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Photogrammetric object description with projective invariants

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Abstract

Projective invariants are helpful for image understanding and object recognition. It is shown that the *cross-ratio of volumes* is the general projective invariant. Consequently, it is proposed as a projective independent description of the object. In order to obtain this description from image observables, a stereopair of the scene is utilized to constitute a 3-D projective model, and a computational algorithm is developed based on a proper decomposition of the fundamental matrix. Numerical computation with real images is given to validate and evaluate the development and algorithm.

Keywords: projective invariant; cross-ratio; object description; image understanding

1. Introduction

An object and its photographic images are traditionally described by Cartesian coordinates with respect to a chosen frame. Although this description is very suitable for positioning and its related calculation, generally it does not *explicitly* show any characteristics of an object after it is imaged. Obviously, this is not beneficial for image understanding and object recognition. The purpose of this article is to construct a projective independent description for an object, which can be fully calculated with its image coordinates.

In photogrammetry and computer vision, there are basically three kinds of genetic elements, i.e., image coordinates, object points and orientation parameters, which may include intrinsic camera unknowns, relative and exterior parameters. A relationship between *only two* types of those elements will constitute an invariant, in which the third class of element disappears. For the purpose of object recognition, the most useful invariant is the IO-type (Image-Object) invariant where orientation parameters are eliminated (Barrett et al., 1995).

The well-known cross-ratio of *signed distances* among four points in a straight line is a 1-D to 1-D IO-type projective invariant (cf. Duda and Hart, 1973). For a 2-D to 2-D projective transformation, namely the object is planar or part of it in question is planar, the projective invariant is the cross-ratio of *signed triangular areas* among five points. Although this fact might not be well-known, it can be inferred from the discussion of Barakat et al. (1994). However, for the 3-D to 2-D projective transformation, which describes the real imaging procedure in photogrammetry and computer vision, no *general* IO-type invariant exists (Weiss, 1993). Therefore, it is of interest to find the so-called model-based invariants, which can only be derived based on some prerequisites or a-priori knowledge of the object to

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be described and recognized (Weiss, 1993; Weinshall, 1993).

As a theoretical generalization, Brill and Barrett (1983) extended the projective transformation to the n-D to n-D case, where it was shown that the *cross-ratio of signed volumes* is invariant. Analogous to the IO-type invariants in 1-D to 1-D and 2-D to 2-D projective transformations, here we define the invariant in 3-D to 3-D projective transformation as the MO-type (Model-Object) invariant, since the 3-D projective transformation could be considered as a *projective model* of the object. Unfortunately, we can not *physically* obtain a 3-D projective model through the photographic procedure; therefore, an algorithm needs to be developed to make this invariant computationally available.

Many recent literatures discuss the computation of projective invariants. The proceedings edited by Mundy et al. (see the source of Barrett et al., 1994) is a comprehensive collection of this topic. Among other things, Barrett et al. (1994) extended their earlier work by using simplified notations and algebraically eliminating orientation parameters from the imaging equations. Preliminary examples for building reconstruction based on projective invariants are given by Barrett et al. (1995). They also extended this method to the image transfer between SAR (Synthetic Aperture Radar) images and photographic images (Barrett and Payton, 1993; cf. also Barakat et al., 1994).

This article characterises the discussion by introducing and, more importantly, decomposing the fundamental matrix of a stereopair. In this way a scalar factor — the length ratio of two conjugate projective rays — is obtained. After the projective invariance of the 3-D cross-ratio is shown, a proper definition of the 3-D projective model then makes available its reconstruction and the calculation of the 3-D cross-ratio from image observables.

The remaining part of this article is organised as follows. Section 2 starts the discussion by introducing the fundamental matrix of a stereopair. Through the decomposition of this matrix, the necessary quantity for computing the MO-type invariant can be obtained. After introducing the 3-D to 3-D projective transformation, Section 3 proves that the *cross-ratio of volumes* is a general projective invariant. Thus, three independent MO-type invariants are defined as *projective coordinates* to describe the object. The reconstruction of a *projective model* for the object in Section 4 makes the computation of the MO-type invariants available from the image observables of a stereopair. This development and algorithm are validated and evaluated in Section 5, with numerical examples of an aerial stereopair. Concluding remarks are given in Section 6.

