# ECE 438.
# Homework 6, due in class Friday, 10/1/2004.

**Problem 1.** Consider sampling the signal below (which is defined for all time, $-\infty < t < \infty$).

$$x_c(t) = \frac{\sin(2\pi \cdot 3000 \cdot t)}{\pi t} \cdot \cos(2\pi \cdot 1500 \cdot t)$$

(a) $x_c(t)$ is sampled at a rate $f_s = 24$ KHz to produce the discrete-time signal $x(n) = x_c(\frac{n}{f_s})$. Plot the magnitude of the DTFT of $x(n)$, $|X(e^{j\omega})|$, over the interval $-\pi < \omega < \pi$. Show as much detail as possible.

(b) What is the Nyquist sampling rate for this signal?

**Problem 2.** The continuous-time signal

$$x_c(t) = \sin(20\pi t) + \cos(40\pi t)$$

is sampled with a sampling period $T_s$ to obtain the discrete-time signal

$$x(n) = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right).$$

(a) Determine a choice of $T_s$ consistent with this information.

(b) Is your choice for $T_s$ in part (a) unique? If so, explain why. If not, specify another choice of $T_s$ consistent with the information given.

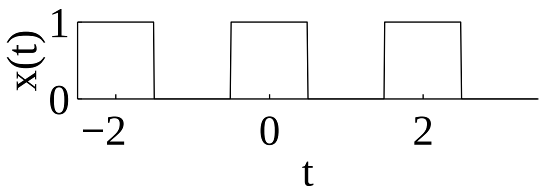



Figure 1: Signal $x(t)$ for Problem 3.

**Problem 3.** Consider the following continuous-time signal illustrated in Fig. 1:

$$x(t) = \begin{cases} 1, & \text{if } 2n - 0.5 \le t < 2n + 0.5, \quad \text{for any integer } n, \\ 0, & \text{if } 2n - 1.5 \le t < 2n - 0.5, \quad \text{for any integer } n. \end{cases}$$

Suppose this signal is ideally sampled (with no prefiltering) at a rate of 1 Hz, to obtain a discrete-time signal $x_d(n)$. The latter is then filtered with a nonlinear discrete-time filter specified by the following input-output relationship:

$$y_d(n) = (x_d(n))^2,$$

where $y_d(n)$ is the response of the filter to the input $x_d(n)$.

(a) Find and plot $x_d(n)$. (**Hint**. Do not do this in the frequency domain. Just recall the definition of sampling and write down $x_d(n)$ directly.)

(b) Find and plot $y_d(n)$.

(c) Suppose we subtract 0.5 from $y_d(n)$ to obtain another discrete-time signal $y(n)$:

$$y(n) = y_d(n) - 0.5.$$

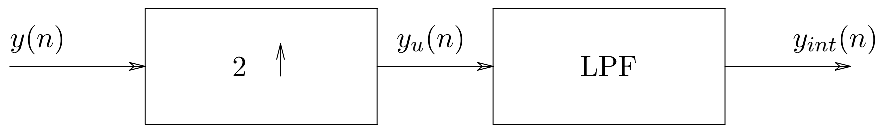


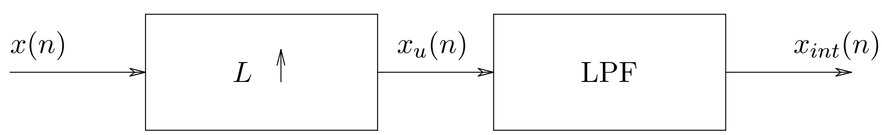

Figure 2: Interpolator for Problem 3c.

Suppose further $y_{int}(n)$ is the result of interpolating $y(n)$ using the scheme illustrated in Fig. 2: first, $y(n)$ is upsampled by a factor of 2 to result in $y_u(n)$; and then $y_u(n)$ is filtered with an ideal discrete-time low-pass filter. For this problem, we assume the following frequency response for the ideal low-pass filter:

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{2\pi}{3} \\ 0, & \frac{2\pi}{3} < |\omega| \le \pi \end{cases}$$

Find $y_{int}(n)$.

**Problem 4.** DT INTERPOLATION WITH SINC FUNCTIONS.
In class, we considered the following system for interpolating a DT signal:




In this system, upsampling by a factor of $L$ is followed by an ideal lowpass filter. For this problem, we assume the following frequency response for the ideal lowpass filter:

$$H(e^{j\omega}) = \begin{cases} L, & |\omega| \le \frac{\pi}{L} \\ 0, & \frac{\pi}{L} < |\omega| \le \pi \end{cases}$$

(a) Use the inverse DTFT formula to show that the impulse response of the ideal lowpass filter is $h(n) = \text{sinc}\left(\frac{n}{L}\right)$.
Argue that therefore interpolating a DT signal $x(n)$ can be represented as a convolution of the upsampled version of $x(n)$ with a sinc:

$$\begin{aligned} x_{int}(n) &= x_u * h(n), \qquad (1) \\ \text{where } h(n) &= \text{sinc}\left(\frac{n}{L}\right), \end{aligned}$$

and where $L$ is the upsampling factor. Therefore, interpolation is achieved by summing up scaled and shifted sinc functions.

