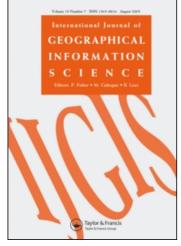
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# **Research Article**

# Fuzzy inference guided cellular automata urban-growth modelling using multi-temporal satellite images

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This paper presents a fuzzy inference guided cellular automata approach. Semantic or linguistic knowledge on urban development is expressed as fuzzy rules, based on which fuzzy inference is applied to determine the urban development potential for each pixel. A defuzzification process converts the development potential to the required neighbourhood development level, which is taken by cellular automata as initial approximation for its transition rules. Such approximations are updated through spatial calibration over townships and temporal calibration with multi-temporal satellite images. Assessment of the modelling results is based on three evaluation measures: fitness and Type I and Type II errors. The approach is applied to model the growth of the city of Indianapolis, Indiana over a period of 30 years from 1973 to 2003. A fitness level of 100  $\pm$  20% with 30% average errors can be achieved for 80% of the townships in urban-growth prediction.

Keywords: Cellular automata; Fuzzy logic; Urban modelling; Calibration

#### 1. Introduction

The complexity of urban growth phenomena makes it extremely difficult to capture its dynamics using traditional mathematical models (Batty and Xie 1994a). Among the dominant dynamic models, the cellular automata (CA) approach is probably the most impressive because of its technological evolution in connection with urban applications (Yang and Lo 2003). It is also attracting more attention from the urban research community recently since cellular automata is able to adapt to the complex urban process with simple transition rules. However, recent work in this area tends to complicate the cellular automata transition rules, such that it is almost impossible to view the link between the interactive development processes and their effect on the output patterns.

Since Tobler (1979) introduced cellular automata to geographical systems, many advances have been achieved. Cellular automata were used to model and explain the built form of French villages and the layout of rooms in houses (Hillier and Hansen 1984). It was also applied to study how different varieties of urban dynamics might arise (Couclelis 1985, 1988, 1989). Though originally not intended to model urban growth, Couclelis' work provided a theoretical and methodological framework for

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using cellular automata to solve complex dynamic geographical problems. A diffusion-limited aggregation (DLA)-based cellular automata model (Batty and Xie 1994a, b) was developed to simulate the growth of urban areas. Their Dynamic Urban Evolutionary Model (DUEM) model was designed to model land-use changes through transition rules. One of the drawbacks in their model was its inability to effectively handle certain types of data such as road network and spatial location. The work of White and Engelen (1993, 1994) allowed for the integration of socio-economic and natural system models in a realistic way (White and Engelen 1997). They used the standard non-spatial models of regional economics and demographics, as well as a simple model of environmental change to predict the demand for future agricultural, residential, and commercial/industrial land-uses (White and Engelen 1997, 2000). Clarke et al. (1997) presented a cell-based urban land-use change model SLEUTH (Slope, Land use, Exclusion, Urban extent, Transportation, Hillshade). The model has four major input data layers: slope, seed layer, transportation, and protected land. It captures four types of urban land-use change: spontaneous growth, new spreading centre growth, edge growth, and roadinfluenced growth. These four growth types are applied sequentially for each growth iteration through five growth coefficients: diffusion, breed, spread, slope resistance, and road gravity. The calibration in the model is meant to find the best set of these five coefficient values to reproduce the urban pattern. SLEUTH was used later to model and predict the urban growth for the San Francisco Bay Area in California and the Washington DC/Baltimore corridor (Clarke and Gaydos 1998). Recently, Yang and Lo (2003) used the SLEUTH model to simulate urban growth in Atlanta, Georgia. The calibration was completed using historical data extracted from a series of satellite images. They established a what-if scenario system to assess the impact of different policies on the urbanization process. Wu and Webster (1998) used the multi-criteria evaluation (MCE) method to define the transition rules. The calibration in their model is meant to find the best weight vectors for the set of input layers. Similarly, Li and Yeh (2002) used neural networks (NN) to calibrate the parameters in cellular automata modelling.

Despite the remarkable achievements in cellular automata urban modelling, its performance can be further improved in many aspects. First, there is no standard method for the definition of transition rules. They are usually chosen based on common understanding about the effects of various natural and social factors on the urban-growth process (Li and Yeh 2004). Wu (2002), however, expressed the transition rules using probability functions, which makes the interpretation of such rules difficult and hence less useful for decision-makers. Cellular automata modelling is all about simplifying a complex process by applying simple rules. Therefore, any cellular automata model, in order to be practical enough, should adapt semantically explicit transition rules that can be easily interpreted by the users. Second, the rules need to be accurately and effectively calibrated. This is an important issue that had been neglected until recent efforts were made to apply the cellular automata method as a reliable procedure for urban development simulation (Wu 2002). The difficulty partially lies in the complexity of the urban development process on the one hand (Batty et al. 1999) and the ability, on the other hand, to find a simplified mathematical model sufficiently reliable to present the urbanization process. In addition, because of the large rule parameter space, there is a need to develop cellular automata models that are computationally efficient and robust enough to explore the most likely subset of the parameter space. This can be achieved by providing good initial values for the rule parameters. Third, many of the cellular automata models use a large number of input variables (e.g. the SLEUTH model). This complicates the selection of transition rules to present the effects of each individual variable on urban growth. Moreover, some variables might be highly correlated, and adding more variables may not necessarily improve the modelling results. Finally, cadastral maps, instead of satellite images, are often used as cellular automata input data (Clarke *et al.* 1997, Wu 2002, Li and Yeh 2004). Despite the popularity and appropriateness of using cadastral maps, we argue that this method may not use comprehensive and contemporary land-use and land-cover information recorded on imagery, which nowadays is becoming widely available at minimal cost. In addition, images are able to clearly reflect certain land use and growth constraints (e.g. water areas) and have more details and contents than highly generalized cadastral maps.

