Quality Analysis of Bundle Block Adjustment with Navigation Data

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ABSTRACT: Precision and reliability are two basic factors for quality analysis of a geodetic or photogrammetric adjustment system. Simulated studies were performed to determine the characteristics of the precision and the reliability of a bundle block adjustment with navigation data. Simulated navigation data with various levels of precision were processed in the bundle block adjustment. This investigation is helpful to the practical application of aerotriangulation with navigation data such as GPS data and/or INS data.

INTRODUCTION

With the fast development of navigation systems, such as the Global Positioning System (GPS) and the Inertial Navigation System (INS) in the 1980s, photogrammetrists are again becoming highly interested in aerotriangulation with navigation data. Ackermann (1984) obtained results in aerotriangulation with a precision of 0.5 m in planimetry, after deleting the linear systematic errors in the raw navigation data. Camera stations with a precision of 0.1 m were simulated with two GPS receivers, one carried on the airplane and the other at a known geodetic station on the ground. The precision of point determination within decimeters, both in planimetry and in elevation, was obtained by bundle block adjustment without ground control (Schwarz, 1984; Lucas, 1986). At the 1986 Symposium of ISPRS Comm. III in Rovaniemi, a simulated thorough study was presented on the practical precision of aerotriangulation with navigation data (Ackermann, 1986; Friess, 1986). Nevertheless, theoretical precision, and especially, reliability still needed to be further investigated.

With respect to the above studies, both the precision and reliability of bundle block adjustment with navigation data were more thoroughly and theoretically investigated with the aid of the computation of **Qxx** and **QvvP**. This investigation demonstrates again the potential applicability of aerotriangulation with navigation data.

TEST DESIGN

Tests were performed with simulated data. The block was made up of ten strips with 21 photos in each strip and nine image points per photo. Other parameters are listed below.

number of object points	441
focal length, f	152mm
photo scale,	1:60,000
relative average flight height, H	approx. 9000m
forward overlap	60%
side overlap	20%
average difference in elevation	approx. 150m
orientation angles	approx. 0
photograph format	23cm by 23cm
root mean square error (RMSE) of image points	5 μm
RMSE of control points	0.1m

The navigation data with various levels of precision were chosen, while considering the 'matching' between the precision of the camera stations and of the orientation angles, i.e.,

$$\sigma_\alpha = \frac{\sigma_s}{H} \cdot \rho'' \qquad (1)$$

where

σ_α and σ_s are the RMSE of the orientation angles and the camera stations, respectively;

H is the relative average flight height; and

$\rho'' = 206264.8$.

The chosen precision of navigation data is shown in Table 1. The test design of navigation data is composed of different precision combinations of the camera station positions and orientation angles.

The distribution of the ground control is illustrated in Figure 1. Version (a) (not shown) refers to no control points. Versions (a) and (b) are for block adjustment with navigation data, while versions (c) and (d) are designed for the conventional block adjustment without navigation data.

The computation was performed with a newly developed combined adjustment program system known as WUCAPS. For the quality analysis, the matrices **Qxx** and **QvvP** are computed by this program system.

Quality analysis includes two aspects, i.e., precision analysis and reliability analysis. They are both discussed below.

THE PRECISION ANALYSIS

Generally, the covariance matrix of object points

$$\mathbf{D}(X) = \sigma_0^2\, \mathbf{Qxx} \qquad (2)$$

TABLE 1. PRECISION OF NAVIGATION DATA IN SIMULATED COMPUTATION

σ_s (m)	0.1	1.0	3.0	6.0	10.0	∞
σ_α (sec)	2.3	22.9	68.8	137.5	229.2	∞

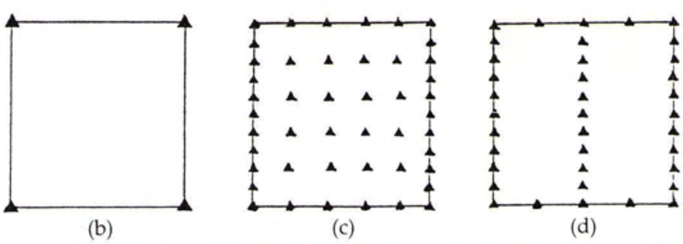

FIG. 1. Distribution of ground control. Version (a), not illustrated, contains no ground control. Versions (a) and (b) are for block adjustment with navigation data, while versions (c) and (d) are designed for the conventional block adjustment without navigation data.

