Construct 3-D Building Topology From Point Primitives

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Abstract
3-D building models are needed for spatial analysis, 3-D database and visualization. In this paper, basic geometric elements (point primitives) collected from stereo image pair are used to reconstruct buildings. This involves collection of only distinct corner points without following specific sequence. The reconstruction procedure consists of two main steps: determine the building boundary and rooftop. The key issue to reconstruct boundary is to find dominant directions of a building and apply sweep line algorithm. Reconstructing rooftop is based on dominant directions and properties of planar graph. The final building models are polygons and edges expressed by points and displayed for 3-D visualization and spatial analysis. Presented in this paper are the developed algorithms, implementation methodology and experimental results along with their images for evaluation.

1. Introduction
The generation and visualization of urban scenes become an important topic for not only a realistic presentation but also various applications such as 3-D geographic information system (GIS), urban planning, telecommunication and environmental studies. Urban area involves many features, such as building, roads, land, and rivers. Among these features, building model is probably the most important and complex one in urban scenes analysis. Typically there are three major steps involved in the generation of 3-D building models: data acquisition, feature extraction, and building reconstruction.

1). Data acquisition: Most 3-D city modeling techniques are guided by data types. Different applications may require different data types and manipulation functions. Most common data types are airborne laser scans and aerial photos. Current laser scanning systems show high potential in automatic building modeling from directly measured 3-D dense point clouds. Many researchers examined this technology to detect and generate building automatically (Maas, 1999a, b; Vosselman, 1999, 2002). However, the performance varies with lidar point spacing. The laser beam samples the surface in certain patterns such that exact edges of buildings may not be measured and its lateral measurement accuracy is not high enough (Brenner, 2003). Aerial images interpreted by

photogrammetric methods are still the most commonly used information for city modeling because of its low cost, well understood and reliable results (Tunc et al., 2004).

2). Feature extraction: Gülch et al. (2004) categorized the process of obtaining building information into four levels according to the degree of human intervention: interactive systems, semi-automatic systems, automatic systems, and autonomous systems. Interactive systems mean that all measurements and modeling are performed manually, while autonomous systems are defined as a fully automatic system, which is still in the research stage. Automatic systems perform main tasks automatically. Numerous feature extraction algorithms are developed to acquire geometric features of buildings such as points, lines, and polygons from images. Most researches need to combine 2-D building detection results, such as linear-corner analysis and image matching, with 3-D digital surface model (DSM) to determine the geometrical parameters of a building (Henricsson et al., 1998; Fischer et al., 1998). Feature extraction from images is efficient, however, there is limited accuracy due to occlusions, image quality, or invisible features, especially in high-density built-up areas. In semi-automatic systems, operators are responsible for feature interpretation, and modeling process is supported by automated tools (Grün and Dan, 1997; Grün and Wang, 1998). So far the most reliable and accurate way for feature collection is still performed manually. Usually an experienced operator can accurately measure target features and avoid the effect of the hidden areas. The drawback is that the process is time consuming and labor intensive.

3). Building reconstruction: Most semi-automatic and automatic systems apply a model-based strategy to reconstruct the buildings. Building models describe the exterior boundary of the building in geometry and topology (Förstner, 1999). Pre-defined building models can be distinguished in different classes according to their topological dimensions and structures. Wire frame models represent a 3-D object by 0-D vertices and 1-D edges without texture or shading information. Surface models describe the surface shapes of objects by vertices, edges and 2-D polygons such as boundary representation model (Mäntylä, 1988). Volumetric model describe the object by vertices, edges, polygons and 3-D, for instance, CSG models and CAD models (Grün and Dan, 1997; Suveg and Vosselman, 2000; Brenner, 2004). There are advantages and disadvantages for different types of models. Because buildings in reality vary in terms of styles, no model can describe all complex buildings accurately.

2.   Related Work

This paper attempts to reconstruct 3-D building models from an unstructured point set collected from aerial images. Several researchers also devote their attention to reconstruct buildings with manually or automatically collected distinct point sets. These related studies are briefly described as below.

Grün and Dan (1997) proposed an automated objects generation system TOBAGO from an unstructured 3-D point set. Each roof unit, which can be a complete roof or a portion of a roof, is processed at one time. The point set is identified in a fixed geometry and

topology among six classes of CAD building models automatically. The system is a semi-automated procedure, where the operator measures points from images in stereo mode.