2. Fundamental matrix

This section will present a brief introduction to the fundamental matrix and thereafter decompose it to obtain the necessary quantities for computing the MO-type invariant.

As a generalization of the essential matrix (Thompson, 1968; Longuet-Higgins, 1981), the fundamental matrix corresponds to the coplanarity condition when the interior orientation is unknown (Faugeras, 1992; Hartley, 1992; Barrett et al., 1995; Shan, 1996):

$$\bar{\mathbf{x}}_{\mathrm{L}}^{T}\bar{E}\bar{\mathbf{x}}_{\mathrm{R}} = 0 \tag{1}$$

where

$$\bar{\mathbf{x}}_{\mathrm{L}} = (\bar{x}_{\mathrm{L}}\,\bar{y}_{\mathrm{L}}\;1)^{\mathrm{T}} \quad \bar{\mathbf{x}}_{\mathrm{R}} = (\bar{x}_{\mathrm{R}}\,\bar{y}_{\mathrm{R}}\;1)^{\mathrm{T}} \tag{2}$$

are homogeneous coordinates of the left and right image in a stereopair, respectively. $(\bar{x}_{\mathrm{L}}\,\bar{y}_{\mathrm{L}})$ and $(\bar{x}_{\mathrm{R}}\,\bar{y}_{\mathrm{R}})$ are left and right image observables in an *oblique* image coordinate system. In Eq. 1 the 3×3 matrix $\bar{E}$ is called the *fundamental matrix* (Faugeras, 1992). It could be *linearly* determined up to a scalar factor with at least *eight* image correspondences (Faugeras, 1992; Hartley, 1992; Barakat et al., 1994).

Decomposing the fundamental matrix will produce the quantities needed for computing the projective invariants defined in next section. As is shown in Shan (1996), the ratios among affine base components $\bar{B}_X$, $\bar{B}_Y$, $\bar{B}_Z$ can be determined by the $\bar{E}$ matrix:

$$\frac{\bar{B}_Y}{\bar{B}_X} = \frac{\bar{e}_{13}\bar{e}_{32} - \bar{e}_{12}\bar{e}_{33}}{\bar{e}_{22}\bar{e}_{33} - \bar{e}_{23}\bar{e}_{32}} \quad \frac{\bar{B}_Z}{\bar{B}_X} = \frac{\bar{e}_{12}\bar{e}_{23} - \bar{e}_{13}\bar{e}_{22}}{\bar{e}_{22}\bar{e}_{33} - \bar{e}_{23}\bar{e}_{32}} \tag{3}$$

where $\bar{e}_{ij}$ are components of the matrix $\bar{E}$. The length ratio $\bar{\lambda}_{\mathrm{R}}/\bar{\lambda}_{\mathrm{L}}$ of two conjugate projective rays meets the relation (Shan, 1996):

$$\frac{\bar{\lambda}_{\mathrm{R}}}{\bar{\lambda}_{\mathrm{L}}}\bar{E}\bar{\mathbf{x}}_{\mathrm{R}} = \bar{B}\bar{\mathbf{x}}_{\mathrm{L}} \tag{4}$$

where

$$\overline{B} = \begin{pmatrix} 0 & -\overline{B}_Z & \overline{B}_Y \\ \overline{B}_Z & 0 & -\overline{B}_X \\ -\overline{B}_Y & \overline{B}_X & 0 \end{pmatrix} \tag{5}$$

Thus, we can obtain the least squares solution of the length ratio $\overline{k}$ from:

$$\overline{k} = \frac{\overline{\lambda}_R}{\overline{\lambda}_L} = \frac{(\overline{E}\overline{\mathbf{x}}_R)^T(\overline{B}\overline{\mathbf{x}}_L)}{(\overline{E}\overline{\mathbf{x}}_R)^T(\overline{E}\overline{\mathbf{x}}_R)} \tag{6}$$

In summary, through decomposing the fundamental matrix, the two ratios of the three affine base components in Eq. 3, and the length ratio of the two conjugate projective rays in Eq. 6 can be obtained.

3. 3-D cross-ratios and projective coordinates

After the 3-D to 3-D projective transformation is introduced, this section generalizes the well-known cross-ratio from 1-D to 3-D. The projective coordinates are then introduced and defined as a measure of the MO-type projective invariant for object description.