Cellular automata controlled by fuzzy logic is a recent development. Most models view a pixel as a binary system or what is called crisp cellular automata. Each pixel is treated as either fully developed or undeveloped. This ignores the fact that a pixel might be partially developed, since the urban growth process is continuous in space. Besides, the role of the pixel development potential in identifying its own development requirement is totally ignored in crisp cellular automata. It is not reasonable to treat a pixel that has an 80% potential to develop in the same way as another pixel with only a 20% potential. The 80% pixel needs a lower urbanization level in the neighbourhood to develop compared with the 20% pixel. Wu (1996, 1998) used fuzzy-logic control in defining the urban transition rules. The fuzzy model captured the feature of land-conversion behaviour, while cellular automata simulated the global pattern from local rules. A set of membership functions for several linguistic variables were defined to represent the input data. Liu and Phinn (2003) modelled urban growth by identifying the state of a pixel using a fuzzy membership function for spatial delimitation of urban, suburban, and rural areas. Their model was based on the assumption that the number of years required for the full urban development of the considered region is known. A fuzzy inference engine was used to resolve the transition rules. An important note considering this model is that fuzzy logic is used to define both the transition rules and the pixel states, which makes the calibration process more complex.

This study presents a fuzzy inference guided cellular automata approach. The role of fuzzy inference is threefold: to apply common semantic or linguistic knowledge to urban modelling; to simplify the definition of the cellular automata transition rules; and to reduce the search space for their calibration. The output from fuzzy inference is used as the initial values for the parameters in the transition rules. Calibration of the transition rules is conducted temporally based on the classification results of historical Landsat satellite images and spatially over a grid system defined by the township map. In addition to the satellite images, the input data also include a digital elevation model (DEM), a road map, and a population census map. The methodology is evaluated by a dataset covering the Indianapolis area, Indiana over a period of 30 years from 1973 to 2003.

#### 2. Principles

A synthetic city, shown in figure 1, is used to describe the principle of the fuzzy inference guided cellular automata. The size of the synthetic city is  $200 \times 200$  pixels. Three factors in this example are considered for urban development: land use,

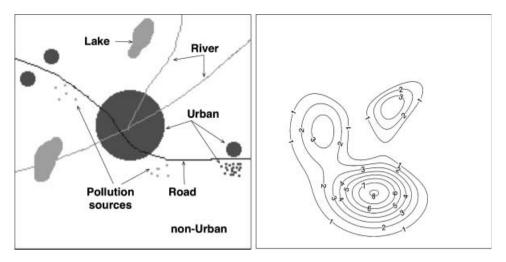


Figure 1. Synthetic city (left) and its elevation (right).

elevation, and distance to the city centre. The land-use image (map) has six classes: lake, river, road, urban, non-urban, and pollution source. The DEM and distance to the city centre are respectively normalized and used as input to the fuzzy inference process.

Fuzzy logic was first introduced by Zadeh (1965, 1971) to model a continuous process as an extension to the crisp set theory. Fuzzy inference allows us to linguistically describe the concepts related to urban growth. In fuzzy inference, the distance to the city centre (d) is a fuzzy variable or linguistic variable. This fuzzy variable may take Close, Medium, or Far as fuzzy values. Fuzzy values are related to their crisp values, i.e. the exact (normalized) distances, through a membership function. Essentially, the membership function categorizes the crisp distance to the city centre when x=d, while figure 2(a) is the corresponding plot. It should be noted that one crisp value may belong to more than one fuzzy value at different degrees. For example, according to the membership function in figure 2(a) or equation (1), d=0.45 belongs to Close with a membership degree 0.5, to Medium with a membership degree 0.25, and to Far with a membership degree 0. The same concept

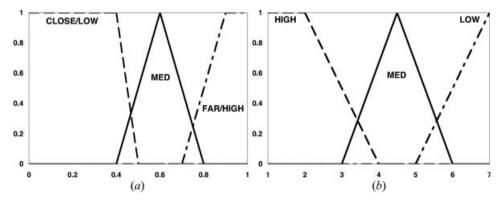


Figure 2. Membership functions of (a) distance and elevation and (b) output.

