ECE 438 Homework 9
due Friday 10/29/2004 at 1:30pm in the VISE Lab MSEE 184.

Note: there will be no class on Friday 10/29. Dan will be collecting the homeworks during his office hour in MSEE 184.

Problem 1. Let $X(n)$ be a wide-sense stationary sequence of random variables with zero mean and autocorrelation function

$$r_{XX}(n) = \begin{cases} 1, & |n| \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Suppose this sequence is filtered to generate the output sequence

$$Y(n) = \frac{1}{3}(X(n) + X(n-1) + X(n-2))$$

(a) Find the mean of the sequence $Y(n)$.
(b) Find the cross-correlation $c_{XY}(n)$ between the sequences $X(n)$ and $Y(n)$.
(c) Find the autocorrelation $r_{YY}(n)$ of the sequence $Y(n)$.

Problem 2. Consider the following random sequence:

$$X(n) = \cos(\omega_0 n + \Theta),$$

where ω_0 is a non-random constant, and Θ is a random variable, uniformly distributed between 0 and 2π:

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta \leq 2\pi \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute the mean sequence, $E[X(n)]$.
(b) Compute the autocorrelation sequence, $r_{XX}(m, n)$.
(c) Is the sequence $X(n)$ wide-sense stationary?

Problem 3. A number n of independent experiments are performed to estimate the probability of an event A as n_A/n where n_A is the number of occurrences of A in the n experiments. Is this an unbiased estimate of $P(A)$ (i.e., is $E[n_A/n] = P(A)$)? Is this a consistent estimate of $P(A)$ (i.e., is $\lim_{n\to\infty} Var(n_A/n) \to 0$)? **Hint**. Use the fact that $n_A = \sum_{i=1}^{n} X_i$, where

$$X_i = \begin{cases} 1, & \text{if A occurred in the } i\text{-th experiment} \\ 0, & \text{otherwise.} \end{cases}$$

Problem 4. Suppose that $X(1), X(2), \ldots$ is a sequence of independent, identically distributed random variables, each of which has pdf $f(x)$. Let

$$Y(n) = \frac{1}{n}\sum_{i=1}^{n} v(2 - X(i)),$$

where $v(t)$ is the continuous-time unit step:

$$v(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0. \end{cases}$$

(a) Express in words the meaning of $Y(n)$. (**Hint**. The random variable $nY(n)$ is the number of occurrences of a certain event.)

(b) Find $E[v(2 - X(i))]$ in terms of $f(x)$.

(c) Find $E[Y(n)]$ in terms of $f(x)$. Is $Y(n)$ an unbiased estimator of the probability of the event $\{X(n) \leq 2\}$?

(d) Find $Var[Y(n)]$ in terms of $f(x)$. What is the limit of $Var[Y(n)]$ as $n \to \infty$?

(e) For this part, let $f(x)$ be the uniform density, between 0 and 10:

$$f(x) = \begin{cases} 0.1, & 0 \leq x \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

Use the MATLAB command *rand* to generate 2000 samples of the random sequence $X(n)$. (**Hint**. This command produces numbers which are uniformly distributed between 0 and 1. In order to get the desired sequence, you will need to scale the output of *rand*.) Compute 2000 samples of the random sequence $Y(n)$. Turn in your code and a plot of $Y(n)$ for $1 \leq n \leq 2000$. Suppose your friend generated another realization of $X(n)$, and calculated the corresponding $Y(n)$. Your task is to pick n_0, $1 \leq n_0 \leq 2000$ (*without* looking at your friend's sequence $Y(n)$), such that $|Y(n_0) - 0.2| < 0.09$. If you are right, he pays you \$10; if not, you pay him \$10. What n_0 should you choose to maximize your chances of winning, and why? Is it a fair bet?