A 3-D to 3-D projective transformation from object to its *model* can be expressed as (cf. Brill and Barrett, 1983):

$$\begin{aligned} U' &= \frac{a_{11}U + a_{12}V + a_{13}W + a_{14}}{a_{41}U + a_{42}V + a_{43}W + a_{44}} \\ V' &= \frac{a_{21}U + a_{22}V + a_{23}W + a_{24}}{a_{41}U + a_{42}V + a_{43}W + a_{44}} \\ W' &= \frac{a_{31}U + a_{32}V + a_{33}W + a_{34}}{a_{41}U + a_{42}V + a_{43}W + a_{44}} \end{aligned} \tag{7}$$

In terms of homogeneous coordinates, Eq. 7 is equivalently expressed as:

$$t\overline{\mathbf{u}}' = A\overline{\mathbf{u}} \tag{8}$$

where

$$\overline{\mathbf{u}} = (U\ V\ W\ 1)^T \quad \overline{\mathbf{u}}' = (U'\ V'\ W'\ 1)^T \tag{9}$$

are homogeneous coordinates of a point in object and model systems, respectively. $A = \{a_{ij}\}$ is the 4×4 transformation matrix, $t \neq 0$ is a scalar factor relevant to this object point. Comparing Eq. 7 with Eq. 8 reveals the fact that a *projective* transformation of non-homogeneous coordinates can be expressed as a *linear* transformation in terms of the corresponding homogeneous coordinates.

Analogous to the 1-D case, where the cross-ratio is referred to the signed distances among $1+3=4$ points (cf. Fig. 1), the volume cross-ratio in 3-D is

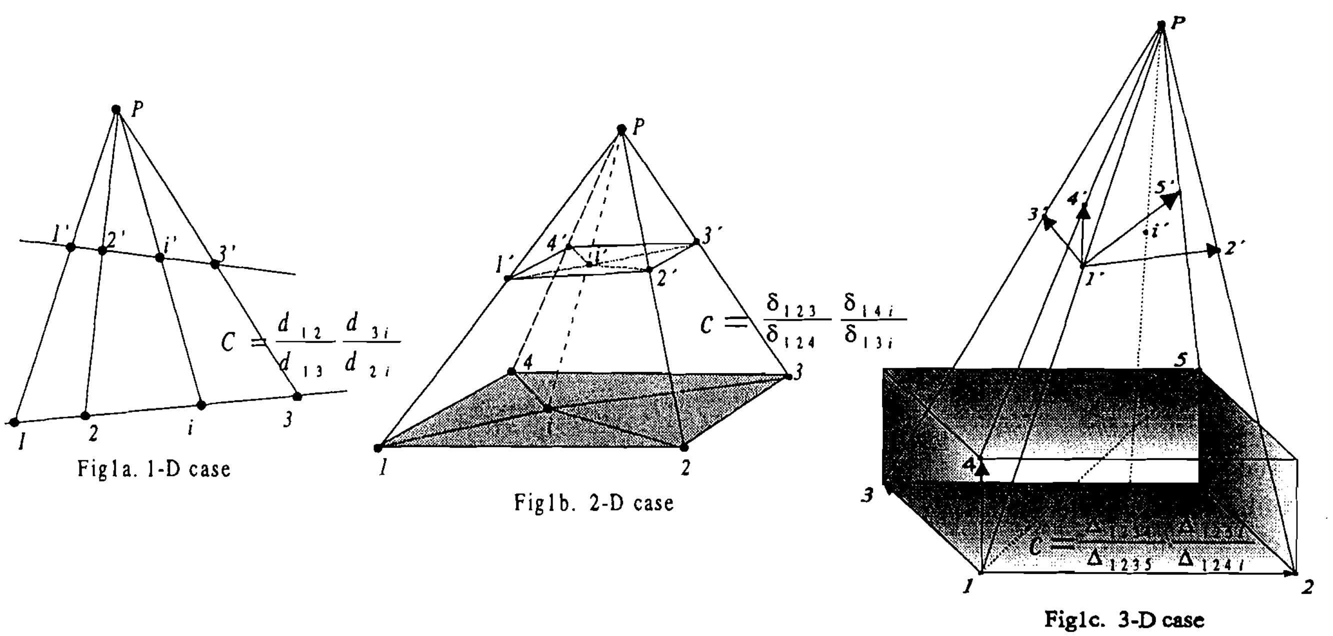

Fig. 1. Perspective transformation and cross-ratios. There are 3, 4, 5 basis points in 1-D, 2-D and 3-D perspective transformations, respectively. *d*, *δ*, *Δ* are distance, area and volume elements, respectively. *C* is the cross-ratio for each case.