can be applied to a DEM whose membership function is also defined by equation (1) when x=h.

$$\mu_{x}(x) = \begin{cases} 1 & 0 \le x \le 0.4 \\ (-10x+5) & 0.4 \le x \le 0.5 \\ 0 & x \ge 0.5 \\ 0 & x \le 0.4 \\ (5x-2) & 0.4 \le x \le 0.6 \\ (-5x+4) & 0.6 \le x \le 0.8 \\ 0 & x \ge 0.8 \\ 0 & x \ge 0.8 \\ 1 & x \ge 0.9 \end{cases}$$
(1)

The output for the fuzzy inference also needs to be defined. In this study, the fuzzy output variable y represents the required neighbourhood development (urbanization) level for a pixel to become urban according to its own development potential. The development potential of a pixel may take three fuzzy values: High, Medium, and Low. The lower the development potential of a pixel, the more urban pixels that are needed in its neighbourhood for its development. The output fuzzy membership function is shown in figure 2(b) and equation (2).

$$\mu_{\text{out}}(y) = \begin{cases} 1 & y \le 2 \\ \left(-\frac{1}{2}y+2\right) & 2 \le y \le 4 \\ 0 & y \ge 4 \\ \\ \text{Medium} \begin{cases} 0 & y \le 3 \\ \left(\frac{2}{3}y-2\right) & 3 \le y \le 4.5 \\ \left(-\frac{2}{3}y+4\right) & 4.5 \le y \le 6 \\ 0 & y \ge 6 \\ \\ \text{Low} \begin{cases} 0 & y \le 5 \\ \left(\frac{1}{2}y-2.5\right) & y \ge 5 \end{cases} \end{cases}$$
(2)

The fuzzy input variables and the fuzzy output variable are associated through fuzzy rules. As an example, we introduce the following two fuzzy rules:

- Rule 1: IF (distance is Medium AND elevation is Medium) THEN output is Medium.
- Rule 2: IF (distance is Close AND elevation is Medium) THEN output is High.

The role of fuzzy rules is to determine the development potential of a pixel. Figure 3 illustrates how this is achieved by evaluating the fuzzy rules through a fuzzy inference process. In the evaluation of the fuzzy rules, the minimum–maximum (Mamdani) method (Mamdani 1974) is adopted. Let d=0.45 and h=0.55 for a given pixel. Its membership to each corresponding fuzzy value can be determined through the membership functions shown in figure 2(*a*) or equation (1). For Rule 1 above, the membership of d=0.45 belonging to Medium is 0.25, and the membership of

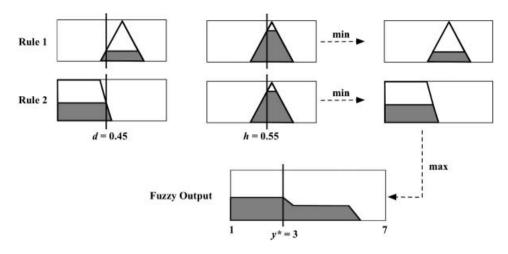


Figure 3. Fuzzy inference process.

h=0.55 belonging to Medium is 0.75. Using the Mamdani method, the minimum of the memberships, 0.25=min(0.25,0.75) is retained as the membership of the fuzzy output Medium. Similarly, for Rule 2 above, the membership of d=0.45 belonging to Close is 0.5, and the membership of h=0.55 belonging to Medium is 0.75. The membership of the fuzzy output High is 0.5=min(0.5, 0.75). As shown in figure 3, the two fuzzy output membership functions (Medium and High in equation (2)) are merged into one by taking the maximum at the corresponding output fuzzy values.

Finally, the fuzzy output function is converted to a single crisp value using the centre of area (COA) method, which considers the area-weighted average or centroid of the membership area, i.e.

$$y^{*} = \frac{\sum_{i=1}^{N} y_{i}^{*} \mu_{\text{out}}(y_{i})}{\sum_{i=1}^{N} \mu_{\text{out}}(y_{i})}$$
(3)

where N is the number of fuzzy graph points and is taken as 100 in this study. In this example, the defuzzified value is  $y^*=3$ . As addressed before, the output value  $y^*$  stands for the required number of urban pixels in the neighbourhood of a test pixel in order for it to develop to urban. It will be used as the initial value for the cellular automata transition rules.

A total of seven fuzzy rules are applied to the synthetic city as summarized in table 1. The rules are defined based on the influence of the two input factors (Distance and Elevation) on urban development. The values in the table refer to the membership functions of the output variable under fuzzy inference. For example, if a test pixel has Low elevation and Close distance to the city centre, then it has High development potential and hence needs a small number of urban pixels in the neighbourhood to develop. The fuzzification–defuzzification process is the same as described above except that there are seven (instead of two) rules to be considered.