Grün and Wang (1998) proposed a semi-automated CC-Modeler system for building reconstruction from point primitives. Such task is formulated as a consistent labeling problem and solved by probabilistic relaxation. The final outcome generates the full topology of the buildings. It utilizing manually measured 3-D point clouds. Those points are labeled as 2-D boundary points and interior points according to their functionality and structure. The boundary points are to be measured in certain sequence, and other points are to be measured without order. This topology builder fits the faces jointly to the given measurements to model buildings. In this approach, each roof unit needs to be processed independently, and the operator can edit and connect roof unit in a post processing step.

Koehl and Grussenmeyer (1998) developed a technique to generate 3-D city objects with geometric, topological and thematic modeling at the same time. 3-D points acquired from a digital image pair are associated with a generic model library by operators for geometric data acquisition. The complex objects are composed and made of basic geometric elements. After that, the 3-D objects are projected to an existing digital terrain model (DTM) to form vertical walls. The data can be stored in a relational database for managing building models. The operator plays a key role in selecting, orienting, and re-dimensioning the 3-D object from predefined generic shape library.

3.   The Proposed Approach

Most methods mentioned above apply CAD models for building reconstruction. Because of the variety of building styles in urban area, pre-defined model library can only handle limited numbers of buildings. Therefore, we propose a semi-automatic approach, which does not require any assumption on building models. In our approach, we ask the operator to digitize minimum necessary distinct vertices for each building from aerial images (Figure 1).

To measure the entire building from the images is sometimes difficult because of many hidden parts, especially the footprint on the ground. Whereas the roof constitutes a crucial part of a building, and therefore the main concern is focused on the determination of the roof. The roof structure represents the exterior surface and its supporting structures on the top of a building. The operator is required to collect 3-D points of a building roof correspond to edge points, corners, junctions or edge intersections. The roof is measured such that no point juts out of eaves, i.e., all interior points are inside contour points.

For complex buildings, a roof is a single unit that larger roof structures such as dormers need to be included in another unit. A multi-story building needs to be separated into multi-units. Small structures such as chimney and small attachments on the facades are omitted. Invisible part of the building has to be estimated by operators. Each vertex includes x, y, z coordinates and a unique id number. The geometrical elements of building objects include points, edges and polygons. The building faces can be represented by

3

polygons, which are described by ordered edges. Edge, which is a connection between two points, represents the border of polygons. Therefore, to successfully reconstruct a building from point primitives, two main steps are needed in our approach. The first step requires finding correct edges from point information including boundary and interior structures. The second step combines edges to form polygons representing faces of a building. The final result will be specified by a topological relationship between points, edges and polygons. This model can also store attributes of all faces of a building to support photo-realistic visualization and spatial analysis.
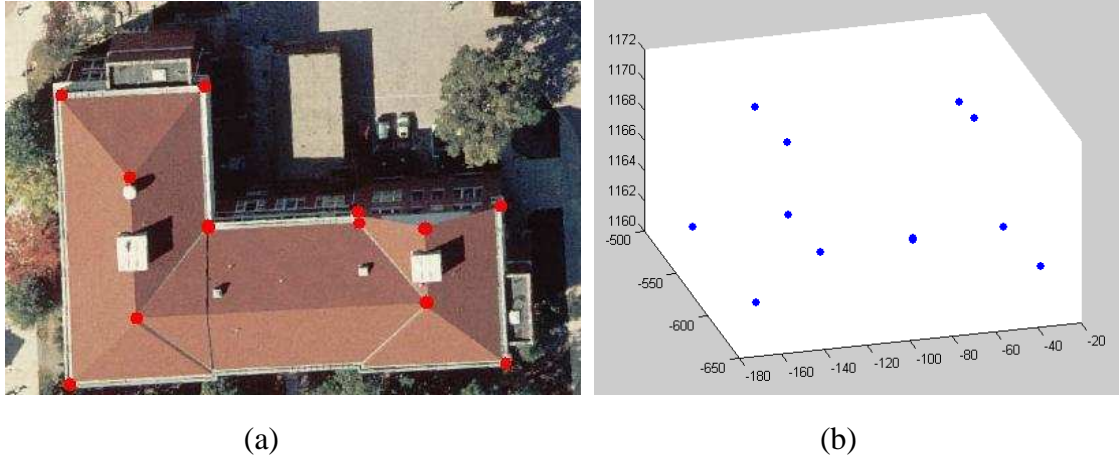

(a)   (b)

Figure 1. A building image (a) and collected roof points (b).