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TABLE 2. AVERAGE PRECISION OF ADJUSTMENT WITH NAVIGATION DATA ($\overline{\mu}|\overline{m}$, σ_o AS UNIT)

<table>
<tr><td>precision σ_s (m)</td><td></td><td colspan="2">0.1</td><td colspan="2">1.0</td><td colspan="2">3.0</td><td colspan="2">6.0</td><td colspan="2">10.0</td><td colspan="2">∝</td></tr>
<tr><td>σ_α (″)</td><td></td><td>X,Y</td><td>Z</td><td>X,Y</td><td>Z</td><td>X,Y</td><td>Z</td><td>X,Y</td><td>Z</td><td>X,Y</td><td>Z</td><td>X,Y</td><td>Z</td></tr>
<tr><td>2.3</td><td>(a)</td><td>0.6|0.6</td><td>1.1|1.1</td><td>0.8|0.8</td><td>1.2|1.3</td><td></td><td></td><td></td><td></td><td>2.2|3.0</td><td>1.2|2.7</td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td>0.6|0.6</td><td>1.1|1.1</td><td>0.8|</td><td>1.2|</td><td>1.0|1.0</td><td>1.3|1.5</td><td>1.0|</td><td>1.5|</td><td>1.0|</td><td>1.7|</td><td>1.0|1.2</td><td>1.8| 1.9</td></tr>
<tr><td>22.9</td><td>(a)</td><td></td><td></td><td>1.0|1.2</td><td>1.6|1.8</td><td>1.3|1.8</td><td>1.9|2.4</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td>0.8|</td><td>1.2|</td><td>1.0|</td><td>1.6|</td><td>1.4|</td><td>1.8|</td><td>2.0|</td><td>1.9|</td><td>2.6|</td><td>1.9|</td><td>3.5|</td><td>1.9|</td></tr>
<tr><td>68.8</td><td>(a)</td><td></td><td></td><td>1.2|1.3</td><td>1.8|2.0</td><td>1.6|2.2</td><td>2.8|3.1</td><td></td><td></td><td>2.9|4.4</td><td>3.7|5.3</td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td>0.8|0.8</td><td>1.2|1.3</td><td>1.1|</td><td>1.8|</td><td>1.5|1.8</td><td>2.8|2.9</td><td>2.2|</td><td>3.2|</td><td>3.4|3.0</td><td>3.3|4.6</td><td>7.4|</td><td>3.6|</td></tr>
<tr><td>137.5</td><td>(a)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td>0.8|</td><td>1.2|</td><td>1.1|</td><td>1.8|</td><td>1.4|</td><td>3.1|</td><td>2.2|</td><td>4.0|</td><td>3.6|</td><td>4.3</td><td>9.2|</td><td>6.5|</td></tr>
<tr><td>229.2</td><td>(a)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>3.2|4.8</td><td>6.1|7.0</td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td>0.8|</td><td>1.2|</td><td>1.1|</td><td>1.8|</td><td>1.5|1.8</td><td>3.3|3.1</td><td>2.2|</td><td>4.4|</td><td>3.6|</td><td>5.1|</td><td>9.8|5.2</td><td>12. |14.</td></tr>
<tr><td>α</td><td>(a)</td><td>1.0|0.9</td><td>1.3|1.4</td><td>1.2|1.3</td><td>1.9|2.0</td><td>1.9|2.2</td><td>3.4|3.4</td><td></td><td></td><td>3.2|5.0</td><td>7.8|7.7</td><td>0.9|0.8</td><td>1.7| 2.0</td></tr>
<tr><td></td><td>(b)</td><td>0.8|0.8</td><td>1.2|1.3</td><td>1.1|</td><td>1.9|</td><td>1.5|</td><td>3.4|</td><td>2.3|</td><td>5.0|</td><td>3.8|3.2</td><td>6.8|6.6</td><td>1.0|0.9</td><td>2.2| 2.5</td></tr>
</table>

is taken as the measure of theoretical precision. In Equation 2, **Qxx** is the cofactor matrix of the object points, and σ_o is the RMSE of the observations with unit weight.