defined among $3+3=6$ points as (cf. Brill and Barrett, 1983):

$$C(1,2,3,4,5,i)=\frac{\Delta_{1234}\,\Delta_{125i}}{\Delta_{1235}\,\Delta_{124i}} \tag{10}$$

where Δ_{1234} is the *volume* of the tetrahedron formed by points 1, 2, 3 and 4, i.e.,

$$\Delta_{1234}=\begin{vmatrix} U_1 & U_2 & U_3 & U_4 \\ V_1 & V_2 & V_3 & V_4 \\ W_1 & W_2 & W_3 & W_4 \\ 1 & 1 & 1 & 1 \end{vmatrix}=|\bar{\mathbf{u}}_1\,\bar{\mathbf{u}}_2\,\bar{\mathbf{u}}_3\,\bar{\mathbf{u}}_4| \tag{11}$$

For other quantities in Eq. 10, similar expressions can be written. In this definition, the indices 1 to 5 stand for the chosen basis points. The meaning of *volume* here is generalized, as it might be proportional to the true volume by a common factor. In this sense, the *cross-ratio of volumes* is equivalent to the term *cross-ratio of determinants* appearing in Barrett et al. (1994, 1995). For a conceptual comparison, Fig. 1 gives an illustration of the perspective transformations and cross-ratios in 1-D, 2-D and 3-D.

Next, it can be shown (Brill and Barrett, 1983) that the cross-ratio defined in Eq. 10 is invariant under a projective transformation, i.e.,

$$C(1,2,3,4,5,i)=\frac{\Delta_{1234}\Delta_{125i}}{\Delta_{1235}\Delta_{124i}}=\frac{\Delta'_{1234}\Delta'_{125i}}{\Delta'_{1235}\Delta'_{124i}}$$
$$=C'(1,2,3,4,5,i) \tag{12}$$

This can be seen by substituting the relation

$$\Delta'_{1234}=|\bar{\mathbf{u}}'_1\,\bar{\mathbf{u}}'_2\,\bar{\mathbf{u}}'_3\,\bar{\mathbf{u}}'_4|=\frac{|A|}{t_1t_2t_3t_4}|\bar{\mathbf{u}}_1\,\bar{\mathbf{u}}_2\,\bar{\mathbf{u}}_3\,\bar{\mathbf{u}}_4|$$
$$=\frac{|A|}{t_1t_2t_3t_4}\Delta_{1234}$$

and the like back to Eq. 10. Since the factor t_i appears the same number of times in the numerator and in the denominator of Eq. 10, this will lead to cancelling all factors t_i, thus Eq. 12 holds true.

A permutation of the indices in Eq. 10 may produce other volume cross-ratios. However, only three of them are independent. This could be shown by a numerical counting. To specify six 3-D points constituting a volume cross-ratio, we need $6\times 3=18$ independent parameters. However, $(3+1)\times(3+1)-1=15$ of them are taken up by the transformation parameters in matrix A of Eq. 8, where -1 takes into account one parameter eliminated in the numerator and denominator of Eq. 7. The remaining $3=18-15$ degrees of freedom refer to the number of independent volume cross-ratios. In this sense, suggested initially by Duda and Hart (1973) for 1-D and generalized by Brill and Barrett (1983) to n-D, the three independent invariants are defined as *projective coordinates* of point i relative to the chosen five basis points. More particularly we specify the following three cross-ratios:

$$C_1(i)=\frac{\Delta_{1234}\,\Delta_{125i}}{\Delta_{1235}\,\Delta_{124i}}\qquad C_2(i)=\frac{\Delta_{1324}\,\Delta_{135i}}{\Delta_{1325}\,\Delta_{134i}}$$
$$C_3(i)=\frac{\Delta_{2314}\,\Delta_{235i}}{\Delta_{2315}\,\Delta_{234i}} \tag{13}$$

as the *projective coordinates* of point i relative to the five chosen basis points 1, 2, 3, 4 and 5.