4.   Finding Edges

After collecting the point primitives, we first classify them into contour points and interior points. These points are separated based on the height. We grouped points in the lowest level as contour points. The rest points are labeled as interior points.

Although a building can be a complex 3-D structure, we only consider polyhedral surface that all eaves form a planar polygon. Likewise, a roof structure can be regarded as an undirected simple planar graph from its top view. To facilitate the discussion, we first review several related concepts in graph theory. A graph is defined as an ordered pair $G = (V, E)$, where V is a set of vertices, $V = \{v_1, v_2, ..., v_n\}$, and E is a set of edges, $E = \{e_1, e_2, ..., e_n\}$. An edge $e_k = \{v_i, v_j\} \in E(G)$ is a connection of v_i and v_j. The vertices v_i, v_j associated with edge e_k are called the end vertices of e_k. The most common representation of a graph is by means of a diagram. The vertices are represented as points and each edge as a line segment joining its end vertices. A simple graph does not contain loops or multiple edges, that is, each edge can be uniquely defined by its two end vertices. A planar graph is one that be drawn on a plane without graph edges crossing, i.e. edges intersect only at their common vertices.

4.1 Finding contour edges

First we find edges considered to be contour edges from contour points. Contour edges represented boundary vertical walls of the building. Boundary of a building can be a simple cycle from its top view. One can determine the cycle to be a set of 2-D contour edges. To find the possible contour edges, first we detect the set of contour points by a convex hull algorithm. The QuickHull algorithm (Barber et al., 1996) is applied here to determine the smallest convex hull. A set of contour points and its initial convex hull is shown in Figure 2a. If all contour points are used to form the convex hull, the contour of the building is determined. Otherwise, if there are contour points within the convex hull, the building contour must be concave and further processing is needed to determine its boundary.

The key strategy to form a concave polygon is to modify the initial convex hull based on the nature of buildings. There is no unique solution to forming a concave polygon from a given set of points, therefore some prior assumptions about building properties should be made for determining the boundary. For example:

1. There are likely parallel edges in a building boundary.
2. Some edges are likely perpendicular to one another.

With these assumptions, the method to form concave polygon is proposed as follows:
1. Find the slope of each edge in the initial convex hull. Sorting these slopes by how many times they occur (Figure 2b, 2c).
2. If there are edges perpendicular to each other, the sweep line algorithm (Bentley and Ottmann, 1979) will be applied along with these perpendicular slopes for points not on the initial convex hull to find adjacent points (Figure 2d, 2e).
3. If no perpendicular slopes exist, we apply the sweep line algorithm based on the sorting results. This process will be stopped until all contour points are connected.
4. Update the edge set such that every point joins with two edges (Figure 2f).

The sweep line algorithm provides a method for ordering points and finding the relationship among them. The common sweep line algorithm is modified and described as below:
1. Apply the sweep line algorithm on the points inside the convex hull. The slopes of sweeping lines are determined from step 2 and 3 above.
2. While sweep line hits points, sorting these points with respect to either x or y coordinate. If even numbers of points admit the line, then pairs of points are connected along the line (Figure 3).
3. Validity check is required during detecting new edges:
   (a) No crossing edge.
   (b) No diagonal edge, which is a line segment connecting two non-adjacent polygon points of a polygon.

(a) Initial convex hull of contour points

(b) Contour edge slope analysis

Slope	Frequency
S_1	2
S_2	2
S_3	1

Orthogonal

(c) Sort slopes based on frequency and list orthogonal slopes

(d) Sweep lines along slope_1 and slope_2.