The theoretical precision in one coordinate direction of the *i*th object point is measured by

$$m_i = \sigma_o \sqrt{(\mathbf{Qxx})ii} \tag{3}$$

The average theoretical precision of *n* object points is

$$\overline{m} = \sigma_o \sqrt{\mathrm{tr}(\mathbf{Qxx})/3n}. \tag{4}$$

For simulated data the practical average precision of object points is represented by

$$\overline{\mu} = \sqrt{\sum(\Delta X^2 + \Delta Y^2 + \Delta Z^2)/3n} \tag{5}$$

where ΔX, ΔY, ΔZ are differences between the adjusted and simulated coordinates of an object point.

Similarly, $\overline{m}_x$, $\overline{m}_y$, $\overline{m}_z$, and $\overline{\mu}_x$, $\overline{\mu}_y$, $\overline{\mu}_z$ are defined to describe the average precision of object points in the *X*, *Y*, *Z* directions respectively.

The theoretical and practical average precisions are listed in Table 2, where lines (a) and (b) indicate the ground control version shown in Figure 1. The figures at the lower right corner of Table 2 were obtained by the conventional bundle block adjustement without navigation data.

The following precision characteristics can be derived from an analysis of Table 2.

(1) If the precision of camera station positions is 1.0m or better, the adjusted coordinates, especially the elevation, are better than those obtained from conventional bundle block adjustment, no matter whether or not ground control or orientation angles are available. The planimetry and elevation precision is 0.6 to 1.1 σ_o and 1.1 to 1.9 σ_o, respectively.

It is very interesting that the adjusted object points are more precise than the navigation data with which the adjustment is performed. That means the precision of point determination can reach submetre level, if GPS can supply the camera stations with an RMSE less than 1.0m.

(2) The same precision as (1) can be reached with orientation angles, but only if they are known to better than several seconds. This shows that the camera station positions are more dominant than the orientation angles for aerotriangulation with navigation data.

(3) In general cases, the use of orientation angles improves planimetric precision to only a limited extent, but it is very effective for precision improvement in elevation. Only when the precision of the camera station positions is rather poor, can planimetric precision be raised with the aid of highly precise orientation angles. Thus, the addition of orientation angles to aerotriangulation is basically beneficial to the improvement of the precision in elevation.

(4) The precision distribution of the object point field in a bundle block adjustment with navigation data is demonstrated in Figure 2, from which we conclude the following:

- For aerotriangulation with navigation data, precision is worst both at corners and edges of the blocks, especially for those blocks without ground control. Therefore, the flight strategy of extra strips and photos in the direction of side overlap and forward overlap around the block is recommended.
- The precision of the adjusted object points in the interior of the block is quite homogeneous, both in planimetry and in elevation.
- Compared with a conventional adjustment without navigation data, the elevation precision in a block adjustment with navigation data is obviously more homogeneous, while the planimetric precision has the same homogeneity. This means that, in an aerotriangulation with navigation data, there is no need for elevation control points in the interior of the block at all.

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A. √Qxy , version b                    B. √Qxy , version a

0.5 - 1.7 - 1.7 - 1.9 - 2.0 -          3.6 - 2.7 - 2.3 - 2.1 - 2.1 -
 |     |     |     |     |              |     |     |     |     |
1.5 - 1.1 - 1.2 - 1.2 - 1.2 -          2.3 - 1.6 - 1.4 - 1.3 - 1.3 -
 |     |     |     |     |              |     |     |     |     |
1.4 - 1.1 - 1.0 - 0.9 - 1.0 -          1.9 - 1.4 - 1.1 - 1.0 - 1.0 -
 |     |     |     |     |              |     |     |     |     |
1.7 - 1.2 - 1.1 - 1.0 - 1.0 -          1.8 - 1.2 - 1.1 - 1.0 - 1.0 -
 |     |     |     |     |              |     |     |     |     |
1.7 - 1.2 - 1.0 - 0.9 - 0.8 -          1.7 - 1.2 - 1.0 - 0.9 - 0.8 -
 |     |     |     |     |              |     |     |     |     |
1.8 - 1.2 - 1.0 - 1.0 - 1.0 -          1.8 - 1.2 - 1.0 - 1.0 - 1.0 -
 |     |     |     |     |              |     |     |     |     |
1.7 - 1.2 - 0.9 - 0.8 - 0.8 -          1.7 - 1.2 - 0.9 - 0.8 - 0.8 -
 |     |     |     |     |              |     |     |     |     |

