

CE 559 Water Quality Modeling - Exam 2 Spring 2009

(Example Problems)

1. Dissolved Oxygen Balance in a Stream. Write the general mass balance equation for D.O. Deficit (D) in a stream, including *all* source and sink terms discussed in class. Express it as a differential equation for deficit ($\partial D/\partial t$). Define all terms. Define all terms and tell whether each term is a source or sink of D.
2. The stream entering a lake (volume of lake = 1000 m³; flow in and out = 50 m³/day) supplies the lake with a steady concentration of alachlor (the herbicide Lasso, made by Monsanto). In this lake, alachlor biodegrades with a pseudo-first-order rate constant of 0.0023/hr. Write the mass balance equation on alachlor in the lake (dC/dt), and solve for the steady state concentration within the lake, in terms of C_{in}.
3. The stream entering a lake supplies the lake with a steady concentration of chemical B after t=0. In this lake, B biodegrades with a pseudo-first-order rate constant of k. Write the mass balance equation on B in the lake (dB/dt), and solve for the **non-steady state** concentration within the lake, in terms of B_o, B_{in}, k, Q, and V.

3a. Defining 95% steady-state as:

$$0.95 = (B - B_o)/(B_{ss} - B_o)$$

Calculate the time it takes to reach 95% steady-state in term of k, V, and Q (or tau) for the lake in problem

4. Compound A is discharged into a stream at point x = 0. Once in the stream, it reacts to form chemical B that in turn reacts to form chemical C. Both reactions are first-order decay reactions.
 - a. Write the time variable equation on A in the stream assuming plug flow with first order decay.
 - b. Solve for A assuming steady-state conditions apply. If the mean stream velocity (μ) is .3 m/sec, and the decay constant, k₁, is 1.0/day, what percentage of A remains 20,000 m down stream.
 - c. Compound A produces compound B, which also decays by a first order process. Write the time variable equation on B in the stream assuming plug flow with first order decay (constant = k₂) and production.
 - d. Under steady-state conditions, where (x =) does the maximum concentration of B occur, in terms of A_o, B, k₁, k₂, and μ .
5. Diagram and label the Lewis-Whitman 2-film theory of gas transfer, and label all parts.

Derive:

$$K_L = \frac{1}{\left(\frac{1}{k_l} + \frac{1}{H k_g} \right)}$$

6. In the summer, Lake A has an epilimnion and hypolimnion. Inflow and outflow from a stream occurs to the epilimnion. Only dispersive transport occurs between the two compartments, and each can be considered completely mixed. Chemical B enters the lake at a constant rate from the stream and decays within both compartments of the lake with the same pseudo-first-order rate constant. Right the 2 ODE that define the temporal mass balance of B in each compartment, assuming constant volume of each compartment, constant flow, and constant cross-sectional area between the two compartments. Write these equations explicitly in terms of the dispersion coefficient (D, m²/day) and cross-sectional area.
7. Define the following terms: pelagic zone, euphotic zone, hypolimnion, epilimnion, littoral, diatoms, phytoplankton.
8. Derive the Streeter-Phelps equation (D =) starting with the mass balance around a control volume, realizing that L = L_o·exp(-k_rt)
9. Derive the equation for the temporal change in dissolved O₂ in a BOD bottle (O₂ =). Variables in your final equation should be [O₂]_o, k, and L_o.
10. For the Streeter Phelps approach, solve for the Critical Deficit (D_c).

5. Define the optimum weighting factor for the y-axis variable for the following relationships, assuming that the variance on all values of C are nearly identical (Be sure to show your work!).
- a) $\ln C = f(t)$ where t is the independent variable
 - b) $1/C = f(t)$ where t is the independent variable
 - c) $\ln (C - C_0) = f(t)$ where t is the independent variable and C_0 is a constant with a variance of 2 %.