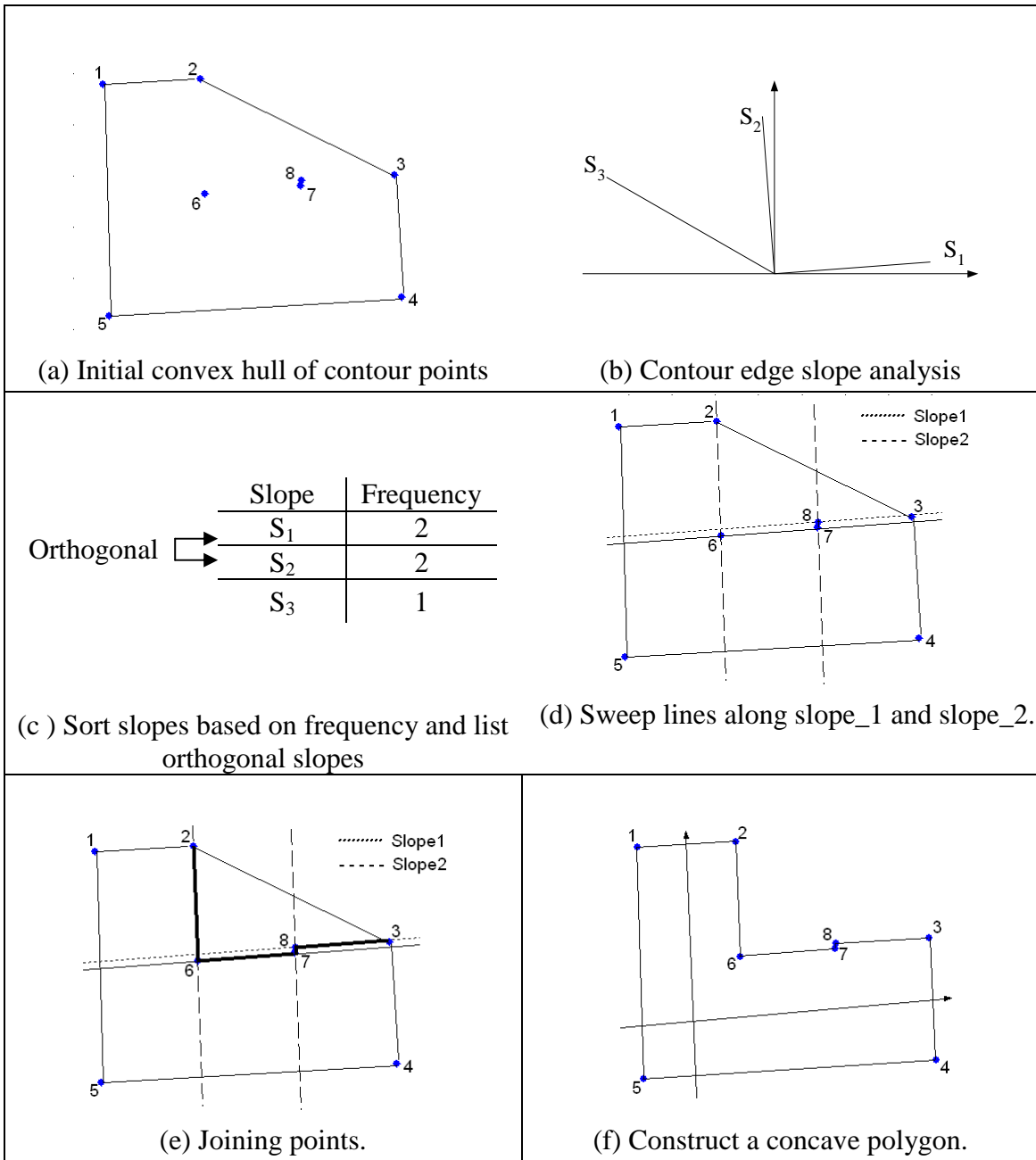(e) Joining points.

(f) Construct a concave polygon.

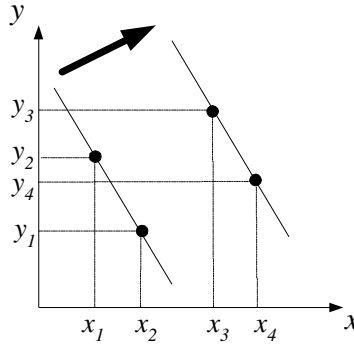Figure 2. Derivation of concave building contour

Figure 3. Sweep line passes through points along its slope. Ordering after a sweep: the intersection along the sweep line in x direction is $x_2 > x_1$, $x_4 > x_3$; in y direction is $y_2 > y_1$, $y_3 > y_4$. The connecting order can be determined by sorting with respect to either x or y coordinate.