A brief description of the principle of cellular automata is helpful for understanding its association with fuzzy inference. Cellular automata, originally

	Elevation					
Distance to city centre	Low	Medium	High			
Close	High	High	Medium			
Medium	NA	Medium	Low			
Far	NA	Low	Low			

Table 1. Fuzzy rules for urban development potential of the synthetic city.

introduced by Ulam and von Neumann in the 1940s as a framework to study the behaviour of complex systems (von Neumann 1966), is commonly defined as a dynamic discrete system in space and time that operates on a uniform grid being controlled by a predefined set of transition rules (Sipper 1997). Its atomic element is a cell or pixel in this study. A pixel can take one of the possible states or values, such as road, vegetation, or urban in the land-use image. The state of a pixel may change over time due to the dynamic nature of the system. Such a state change is governed by a set of transition rules defined for the neighbourhood of the pixel. In this study, a  $3 \times 3$  neighbourhood is used. For example, a transition rule can state that a vegetation pixel will change its state to urban if three or more of its neighbouring pixels are urban. Such an evaluation of the transition rules is carried out pixel by pixel over the entire image and is repeated until certain criteria are met. The number of iterations or the differences between the cellular automata outcome and the ground data are often used as iteration criteria.

As addressed earlier, fuzzy inference and cellular automata are coupled by using the output of the fuzzy inference as the input to the cellular automata. Specifically, the fuzzy inference determines the required number of urban pixels in the neighbourhood for a pixel to develop to urban. The cellular automata use this number as a threshold value to test if there are enough urban pixels in the neighbourhood for a pixel to become urban. The following are the transition rules used for the synthetic city, where  $y^*$  is the fuzzy output:

- IF a pixel is urban, river, road, or lake, or has pollution source in its neighbourhood, THEN there is no change in its state.
- IF a non-urban pixel has equal or more than y\* urban pixels in its neighbourhood, THEN change it to urban.
- IF a non-urban pixel has a road or lake in its neighbourhood AND has equal or more than (*y*\*–2) urban pixels in its neighbourhood, THEN change it to urban.

The transition rules are designed to enforce some growth constraints on certain land uses, such as water resources, that the modeller wishes to exclude from the urbanization process. On the other hand, some land-use classes, such as roads and lakes, are considered as favourable factors for urban growth in the transition rules.

The cellular automata model is run for 0, 25, 50, and 60 times using the above transition rules. The modelling results are shown in figure 4, which clearly illustrates the effects of the fuzzy-coupled transition rules on urban growth. The lower region of the city is characterized by high elevations (figure 1), and thus it is difficult for urban development as a result of using the above fuzzy rules. Similarly, the city grows gradually from the city centre because the fuzzy rules are in favour of locations near the city centre. In the mean time, the transition rules guide the city development along the road, river, and away from the pollution sources. All the

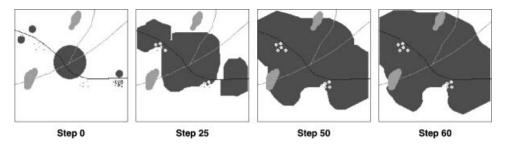


Figure 4. Synthetic city modelling results from fuzzy inference guided cellular automata.

above results suggest that the proper use of the fuzzy rules and transition rules can lead to a desired urban development pattern. Conversely, if a real urban development pattern is used as ground data to calibrate the rules, i.e. to determine the parameter values or thresholds in the transition rules, then they will be able to model the real urban development process.

## 3. Urban growth modelling of Indianapolis

### 3.1 Data and algorithm design

The above principles are applied to model the growth of the Indianapolis area, Indiana. Five images from the years 1973, 1982, 1987, 1992, and 2003 over a period of 30 years provide the temporal land-use information. The image of year 1973 is from Landsat MSS with four spectral bands (60-m resolution), while the other images are from Landsat TM with seven spectral bands (30-m resolution). All the images are spatially registered to the same reference system of UTM NAD 1983. Based on the USGS classification system (Anderson et al. 1976), seven classes are identified in the images: water, road, residential, commercial, forest, pasture, and row crops. The commercial and residential classes represent the urban class of interest in this study. Training samples on the images are selected with reference to the 1-m resolution black and white ortho-photographs (1998 photography) and the Indiana Geological Survey land-cover classification maps (produced based on 1999 TM images). As a result of the maximum-likelihood classification using all spectral bands, five historical land-use maps are created. The urban modelling is carried out on the resampled classification images at 60-m resolution, since this is the lowest resolution in the input images; and upsampling would actually not be meaningful in terms of fidelity and precision, whereas further downsampling would unnecessarily ignore the details in the input images. In addition, this choice would also balance the modelling error and computational cost, since the latter is exponentially proportional to the image resolution. On the other hand, a larger pixel size or neighbourhood size would dramatically increase the urban growth simulation rate (Al-kheder and Shan 2005), which would cause the modelling process to be less controllable.

