

ECE 438
Homework 2, due in class Friday, 9/3/2004.

Problem 1. Each discrete-time (DT) system below is defined by an input-output relation (the response to the input signal x is the signal y). For each system, determine whether or not it is:

- (i) linear,
- (ii) time-invariant,
- (iii) causal,
- (iv) BIBO stable.

For each of the above properties, if you think it holds, prove it. Otherwise, find a counter-example. In addition, find the impulse response (i.e., the response of the system to the unit impulse $\delta(n)$).

- (a) $y(n) = (n + 1) \cdot x(n - 1)$ for all integer n .
- (b) $y(n) = x(n + 1) \cdot x(n) \cdot x(n - 1)$ for all integer n .
- (c) $y(n) = x(2n)$ for all integer n .

Problem 2. For each of the following signals, determine whether or not it is periodic, and if it is, find its period.

- (a) $\sin(3n)$.
- (b) $\cos(\pi n/2) + 0.2 \sin(17\pi n/31)$.
- (c) $(-1)^n$.
- (d) $(-2)^n$.

Problem 3. For all parts of this problem, S is a DT system whose response to the unit step signal $u(n)$ is the following signal: $(n + 1)u(n)$.

- (a) If the system S is LTI, find its impulse response.
- (b) If the impulse response of the system S is $\delta(n)$, can S be an LTI system?
- (c) If the impulse response of the system S is $\delta(n)$, can S be BIBO stable?

- (d) If the system S is LTI, can it be noncausal?

Fully justify your answers.

Problem 4. For each of the following signals, determine whether or not the signal

- (i) is a finite duration signal,
- (ii) is bounded,
- (iii) is absolutely summable,
- (iv) has finite energy.

(a)

$$x(n) = \begin{cases} \frac{1}{n}, & n \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Hint. To solve (iii) and (iv) for this signal, try to compare $\sum_{n=1}^{\infty} \frac{1}{n}$ to $\int_1^{\infty} \frac{1}{t} dt$ and compare

$$\sum_{n=2}^{\infty} \frac{1}{n^2} \text{ to } \int_1^{\infty} \frac{1}{t^2} dt.$$

- (b) $x(n) = \sin(\pi n)$.
- (c) $x(n) = 2^{-n}(u(n) - u(n - 10))$.
- (d) $x(n) = 2^{-n}$.

Fully justify your answers.

Problem 5.

- (a) Can an LTI system with bounded impulse response be BIBO unstable?
- (b) Can an LTI system whose impulse response has finite energy be BIBO unstable? **Hint.** Use your solution to Problem 4(a).
- (c) Can a system whose impulse response is $u(n)$ be BIBO stable? Can it be both BIBO stable and LTI?
- (d) Can a system whose impulse response is $\delta(n)$ be BIBO unstable? Can it be both BIBO unstable and LTI?

Fully justify your answers.

Problem 6. Suppose the impulse response of an LTI system is $h(n) = 2^{-n}u(n)$. Find its responses to the following input signals:

(a) $x(n) = u(n) - u(n - 3)$.

(b) $x(n) = 2^n u(-n)$.