Figure 2 explains how the algorithm works. For example, two perpendicular lines sweep the points within the convex hull. When the sweep line with slope_1 hits point (3, 8) and (6, 7), two new edges are formed. The process repeats to form edge (2, 6). For contour points, each is associated with two edges. By deleting edges (2, 3) from further consideration, new polygons are constructed successfully. In the meantime, the dominant directions of a building are also determined by slopes of sweep lines. Dominant directions reflect the main axis or axes of a building (Figure 2f). Edges of a building tend to be parallel to its dominant directions. In this example, dominant directions are directions with slope_1 and slope_2.
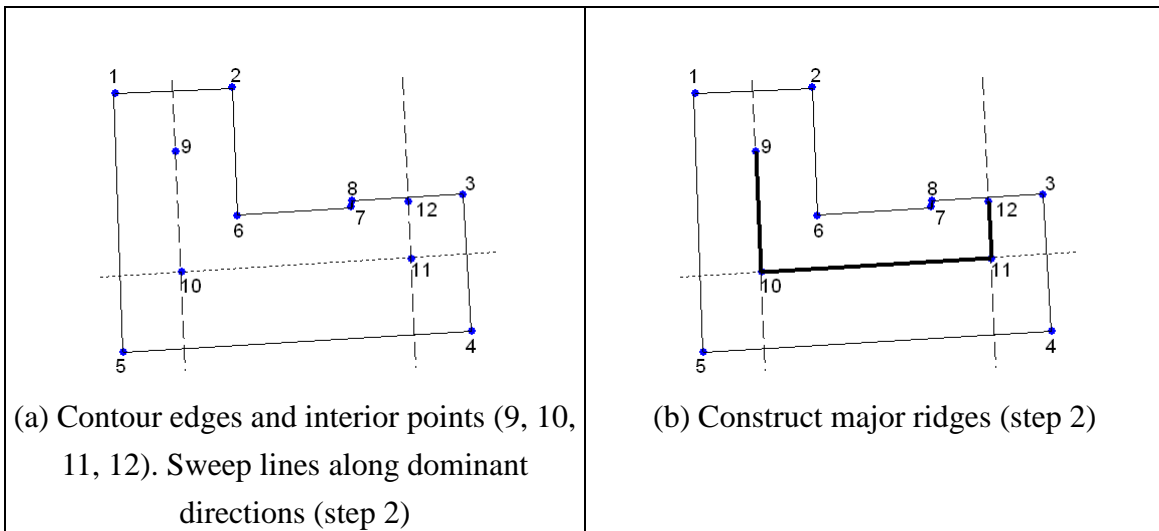
4.2 Finding roof edges

After finding contour edges, the next step is to determine the roof edges. Roof edges representing the roof top are composed of interior points and contour points. In this study, we classify roof edges to two types: major ridge and corner ridge (Figure 4g). Properties of roof edges described below can serve as a basis for determining the roof edges in 3-D space. For example:

1. Major ridges, which represent the trend of the main structure, follow the dominant directions.
2. The major ridges are parallel to the ground. In other words, points forming a major ridge are in the same height.
3. The rest of the edges belong to corner ridges. If a major ridge exists, the length of corner ridge from the same points should be shorter than the major ridge.

Because of the diversity of roofs in different areas, the above list of properties can be modified according to the areas. In this research, any edge satisfying the above criteria is a roof edge candidate. The process of finding roof edges is described as the following steps:

7

1. First we consider the major ridges. These interior points are separated into different levels based on height.
2. Apply the sweep line algorithm along dominant directions for interior points in different height levels. When sweep line meets interior points, these points can be sorted with respect to either x or y coordinate and connected. This step determines major ridges parallel to the dominant directions.
3. Once the major ridges are established, the next step is to find corner ridges. Since no model is applied in this work, all possible edges among all points are computed.
4. Remove edges that intersect with contour edges to enforce the planar graph assumption.
5. If intersections exist among major ridges, these edges need to be removed.
6. For edges which are joining to major ridges, its length should be no longer than major ridges.
7. Remove remaining edges from step 6 which intersect with each other.