Besides the images, the road network, the DEM, and the year 2000 census population map are also used, as shown in figure 5. An image showing the distance to the closest road is created from the road-network input. For the year 2000 census population map, the distance is computed between each population tract's centroid and the overall centroid. Population density (population per square km) is then

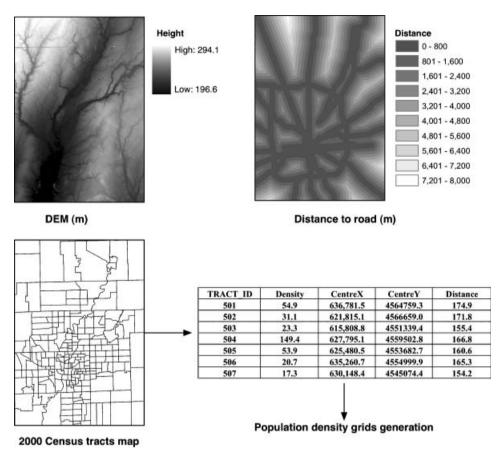


Figure 5. Input data for Indianapolis urban growth modelling.

modelled as an exponential function of the distance to the overall centroid.

$$Population\_density = A \times e^{-B \times Distance}$$
(4)

To reduce the variation in the census data, the population densities for tracts within a distance interval of 2 km are averaged. Figure 6 shows the population density as a function of distance for the year 2000. By comparing with the population data in 1990, it is identified that the population density parameters A and B in equation (4) vary by approximately 1% and 3% per year, respectively. By using this relationship, we calculate the population density at each pixel based on its distance from the overall centroid for each year in the simulation period. It should be noted that the three factors (road, population, DEM) are selected according to our historical growth observation of the city from satellite images. In addition, previous research, such as that of Wu (1998) and Liu and Phinn (2003), emphasized the importance of such factors in controlling the urban growth process. Furthermore, our model design is general enough to be capable of including other social or environmental factors, such as income level, which can be considered in the same way as population data.

Fuzzy inference concepts are applied to the Indianapolis data. The membership functions of the normalized DEM and distance to roads, and the population density

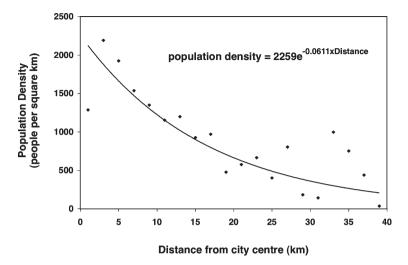


Figure 6. Population density (pop/sq km) as a function of distance for year 2000.

are defined as equation (1). Table 2 summarizes the fuzzy rules being used in this study. They are chosen based on the effects of the input data on the urban development. The fuzzy output defined in equation (2) is converted to a crisp value, which represents the required number of urban pixels in the neighbourhood for a pixel to develop according to its own development potential. Pixels with a high development potential require fewer urban pixels in their neighbourhood to develop than do pixels with a low development potential. This crisp value will be used as the initial threshold in the transition rules and will be updated through calibration.

The cellular automata transition rules are defined as a function of the land use, growth constraints, and fuzzy output:

- IF a pixel is road, water, commercial, or residential, THEN no change.
- IF a non-urban (forest, pasture, or row crops) pixel has equal or more than  $Y_R$  residential pixels in its neighbourhood, THEN change it to residential.
- IF a non-urban pixel has equal or more than  $Y_{\rm C}$  commercial pixels in its neighbourhood, THEN change it to commercial.
- IF the sum of commercial and residential pixels of a non-urban pixel in its neighbourhood is equal to or more than  $Y_S$  pixels, THEN change it to whichever is greater.

Table 2.	Fuzzy rules	for urban	development	potential of	of Indianapolis.

		F	Population density			
		Low	Medium	High		
Distance to road	Close	Medium	Medium	High		
	Medium	Medium	Medium	Medium		
	Far	Low	NA	Medium		
Elevation	Low	NA	NA	High		
	Medium	NA	High	NĂ		
	High	Low	Medium	High		
Medium Pop. Dens	sity & Low Elevati	ion & Far Distance	$\rightarrow$ Medium	C		

 $Y_{\rm R}$ ,  $Y_{\rm C}$ , and  $Y_{\rm S}$  are the threshold parameters for residential, commercial, and the sum of these two classes in order for a test pixel to become urban as described in the above transition rules.

#### 3.2 Calibration and evaluation

Calibration is essentially to determine the values for  $Y_R$ ,  $Y_C$ , and  $Y_S$  such that the CA modelling outcome can best represent the real urban development. For this purpose, the fuzzy output  $y^*$  is used as an initial value subject to certain corrections, i.e.