As is shown in Eq. 12, the projective coordinates in Eq. 13 are MO-type invariants, i.e., they can be equivalently calculated from either object or *model* coordinates. As a generalization of the *similarity model* in conventional photogrammetry, the collection of all the model coordinates $(U'\,V'\,W'\,1)$ in Eq. 8 will constitute a *projective model* of the object. However, since they are not observables, the projective coordinates in Eq. 13 can not be directly applied to computation.

4. Computation of projective coordinates

Following the discussion in the last section, we will define the projective model of the object in such a way that the projective coordinates are computationally available.

The 3-D to 2-D projective transformation (collinear equation) for the left and right images are written as:

$$\bar{A}_{\mathrm{L}}\bar{\mathbf{u}}=\bar{\lambda}_{\mathrm{L}}\bar{\mathbf{x}}_{\mathrm{L}}\qquad \bar{A}_{\mathrm{R}}\bar{\mathbf{u}}=\bar{\lambda}_{\mathrm{R}}\bar{\mathbf{x}}_{\mathrm{R}} \tag{14}$$

where $\bar{A}_{\mathrm{L}}$ and $\bar{A}_{\mathrm{R}}$ are 3×4 transformation matrices for the left and right image, respectively. There are six equations in Eq. 14 altogether. Any four of them will constitute a projective model of the object, as long as the corresponding 4×4 transformation matrix is not singular. To keep the formulae symmetric, the first two equations of each set are chosen, namely:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} U \\ V \\ W \\ 1 \end{pmatrix} = \begin{pmatrix} \overline{\lambda}_{\mathrm{L}}\overline{x}_{\mathrm{L}} \\ \overline{\lambda}_{\mathrm{L}}\overline{y}_{\mathrm{L}} \\ \overline{\lambda}_{\mathrm{R}}\overline{x}_{\mathrm{R}} \\ \overline{\lambda}_{\mathrm{R}}\overline{y}_{\mathrm{R}} \end{pmatrix} =$$

$$\overline{\lambda}_{\mathrm{L}} \begin{pmatrix} \overline{x}_{\mathrm{L}} \\ \overline{y}_{\mathrm{L}} \\ \overline{k}\overline{x}_{\mathrm{R}} \\ \overline{k}\overline{y}_{\mathrm{R}} \end{pmatrix} = \overline{\lambda}_{\mathrm{L}}\overline{\mathbf{p}} \tag{15}$$

where $\{a_{ij}\} = A$ is the transformation matrix.

A comparison between Eq. 15 and Eq. 8 shows that the homogeneous coordinates

$$\overline{\mathbf{p}} = (\overline{x}_{\mathrm{L}}\,\overline{y}_{\mathrm{L}}\,\overline{k}\overline{x}_{\mathrm{R}}\,\overline{k}\overline{y}_{\mathrm{R}})^{\mathrm{T}} \tag{16}$$

are essentially a projective transformation of the object. The whole set of $\overline{\mathbf{p}}_i$ constitutes a *projective model* of the object. Therefore, following from the discussion in Section 3 and letting $\overline{\mathbf{u}} = \overline{\mathbf{p}}$ in Eq. 8, the projective coordinates of point i can be calculated with following formulae:

$$\begin{aligned} C_1(i) &= \frac{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_4|}{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_5|}\,\frac{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_5\,\overline{\mathbf{p}}_i|}{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_4\,\overline{\mathbf{p}}_i|} \\ C_2(i) &= \frac{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_4|}{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_5|}\,\frac{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_5\,\overline{\mathbf{p}}_i|}{|\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_4\,\overline{\mathbf{p}}_i|} \\ C_3(i) &= \frac{|\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_4|}{|\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_1\,\overline{\mathbf{p}}_5|}\,\frac{|\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_5\,\overline{\mathbf{p}}_i|}{|\overline{\mathbf{p}}_2\,\overline{\mathbf{p}}_3\,\overline{\mathbf{p}}_4\,\overline{\mathbf{p}}_i|} \end{aligned} \tag{17}$$

where $\overline{\mathbf{p}}_j$ ($j = 1, 2, 3, 4, 5$) refers to the $\overline{\mathbf{p}}$ vector of basis points in Eq. 16. As described by the algorithm in Section 2, the length ratio of two conjugate projective rays $\overline{k}$ is available if the fundamental matrix is obtained and properly decomposed. Thus, Eqs. 16, 17 can be used to compute the projective coordinates.