The result is a set of edges including major ridges and corner ridges generated between interior points and contour points. Operators can compare the automatic reconstructed edges with images. Due to the complexity of buildings, if the strategy cannot handle all types of buildings, interaction is necessary to assure the completeness of edges. The edges can be edited manually, if needed (Figure 4h). After finding out the correct edge set, we can start to reconstruct polygons from edges.
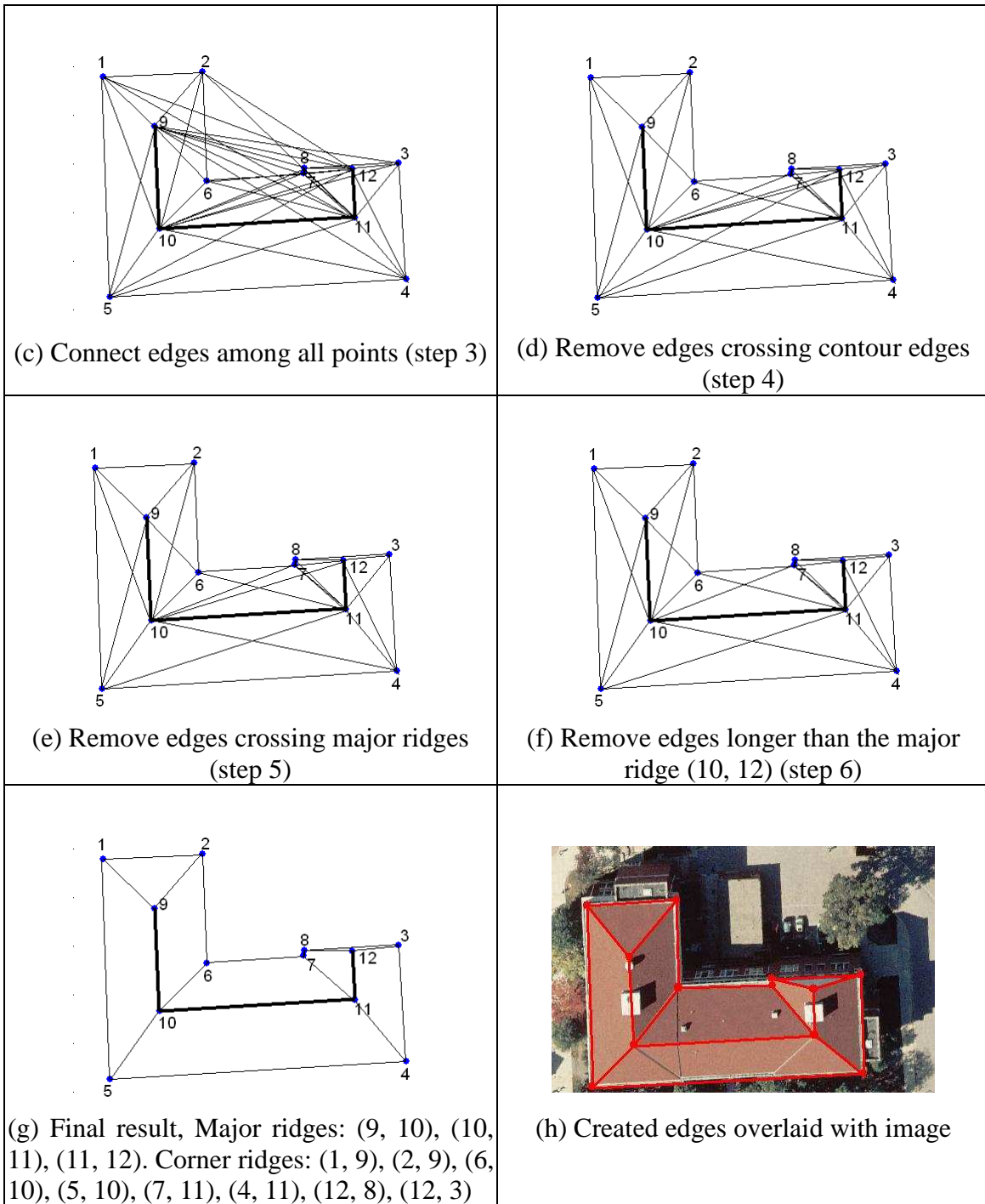
(a) Contour edges and interior points (9, 10, 11, 12). Sweep lines along dominant directions (step 2)	(b) Construct major ridges (step 2)