C. √Qz , version b                     D. √Qz , version a

0.3 - 1.8 - 1.7 - 1.8 - 1.8 -          3.4 - 2.5 - 2.1 - 2.0 - 1.9 -
 |     |     |     |     |              |     |     |     |     |
2.5 - 1.5 - 1.4 - 1.5 - 1.6 -          2.7 - 1.9 - 1.7 - 1.5 - 1.6 -
 |     |     |     |     |              |     |     |     |     |
1.4 - 1.0 - 0.8 - 0.8 - 0.8 -          2.0 - 1.4 - 1.0 - 0.9 - 0.8 -
 |     |     |     |     |              |     |     |     |     |
2.4 - 1.4 - 1.3 - 1.3 - 1.2 -          2.5 - 1.4 - 1.3 - 1.3 - 1.2 -
 |     |     |     |     |              |     |     |     |     |
1.7 - 1.1 - 0.8 - 0.7 - 0.7 -          1.7 - 1.1 - 0.9 - 0.7 - 0.7 -
 |     |     |     |     |              |     |     |     |     |
2.5 - 1.4 - 1.3 - 1.3 - 1.3 -          2.5 - 1.4 - 1.3 - 1.3 - 1.3 -
 |     |     |     |     |              |     |     |     |     |
1.7 - 1.1 - 0.8 - 0.7 - 0.7 -          1.7 - 1.1 - 0.8 - 0.7 - 0.7 -
 |     |     |     |     |              |     |     |     |     |
```

FIG. 2. Weight coefficients of adjusted object points (upper left corner of the block, σ_s = 0.1 m, σ_α ∝).

TABLE 3. THE AVERAGE LOCAL REDUNDANCIES OF VARIOUS OBSERVATIONS

σ_s		0.1 m		0.1 m		1 m		3 m		10 m		10 m		∞	
σ_α		2."3		∞		22".9		68".8		68",8		68".8		∞	
Version		(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(c)	(d)
(1)	$\bar{r}_x$	0.49	0.49	0.39	0.40	0.31	0.31	0.28	0.28	0.28	0.28	0.27	0.27	0.29	0.29
	$\bar{r}_y$	0.59	0.60	0.51	0.51	0.39	0.39	0.35	0.35	0.34	0.34	0.33	0.33	0.36	0.36
(2)	$\bar{r}_s$	0.22	0.22	0.09	0.09	0.79	0.79	0.92	0.93	0.98	0.98	0.96	0.97	-	-
	$\bar{r}_\alpha$	0.34	0.34	-	-	0.87	0.88	0.96	0.97	0.94	0.95	-	-	-	-
(3)	$\bar{r}_X$	-	0.15	-	0.04	-	0.02	-	0.004	-	0.002	-	0.001	0.017	0.015
	$\bar{r}_Y$	-	0.14	-	0.03	-	0.02	-	0.004	-	0.001	-	0.001	0.017	0.015
	$\bar{r}_Z$	-	0.08	-	0.03	-	0.01	-	0.003	-	0.002	-	0.001	0.006	0.005

(1) Image corrdinate observations. (2) Navigation data observations. (3) Control point observations.

THE RELIABILITY ANALYSIS

The concept of reliability includes internal reliability and external reliability. Internal reliability refers to the ability to discover blunders in one particular observation. The effect of an undected blunder in the observation on unknown parameters is measured by external reliability.

For the analysis of reliability, the local redundancy

$$r_i = (\mathbf{Qvv}^*\mathbf{P})_{ii} \qquad (6)$$

needs to be computed, where **Qvv** is the cofactor matrix of vector **V**, the residuals of the observations; and **P** is the weight matrix of observations.

Generally, only one single blunder is assumed. The internal and external reliability are expressed with the following two formulas, respectively (Förstner, 1983,; Li, 1986):

$$\nabla_o L_i = \sigma_{L_i} \cdot \frac{\delta_o}{\sqrt{r_i}} = \sigma_{L_i} \cdot \delta_{o,i} \qquad (7)$$

$$\bar{\delta}_{o,i} = \delta_o \cdot \sqrt{\frac{\bar{r}_i}{r_i}} = \sqrt{1-r_i} \cdot \delta_{o,i} \qquad (8)$$

in which

σ_{L_i} is the RMSE of the ith observation L_i;

δ_o is the non-centrality parameter;

$\delta_{o,i} = \delta_o/\sqrt{r_i}$ and is the internal reliability factor of the observation L_i;

