ECE 438 Homework 7, due in class Friday, 10/15/2004.

Problem 1. Prove the following z-transform properties:

1. Linearity:
$$a_1 x_1(n) + a_2 x_2(n) \leftrightarrow a_1 X_1(z) + a_2 X_2(z).$$
2. Delay:
$$x(n - n_0) \leftrightarrow z^{-n_0} X(z).$$
3. Modulation:
$$z_0^n x(n) \leftrightarrow X\left(\frac{z}{z_0}\right)$$
4. Differentiation in z-domain:
$$nx(n) \leftrightarrow -z\frac{dX}{dz}$$
Hint. Differentiate $\sum x(n)z^{-n}$ termwise with respect to z, and then multiply the result by $-z$.
5. Convolution:
$$x * y(n) \leftrightarrow X(z)Y(z)$$
Hint. Assume that you can change the order of summations in
$$\sum_{n=-\infty}^{\infty}\left(\sum_{k=-\infty}^{\infty} x(k)y(n-k)\right) z^{-n}.$$

Problem 2. For each of the signals below, find the z-transform, specify the region of convergence of the z-transform, and sketch its poles and zeros in the complex plane.

(a) $\delta(n)$.

(b) $a^n u(n)$, $a \neq 0$.

(c) $-a^n u(-n-1)$, $a \neq 0$.

(d) $na^n u(n)$, $a \neq 0$.
Hint. Apply Property 4 form Problem 1 to Part (b).

(e) $-na^n u(-n-1)$, $a \neq 0$.

Problem 3. Consider the z-transform

$$X(z) = \frac{-2.5z}{z^2 - 3.5z + 1.5}$$

Sketch the three different ROC's that are possible for this z-transform. For each ROC, find the corresponding signal $x(n)$.

Problem 4.

(a) An LTI, causal system has the following transfer function:

$$\frac{1 - \frac{1}{2}z^{-1}}{(1 - 16z^{-1})(1 + 0.3z^{-1})(1 - 7z^{-1})}$$

Can this system be BIBO stable?

(b) An LTI, BIBO stable system has the following transfer function:

$$\frac{438 + 2001z^{-1}}{(1 - 438z^{-1})(1 + z^{-1} + 8z^{-2} + 19z^{-3} + 23z^{-4})}$$

Can this system be causal?

(c) A BIBO stable system has the following impulse response:

$$h(n) = 1, \quad \text{for } -\infty < n < \infty.$$

Can this system be LTI?

Problem 5. An LTI system has the following transfer function:

$$\frac{2.96 - z^{-1}}{(1 - 0.2z^{-1})(1 - 0.1z^{-1})} \qquad \text{ROC: } |z| > 0.2.$$

Find the response of this system to the following input signal:

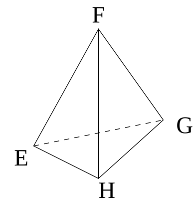
$$x(n) = \left(\frac{1}{2}\right)^n, \quad -\infty < n < \infty.$$

Hint. The easiest way is to use the fact that exponentials are eigenfunctions of LTI systems.

Problem 6. Suppose the z-transform of $x(n)$ is:

$$X(z) = e^z; \text{ region of convergence is the whole } z\text{-plane.}$$

Find $x(n)$.
(**Hint.** The function $X(z)$ possesses a Taylor series about the origin which converges to $X(z)$ for all z.)



Problem 7. The face EGH of the tetrahedron FEGH is painted in three colors: red, green, and blue. The face EFH is painted red. The face HFG is painted green. The face GFE is painted blue. Define the following events:
$A_r = \{$a face picked at random has red on it$\}$
$A_g = \{$a face picked at random has green on it$\}$
$A_b = \{$a face picked at random has blue on it$\}$
Are A_r, A_g, A_b pairwise independent? Are they independent?