(c) Connect edges among all points (step 3)

(d) Remove edges crossing contour edges (step 4)

(e) Remove edges crossing major ridges (step 5)

(f) Remove edges longer than the major ridge (10, 12) (step 6)

(g) Final result, Major ridges: (9, 10), (10, 11), (11, 12). Corner ridges: (1, 9), (2, 9), (6, 10), (5, 10), (7, 11), (4, 11), (12, 8), (12, 3)

(h) Created edges overlaid with image

Figure 4.   Reconstruction of roof edges.

5. Polygon Reconstruction

After forming edge sets among points in previous section, next step is to find polygon sets composed of edges. According to our assumption, the building is a simple planar graph from its top view. Therefore, to construct polygons from edge segments is similar to find minimum cycle basis in a graph (Ferreira, 2004). The definition of a cycle is a sequence of edges of a graph over which one could trace a closed path, where the internal vertices are distinct and its start and end vertices are identical. Ferreira suggests a polygon-detection algorithm that applies the Minimum-Cycle-Basis (MCB) algorithm proposed by Horton (1987). According to the algorithm presented by Horton, algorithms for searching shortest path can be applied to look for minimum cycle basis. The original idea is to find minimal cycles between pair of vertices of each edge. Therefore, we start search minimum cycles from contour edges (E_c) and then examine roof edges (E_r). Each edge should contribute itself to two polygons. Every roof edge should contribute itself to two roof polygons. The searching starts from contour edges and then moves to roof edges. The contour edge represents the turning edge of the vertical wall and the roof, hence it participates in one roof polygon only. The polygon detecting algorithm is described in the right box.

The Breadth-First Search algorithm is applied here for searching shortest path because the graph is an undirected, unweighted simple graph. The result can be verified by Euler's polyhedron formula. If G is a connected plane graph with n vertices, m edges and p polygons, then: $p = m - n+2$.

```
Ec = {e_c}, the set of contour points
Ei = {e_i}, the set of interior points
for e ∈ Ec
    count (e) = 1;
end


for e ∈ Ei
    count (e) = 2;
end


for e ∈ Ec

    Φ =  MCB(e)

        for e ∈ Φ

            count (e) = count (e)-1;
        end
end

for e ∈ Er
    if count (e)>0

        Φ =  MCB(e)

    endif

    for e ∈ Φ

            count (e) = count (e)-1;
    end
end
```

After reconstructing the roof completely, the roof structure is stored as a topological relationship shown in Table 1. The data structure provides information about surface shapes, positions and how they are joined together. This table stores the 3-D topological relationships among the points, edges and the polygons. The 3-D object in Figure 3g is labeled. In this table, polygons are described by a sequence of points, and edges are represented by points without orders.

Table 1. Topological relationship of Figure 4g

Point list				Edge list	Polygon list
Point	x	y	z	(Point, Point)	Points
1	836	476	1161	9, 10	1, 2, 9
2	881	478	1161	10, 11	2, 6, 10, 9
3	971	440	1161	11, 12	1, 9, 10, 5
4	974	393	1161	1, 9	6, 7, 11, 10
5	839	385	1161	2, 9	7, 8, 12, 11
6	883	433	1161	6, 10	8, 3, 12
7	927	436	1161	5, 10	12, 3, 4, 11
8	927	438	1161	7, 11	10, 11, 4, 5
9	859	455	1170	4, 11	
10	861	413	1170	12, 8	
11	951	418	1170	12, 3	
12	949	438	1170	1, 2	
				2, 6	
				6, 7	
				7, 8	
				8, 3	
				3, 4	
				4, 5	
				1, 5	

