S(n-1) + a_2 S(n-2) + a_3 S(n-3).$$

Find the coefficients a_1, a_2, and a_3 which minimize the mean square error, $E[(S(n) - \hat{S}(n))^2]$.

Problem 4. In Fig. 1, all the images are 20×20 discrete-space, continuous-valued, grayscale images. Each pixel can have values from 0 (black) to 3 (white). The image in the top row depicts a light square on a dark background. Within the square, the image has a constant value of 2; within the background, it has a constant value of 1. (Warning: if you print this out, the image may look somewhat grainy and non-constant on your printout because of imperfections of printing and copying devices.) Each of the six images in the second and third rows was obtained by modifying the square image in the top row. The objective of this problem is to determine what operations resulted in the six images.

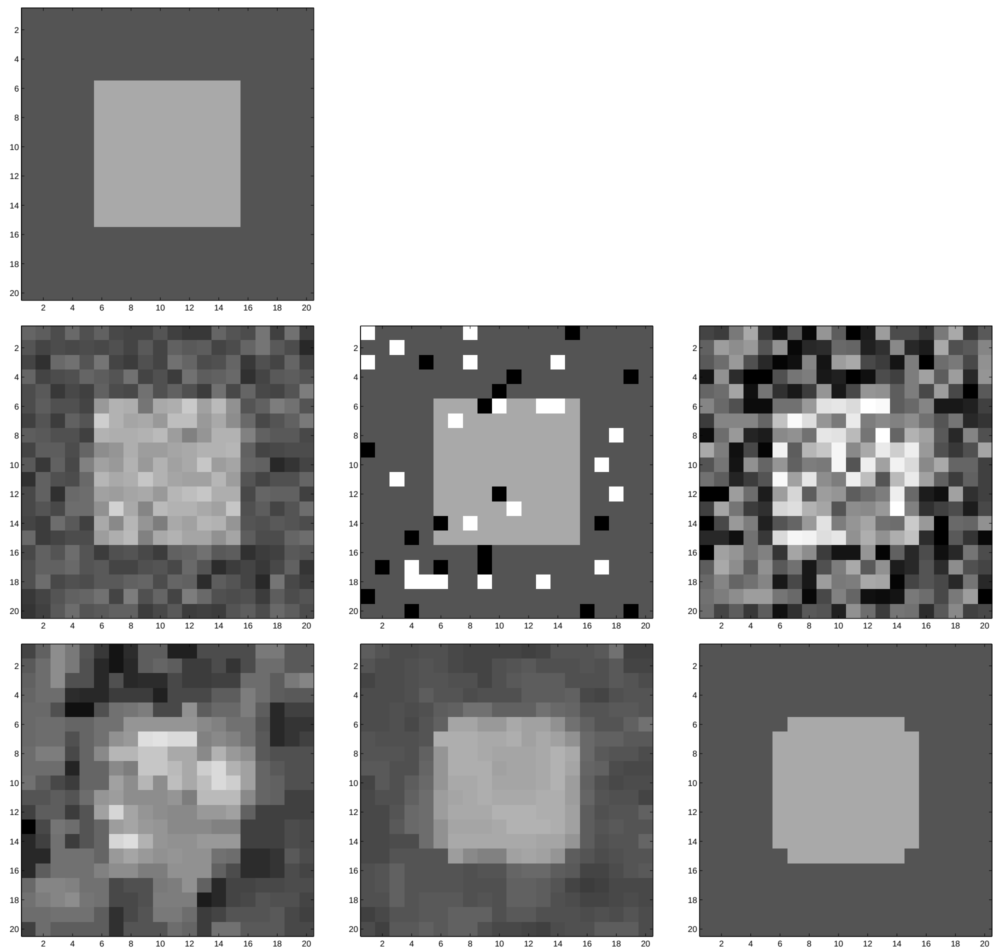

Figure 1: Images for Problem 4.

(a) One of the images in the second row was obtained from the square image in the top row by adding white zero-mean noise with marginal density which is uniform between -1 and 1. One of the images in the second row was obtained from the square image in the top row by adding white zero-mean Gaussian noise with standard deviation 0.2. One of the images in the second row was obtained by corrupting the square image in the top row with "occlusive" noise: each pixel value stayed the same with probability 0.9, was replaced by 0 with probability 0.05, and was replaced by 3 with probability 0.05. Indicate which image in the second row corresponds to each type of noise noise (uniform, Gaussian, occlusive), and fully explain your answers.

(b) The three images in the third row were obtained by applying a median filter with a 3×3 window to the three images in the second row. Indicate which image in the third row corresponds to each image in the second row, and fully explain your answers.

Problem 5.

(a) Let $f_1(m, n)$ and $h(m, n)$ be 3×3 images defined as follows:

$f_1(m,n) =$

1	2	3
2	3	4
3	4	5

$h(m,n) =$

0	1	0
1	-4	1
0	1	0

Assume that in both cases, the location of the central pixel is $(0,0)$, and assume that $f_1(m,n) = h(m,n) = 0$ when $|m| \geq 2$ or $|n| \geq 2$. Calculate $g_1(m,n) = f_1 * *h(m,n)$, the 2-D convolution of f_1 and h. Calculate only the values for the nine pixels of $g_1(m,n)$ for which $|m| \leq 1$ and $|n| \leq 1$.

(b) Calculate the 2-D convolution of $h(m,n)$ defined in Part (a) and the image $f_2(m,n) = m^2$ defined for all pairs of integers m and n.

Note that the filter with impulse response $h(m,n)$ can be used as a numerical approximation to a certain differentiation operation. Precisely describe this operation.

(c) Find the 2-D discrete-space Fourier transform of $h(m,n)$ defined in Part (a), make the mesh plot of its absolute value using Matlab, and determine whether a filter with impulse response $h(m,n)$ is a lowpass, a highpass, or a bandpass filter.