$$Y_{\rm R} = y^* + \varepsilon_{\rm R}$$
  

$$Y_{\rm C} = y^* + \varepsilon_{\rm C}$$
  

$$Y_{\rm S} = y^* + \varepsilon_{\rm S}$$
(5)

where  $(\varepsilon_{\rm R}, \varepsilon_{\rm C}, \varepsilon_{\rm S})$  are the corrections or the calibration coefficients to be determined through calibration. For this objective, we use multi-temporal images as ground data for calibration over time. In addition, the entire study area is divided into 24 townships using a township layer as shown in figure 7. A township has an average size of  $93 \text{ km}^2$  and represents the ownership of land which can be sold or acquired for different urban development uses. For this reason, each township may potentially have its own urbanization properties and process. The same cellular automata transition rules are defined for all townships; however, different townships can have different rule values. In this way, the transition rules will also be calibrated spatially to consider local growth properties and pattern. A small integer search space, from -3 to 3 for the first two calibration coefficients ( $-3 \le \varepsilon_{\rm R}, \varepsilon_{\rm C} \le 3$ ) and from 0 to 3 for the third one  $(0 \le \varepsilon_S \le 3)$  with an increment of 1, is defined. For example, a combination of ( $\varepsilon_{\rm R}=0$ ,  $\varepsilon_{\rm C}=0$ ,  $\varepsilon_{\rm S}=0$ ) for the calibration coefficients means that the fuzzy output is used as the threshold in the transition rules without any modification. Another example ( $\varepsilon_{\rm R} = -3$ ,  $\varepsilon_{\rm C} = 0$ ,  $\varepsilon_{\rm S} = 0$ ) means that only the residential threshold parameter is changed to be three pixels less than the fuzzy output. The best results from the above search space ( $\varepsilon_{\rm R}$ ,  $\varepsilon_{\rm C}$ ,  $\varepsilon_{\rm S}$ ) will be retained as the final modelling outcome.

Three evaluation measures are designed to select the best outcome from the calibration process. The fitness measure defined in equation (6) is the ratio of the number of urban pixels in the simulated image to the urban count in the ground-data image

$$Fitness\% = \frac{Simulated\_urban\_count}{Ground\_data\_urban\_count} \times 100$$
(6)

The other two measures, Type I and Type II errors, count the mismatch pixel by pixel between the simulated and real images. The Type I error counts the pixels that are urban in the ground data image but non-urban in the simulated image, while the Type II error counts the pixels that are non-urban in the ground data but urban in the simulated. Among the three quality measures, fitness measures the success in reproducing the real urbanization level. A fitness of more than 100% suggests an overestimation of the urbanization level in the modelling process, while a fitness of less than 100% indicates an underestimation of the urbanization level. Type I and Type II errors represent the pixel-by-pixel differences between the simulation results and the ground data. They also provide a strict measure for the mismatch between the simulated and real urban patterns. Such errors need to be minimized for

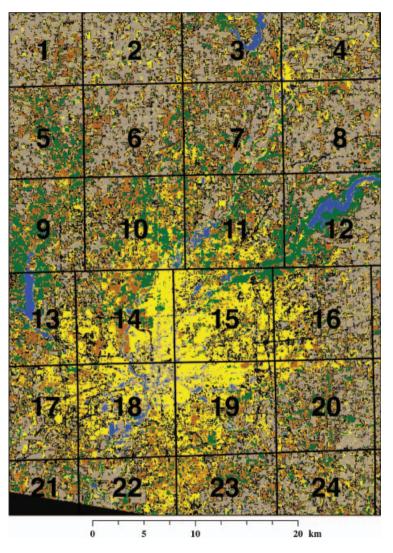


Figure 7. Study area of Indianapolis with township grid overlaid.

accurate modelling. The weighted average of Type I and II errors based on the urban and non-urban counts represents the overall modelling error. Out of all the simulation results in the search space, the final outcome is selected as the one that produces the minimum possible average error with a fitness level within  $100 \pm 10\%$ . It should be noted that all the measures are computed and summarized based on townships, and each township is evaluated separately.

Our modelling study involves two aspects: simulation and prediction. Simulation means running the model between two historical images (representing two growth years) where calibration (finding the best rule values) is performed at the end year to find the best rule values to produce the urban growth (level and pattern) between these years. However, the prediction means that the best rules found at a certain calibration year are used to extrapolate the urban growth pattern to identify the future urban growth. The oldest image of 1973 is used as the first input to the

cellular automata simulation. The computation is run for all possible combinations of  $(Y_{\rm R}, Y_{\rm C}, Y_{\rm S})$  from 1973 to 1982. The best parameter set is selected for each township based on the above criteria by comparing the simulation results with the 1982 image. In the next step, the selected rule values and the 1982 image are used to produce the simulation for 1987. The same procedure is repeated in 1987 to find the best set of calibration coefficients.

As addressed above, the term prediction refers to modelling without calibration at the destination year, i.e. one set of rule values is used for the modelling without modification. Prediction is carried out for the years 1992 and 2003, respectively. Once the best sets of calibration coefficients in 1987 are selected, the same rules are used to predict the year 1992. This represents a short-term prediction of a 5-year interval. The next prediction is performed for 2003 for a long-term period of 11 years starting from 1992 with its calibration results. A larger search space from -4to 4 for all calibration coefficients ( $\varepsilon_{\rm R}$ ,  $\varepsilon_{\rm C}$ ,  $\varepsilon_{\rm S}$ ) is used for this long-term prediction to investigate more combinations. The prediction results in 1992 and 2003 are evaluated respectively against their ground-data images. The final stage of prediction is future prediction beyond the available ground-data images. After calibration for the year 2003, the same rules are used to predict the year 2010 and 2020 under the same urbanization development conditions.