$\nabla_o L_i$ is the minimum blunder which can be discovered statistically; and

$\bar{\delta}_{o,i}$ is the external reliability factor of observation L_i.

With the significant level α = 0.1 percent and the test power β = 80 percent, the non-centrality parameter δ_o is then equal to 4.13.

The average local redundancy is accepted as a measure of the overall reliability of one particular group of observations. It is defined as

$$\bar{r}_k = \sum_{j=1}^{nk} r_j/n_k \qquad (9)$$

where n_k is the number of the kth group of observations, and r_j is the local redundancy of the jth observation in the kth group.

The average local redundancies of various kinds of observations are listed in Table 3. Based on an analysis of Table 3, the following conclusions are drawn:

(1) In an aerotriangulation with navigation data, the average reliability of image points and navigation data would not be affected practically by ground control points at the corners of the block (see column (a) and (b) in Table 3). Therefore, in view of the reliability, ground control points would not necessarily be needed in an aerotriangulation with navigation data.

(2) In an aerotriangulation with navigation data, the average reliability of image points is generally not less than that in the conventional adjustment without navigation data. With more precise navigation data, the image points become more reliable (see Figure 3). If the navigation data reach even higher precision, the reliability of the image points can be improved quite efficiently, whereas it would not be affected by control points in the conventional adjustment without navigation data.

(3) The more precise the navigation data are, the smaller their local redundancies become. But, compared with the poor reliability of the control points ($\bar{r}$ = 0.02 to 0.20) in the conventional adjustment, the reliability of navigation data ($\bar{r}$ = 0.2 to 0.9) is absolutely much better and more homogeneous over the entire block. While it is well known that the reliability of ground control points in a conventional adjustment without nacigation data is dependent on their locations in the block (see Figure 4). Therefore, the poor reliability of the ground control points in a conventional adjustment can be thorougly avoided by the replacement of the ground control points with navigation data in the aerotriangulation.

(4) Blunders in navigation data, which cannot be found statistically, do not significantly affect the adjusted object points

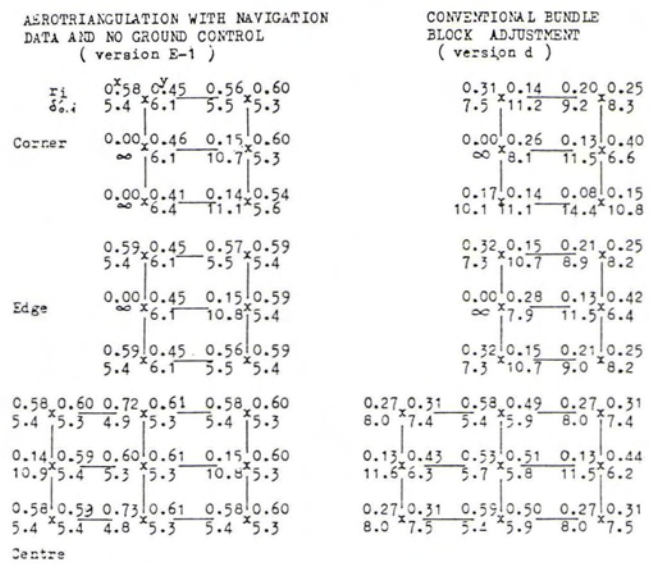

FIG. 3. Local redundancy and reliability factor of image points.

TABLE 4. EXTERNAL RELIABILITY OF OBSERVATIONS

σ_s / σ_α		0.1 m / 2".3		0.1 m / ∞		1 m / 22".9		3 m / 68".8		10 m / 68".8		10 m / ∞		∞ / ∞	
Version		(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(c)	(d)
(1)	$\bar{\delta}_x$	4.2	4.2	5.1	5.1	6.1	6.2	6.6	6.6	6.7	6.6	6.8	6.7	6.3	6.4
	$\bar{\delta}_y$	3.4	3.3	4.1	4.1	5.1	5.2	5.7	5.7	5.8	5.8	5.9	5.9	5.5	5.6
(2)	$\bar{\delta}_s$	7.8	7.8	13.0	13.0	2.2	2.1	1.2	1.2	0.6	0.6	0.8	0.8	-	-
	$\bar{\delta}_\alpha$	5.8	5.8	-	-	1.6	1.5	0.8	0.8	1.0	0.8	-	-	-	-
(3)	$\bar{\delta}_{XY}$	-	10.2	-	23.2	-	32.5	-	64.3	-	106.0	-	123.6	30.6	33.4