(b) In MATLAB, create the following vector of time points from 0 to 32 seconds:

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t = 0:0.05:32
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Using this vector, create a sinusoid

$$s(t) = \sin\left(\frac{\pi}{8}t\right).$$

Create another signal, $s_1(n)$, which is the sampling of $s(t)$ with sampling period 1 second. Create $s_2(n)$, which is the sampling of $s(t)$ with sampling period 2 seconds.
Now interpolate $s_2(n)$, using Eq. (1) with $L = 2$.
Step 1. Upsample $s_2(n)$ by inserting a zero after each sample.
Step 2. Using MATLAB's *sinc* function, create $h(n)$:

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h=sinc([-A:A]/L);
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where A is some positive integer. Make sure that the value of $A$ is large enough, so that the smallest side lobe of the sinc is small compared to the main lobe.
Step 3. Using MATLAB's *conv* function, convolve the upsampled version of $s_2(n)$ with $h(n)$, to get $s_{2int}(n)$.
Step 4. Truncate $s_{2int}(n)$, to make its first sample correspond to $s(t)|_{t=0}$, and its last sample correspond to $s(t)|_{t=32}$.

Using *subplot*, display a plot of $s(t)$ and stem plots of $s_1(n)$, $s_2(n)$, and $s_{2int}(n)$ in one window. Label the horizontal axes correctly. Make sure that $s_1(n)$ and $s_{2int}(n)$ are similar. Submit your MATLAB code and the printout of the plots. (Use *orient tall* before you print.)

(c) Repeat part (b), with the same vector $t$ and

$$s(t) = \begin{cases} 1, & 8 \le t \le 24 \\ 0, & \text{otherwise.} \end{cases}$$

Again, submit a plot of $s(t)$ and stem plots of $s_1(n)$, $s_2(n)$, and $s_{2int}(n)$, as well as your MATLAB code. Can you notice any dissimilarities between $s_{2int}(n)$ and $s_1(n)$? If so, write a very brief explanation for these dissimilarities.

**Problem 5.** DT INTERPOLATION WITH BOX FUNCTIONS.
One alternative to the scheme considered in Problem 4 would be to convolve with a box function (instead of convolving with a sinc), namely, with

$$h(n) = \begin{cases} 1, & n = 0, 1, \ldots, L-1 \\ 0, & \text{otherwise} \end{cases}$$

Repeat Parts (b) and (c) of Problem 4 for this new $h(n)$. You only need to submit the stem plots for the two new signals $s_{2int}(n)$. Comment on the differences between the results you get and your results from Problem 4.

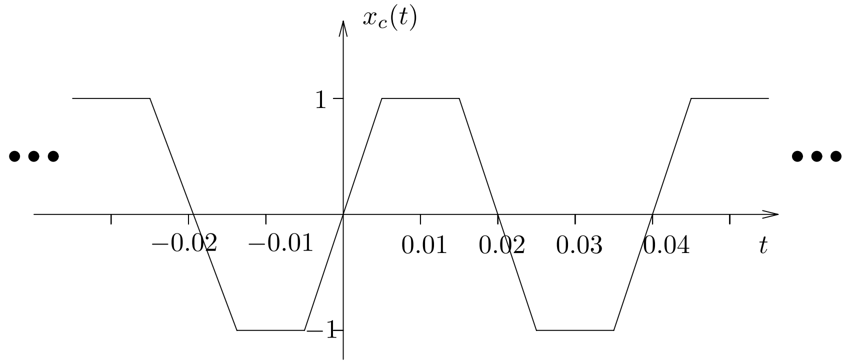


Figure 3: Illustration for Problem 6.

**Problem 6.** The CT signal $x_c(t)$ depicted in Fig. 3 is periodic with fundamental period 0.04. This signal is ideally sampled at 100 Hz (without prefiltering) to obtain a DT signal $x(n)$.

(a) Sketch $x(n)$. Carefully label the axes.

(b) Find such coefficients $X_0$, $X_1$, $X_2$, and $X_3$ that

$$x(n) = X_0 + X_1 e^{\frac{j\pi n}{2}} + X_2 e^{j\pi n} + X_3 e^{\frac{j3\pi n}{2}} \text{ for all integer } n.$$

(c) Suppose that a CT signal $y(t)$ is reconstructed from $x(n)$ using the ideal scheme considered in class. Specifically, $x(n)$ is converted into a CT sequence of impulses,

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(n)\delta\left(t - \frac{n}{f_s}\right),$$

and then $x_s(t)$ is filtered with an ideal low-pass filter whose frequency response is:

$$H(f) = \begin{cases} A, & -\frac{f_s}{2} \le f \le \frac{f_s}{2} \\ 0, & \text{otherwise,} \end{cases}$$

where $f_s = 100$ Hz and $A$ is a constant such that $y(0.01) = 1$. Find $y(t)$ and sketch it.