#### 4. Results and analysis

This section studies the calibration results and the modelling outcome. For this purpose, the best ( $\varepsilon_{\rm R}$ ,  $\varepsilon_{\rm C}$ ,  $\varepsilon_{\rm S}$ ) calibration coefficients and quality measures for 1982 and 1987 are presented in table 3. Table 4 lists the numerical results for the prediction years 1992 and 2003. First, we examine how the final calibrated values vary from the fuzzy approximations. As a general comment, they are farther away from the fuzzy output for townships away from the city centre (e.g. township #2), and closer to the fuzzy output for townships closer to the city centre (e.g. township #13). This suggests that simple fuzzy rules are unable to accurately model the urban growth, and certain corrections must be applied, especially for places away from the city centre. A closer look at table 3 reveals that the residential rule calibration result is the closest to the fuzzy output. This means that the fuzzy output provides a better initial approximation for the residential rules needed for reliable modelling compared with the other two rules, and there tends to be a positive correction for the calibration in all years. This indicates that the approximation provided by the fuzzy inference overestimates the urban development. As a result, the transition rules are tightened by the calibration by implementing positive corrections to the fuzzy output to match the real urban pattern. As an example, figure 8 plots the residential rule corrections and the residential pixel counts. It is seen that less developed townships usually have more restricted rules (large rule values) compared with more developed townships. This result is due to the fact that developed townships usually have a faster development rate than underdeveloped ones, so certain restrictions on their transition rules are needed to match the actual development pace.

As discussed earlier, a fitness measure is meant to ascertain that the correct urban development level is achieved. Figure 9, along with Tables 3 and 4, investigates the fitness variation over the townships for simulation years 1982 and 1987 and prediction years 1992 and 2003. Most ( $\sim$ 3/4) calibration fitness results are within the specified range of 100±10%. For the prediction years (figure 9(c)), most townships

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nive	Township	Rules ( $\varepsilon_{\rm R}, \varepsilon_{\rm C}, \varepsilon_{\rm S}$ )	
e Ú	1	(1,0,2)	
rdu	2	(3,3,3)	
[Pu:	2 3 4 5	(0,3,1)	
ï.	4	(3,3,3)	
B B	5	(3,3,0)	
dec	6	(3,3,0)	
loa	7	(0,0,3)	
umo	8	(0,3,3)	
Ā	9	(0,3,3)	
	10	(0,3,1)	
	11	(0, -1, 1)	
	12	(0,-1,1)	
	13	(0, -1, 1)	
	14	(3, -2, 1)	
	15	$(2 \ 2 \ 0)$	

Table 3. Calibration results for 1982 and 1987.

	1982				1987					
	Error %			Error %						
Township	Rules ( $\varepsilon_{\rm R}, \varepsilon_{\rm C}, \varepsilon_{\rm S}$ )	Fitness %	Type I	Type II	Average	Rules ( $\varepsilon_{\rm R}$ , $\varepsilon_{\rm C}$ , $\varepsilon_{\rm S}$ )	Fitness %	Type I	Type II	Average
1	(1,0,2)	91.63	64.64	16.63	26.71	(3,3,3)	148.82	56.86	17.54	22.59
2	(3,3,3)	97.36	60.40	17.76	26.83	(3,3,3)	110.70	51.89	16.97	23.11
3	(0,3,1)	90.78	40.70	20.37	26.42	(2,1,3)	103.91	38.36	22.43	26.92
4	(3,3,3)	99.53	39.59	20.08	25.40	(3,3,3)	100.47	43.02	19.35	25.35
5	(3,3,0)	92.86	57.90	19.52	29.01	(2,3,3)	116.67	44.27	18.71	23.74
6	(3,3,0)	88.71	50.85	18.68	27.30	(2,3,2)	104.28	42.31	20.04	25.58
7	(0,0,3)	91.17	41.19	25.56	31.14	(1,3,1)	106.48	33.77	30.28	31.58
8	(0,3,3)	96.68	41.31	19.31	25.05	(3,3,3)	103.43	49.33	19.84	26.85
9	(0,3,3)	91.85	49.99	17.83	27.00	(0,0,2)	91.38	31.50	23.50	26.47
10	(0,3,1)	89.67	26.88	29.91	28.32	(3,3,1)	93.56	18.23	33.82	24.26
11	(0, -1, 1)	90.89	25.61	33.58	29.22	(0,3,1)	102.53	14.17	43.90	25.36
12	(0, -1, 1)	97.99	48.12	19.82	27.00	(1,0,3)	107.44	45.52	26.73	32.21
13	(0, -1, 1)	93.43	22.65	27.53	25.08	(1,0,3)	94.85	17.54	31.27	23.90
14	(3,-2,1)	90.61	14.66	31.65	19.50	(0,3,1)	101.76	6.54	57.28	17.85
15	(3,-2,0)	101.17	2.16	60.05	11.59	(0, -3, 3)	103.79	0.54	81.48	11.42
16	(0,3,1)	98.17	24.51	29.38	27.23	(1,1,2)	98.57	22.98	27.63	25.53
17	(3,3,0)	95.57	15.61	33.82	23.40	(2,0,3)	101.79	16.03	41.16	26.87
18	(-3, -3, 3)	99.69	0.08	76.38	8.90	(-2,3,3)	101.54	1.22	81.58	11.80
19	(3,3,0)	94.40	9.06	46.54	17.00	(3,0,3)	100.97	7.25	46.31	16.87
20	(0,3,2)	91.33	39.97	23.79	29.65	(3,1,2)	106.27	35.67	27.54	30.34
21	(0, -1, 1)	86.41	40.52	26.84	32.13	(3,3,3)	120.79	27.17	25.74	26.16
22	(0,0,3)	90.31	28.11	34.02	30.88	(0,3,3)	107.83	17.18	40.35	28.09
23	(0, -1, 1)	91.67	28.14	32.99	30.51	(3,3,3)	107.56	17.23	29.10	23.81
24	(0,3,3)	90.34	47.38	26.44	33.44	(3,3,3)	128.73	44.75	26.78	31.31
Average		93.43	34.17	29.52	25.78	· · · · /	106.84	28.47	33.72	24.50