As pointed out in Section 3, C_1, C_2 and C_3 are of an object description independent of the projective transformation. That means their values remain *unchanged* under projective transformation. Obviously this is beneficial for object recognition and image understanding. Since there are no transformation parameters involved, the projective coordinates can be calculated solely with the image observables of a stereopair. Therefore, by choosing five basis points, either globally or locally, the object or part of the object can be described with its projective coordinates C_1, C_2 and C_3.

5. Tests and analyses

To validate and evaluate the development and algorithm for object description, tests were implemented with an aerial stereopair. Its primary parameters are shown in Table 1. Five known object points were chosen as the basis points. Four of them form the maximum volume and any four of them are not coplanar, i.e, $\Delta \neq 0$.

The image coordinates, as well as their 3-D object coordinates were measured using an analytical plotter. Thus, for each object point, its projective coordinates can either be computed with its object coordinates via Eq. 13, or with its image coordinates via Eq. 17. These two results were then compared statistically in terms of root mean square errors (RMSE), given in Table 3. To further verify the invariance of the projective coordinates for object description, various affine image transformations were added to the original image observables. The tests are numbered as 0–5 in the left column of Table 2, where test No. 0 was carried out by original image observables. Among the transformation parameters listed in

Table 1
Photographic parameters

Flight height:	ca. 2250 m
Principal length:	88.94 mm
Frame size:	230 mm×230 mm
Camera:	RC-10
Overlap:	ca. 65%

Table 2
Image transformation parameters

No.	x			y		
	s_x	α_x (deg.)	x_0 (mm)	s_y	α_x (deg.)	y_0 (mm)
0	1.0	0.0	0.0	1.0	0.0	0.0
1	−1.3	−15.0	−10.0	1.5	−15.0	−15.0
2	1.6	15.0	10.0	−1.3	15.0	−15.0
3	1.3	−10.0	−15.0	−1.4	−10.0	15.0
4	1.3	10.0	15.0	1.4	10.0	10.0

Table 3
RMSE of projective coordinates obtained from object and images

No.	C_1	C_2	C_3
0	0.172	0.110	0.170
1	0.172	0.057	0.177
2	0.172	0.052	0.170
3	0.170	0.044	0.175
4	0.173	0.063	0.181

Table 2, s_x, s_y are scale factors, α_x, α_y are rotation angles, x_0, y_0 are translation components of principal point. The notations x and y on the top row of Table 2 indicate respectively the parameters in x and y directions on the image plane. The parameters for the left image take the opposite signs of the parameters for the right image.

A row-by-row analysis of Table 3 first reveals that the projective coordinates calculated from the stereopair are practically consistent to the ones directly obtained from the object itself. A comparison among the rows of Table 3 shows the invariance of the projective coordinates, i.e, they are independent of the linear transformation of the image. These facts validate the development and algorithm described in the previous sections. Therefore, they can be utilized for obtaining the projective-independent description in the invariant-based image understanding and object recognition.It should be noted that there exists an instability in the computation of the projective coordinates. Table 3 is based on the statistics of twenty-two object points from a total of thirty. The other eight object points are nearly located in one plane with the three-pointwise combinations of the five basis points. Obviously, in this case the volumes are vanishing and the projective coordinates become infinite. Actually, in this case it is a 2-D to 2-D projective transformation. The *cross-ratio of areas* or the projective coordinates degenerating from 3-D into 2-D can be directly adopted as a projective-independent measure to describe the object in a similar way.

6. Concluding remarks

3-D cross-ratio or projective coordinate is the general invariant of the projective transformation. It can be computed from the 3-D projective model of the object if the fundamental matrix of the stereopair is determined and decomposed in a proper way. Five points, either globally or locally, should be chosen as a basis for obtaining the projective-independent description. Numerical tests validated the development and the suggested algorithm. Future work will be applying the projective invariants to photogrammetric object recognition.

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