6. Results

The proposed approach is tested with a pair of stereo images of Purdue University campus at the scale of ~1:4,000. During the measurement of distinct points, users need to estimate the location of points in hidden areas, and all roof corners must be completely denoted. To obtain the complete building, one footprint of the building on the ground is needed to define the building base height. Vertical walls of the building can be acquired by projecting the contour edges to the ground defined by the base height. A building can be separated into several units, with each being a polyhedron. In reality, data measurement from images will not be perfectly accurate, therefore, a 0.5 m tolerance is applied for the sweep line algorithms. The output is a list of ordered points representing edges and polygons. The result can be organized in different formats for display or input into a database. Figure 5 shows a number of reconstructed 3-D buildings and the corresponding aerial image.
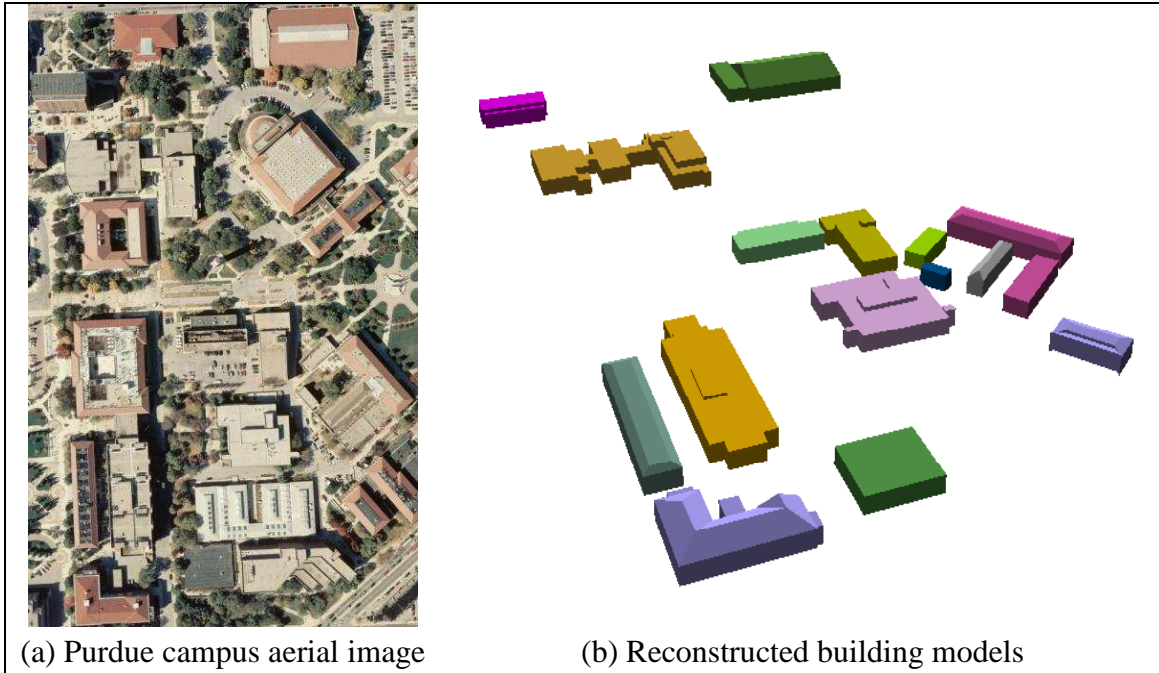
| (a) Purdue campus aerial image | (b) Reconstructed building models |

Figure 5. Examples of reconstructed 3-D buildings

7. Conclusions

This paper presents an approach to reconstruct the 3-D building objects from distinct unstructured points. Our intention is to devise a methodology such that it can be applied on unstructured distinct point primitives without using building models. The major features of the approach are summarized as follows. A complex building is separated into several polyhedron units, which are reconstructed one by one. For each building unit, the boundary is initially approximated by a convex hull, which is then evaluated to obtain the dominant directions. The common sweep line algorithm is modified to determine building edges based on the detected dominant directions. As for the roof reconstruction, major ridges paralleled the dominant directions are determined by the modified sweep line algorithm. After determining major ridges, all possible edges followed the planar graph properties are connected to decided corner ridges. The MCB algorithm detecting minimum cycles from edges are applied for searching polygons.

Through these processes, the unstructured point primitives are reconstructed step by step from rooftop identification to the complete building topological map. The result can be represented in different formats to integrate to 3-D GIS, and support various visualization and spatial analysis applications. It also provides a basis for texture mapping and 3-D topological analysis. The experimental results demonstrate that complex buildings can be successfully reconstructed by using the computational geometry principles as proposed in this study. Future work will be focused on topological mechanism analysis between spatial objects.

12

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