	$\bar{\delta}_Z$	-	13.9	-	24.7	-	37.0	-	70.5	-	106.5	-	152.8	51.1	60.5

(1) Image coordinate observations
(2) Navigation data observations
(3) Control point observations

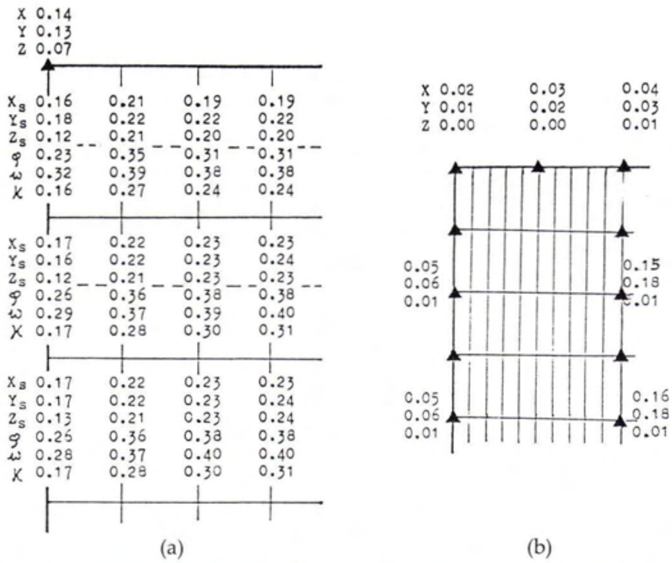

FIG. 4. Reliability comparison of the block adjustment with and without navigation data. (a) Local redundancy of navigation data in the block adjustment with navigation data (σ_α = 2".3, σ_s = 0.1m). (b) Local redundancy of control points in the block adjustment with ground control (ground control version (d)).

(see Table 4). The external reliability of navigation data is almost always much better than that of the ground control points at edges of the block in the conventional adjustment. The Z-coordinates of the ground control points in a conventional adjustment are always less reliable than are the navigation data.

SUMMARY AND CONCLUSIONS

From the above simulated computation and the quality analysis, the following conclusions can be summarized:

(1) With aerial photographs of the proper scale, aerotriangulation is applicable to mapping at scales ranging from 1:100,000 to 1:10,000 if navigation systems can supply camera stations with a precision of around 1m. The precision of the adjusted object points is higher than that of the camera stations.

(2) In aerotriangulation with navigation data, the camera stations are usually dominant. The elevation precision of the adjusted object points can be improved significantly with the aid of orientation angles, when the camera stations are not precise enough.

(3) Compared with a conventional adjustment without navigation data, the elevation precision in a block adjustment with navigation data is obviously more homogeneous, while the planimetric precision retains the same homogeneity. This means that there is no need for elevation control points in the interior of the block at all, if the navigation data are available.

(4) The reliability of a bundle block adjustment with navigation data is quite good. Compared with the reliability of image points in a conventional adjustment, the reliability of image points in an aerotriangulation with navigation data is at the same level or even better. The navigation data are always much more reliable than ground control points in a conventional adjustment. This shows the practical potential of aerotriangulation with navigation data to replace the coventional adjustment with ground control.

We have gained experience in the use of the statoscope in medium- and small-scale mapping. Recent tests (Ackermann, 1988) demonstrated that GPS data can supply camera stations with a precision of several centimetres. This indicates the great potential of aerotriangulation with navigation data in the widely practical uses. The practicality is confirmed both in precision and in reliability through the quality analysis in this paper.

Systematic errors in navigation data are unavoidable. Therefore, the characteristics of systematic errors in navigation data, especially their separability from systematic errors in image points and in ground control points, should be investigated further.

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