	1992				2003				
	Error %			Error %					
Township	Fitness %	Type I	Type II	Average	Fitness %	Type I	Type II	Average	
1	101.34	77.72	11.22	19.80	115.55	79.94	11.83	19.43	
2	112.10	67.96	14.82	23.50	82.29	70.36	13.57	24.74	
3	94.79	52.49	22.56	32.69	89.47	45.82	22.70	31.93	
4	99.15	57.13	19.51	29.46	81.62	57.48	19.65	32.00	
5	95.52	62.62	15.17	25.58	106.07	50.10	18.44	26.22	
6	87.82	53.07	17.12	28.51	101.46	32.82	35.88	34.34	
7	104.18	36.98	33.00	34.90	91.01	30.48	34.92	32.23	
8	93.16	60.75	19.20	30.32	60.69	59.18	18.22	36.81	
9	138.55	25.76	41.53	34.65	86.31	27.88	19.97	23.91	
10	111.63	11.75	43.59	20.72	107.36	6.92	73.92	19.61	
11	107.12	10.72	48.69	18.99	117.95	0.43	89.98	15.73	
12	94.57	45.25	28.01	35.79	78.26	33.60	25.72	30.76	
13	99.30	21.06	29.80	23.83	99.56	11.08	38.94	18.51	
14	114.56	3.40	71.34	13.01	111.50	1.04	86.59	13.68	
15	114.71	0.01	95.75	6.78	103.58	0.22	82.20	6.77	
16	117.54	22.65	29.58	26.35	75.88	30.78	17.87	25.87	
17	96.80	21.50	39.15	26.32	99.98	12.96	36.79	18.54	
18	115.03	0.57	95.89	7.19	106.76	0.17	84.35	9.23	
19	105.79	10.93	51.67	18.26	94.15	8.86	40.45	13.75	
20	123.43	37.24	27.14	30.49	71.99	42.37	16.68	28.90	
21	77.11	46.34	13.99	27.11	117.40	34.21	37.29	36.00	
22	120.52	13.30	37.90	23.62	95.55	20.11	36.85	25.59	
23	78.61	33.99	17.60	27.49	88.37	22.10	40.72	26.76	
24	104.10	58.75	20.21	29.81	68.33	58.64	17.68	32.40	
Average	104.48	34.66	35.19	24.80	93.80	30.73	38.38	24.32	

Table 4. Year 1992, 2003 prediction results (5-, 11-year interval).

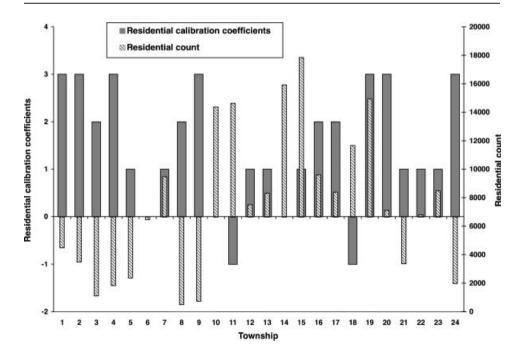


Figure 8. 1992 residential calibration coefficients and residential urban counts.

achieve fitness within the  $100 \pm 20\%$  range, and about one-half of the townships are within the  $100 \pm 10\%$  range. As expected, short-term prediction fitness results (5 years, from 1987 to 1992) are closer to 100% as compared with the long-term results (11 years, from 1992 to 2003). Most fitness results have a fairly random distribution around the 100% line. Less developed townships are more sensitive to the changes in the transition rules, since the number of urban pixels is small, and any small change can affect the fitness level, as can be seen in the townships remote from the city centre. In addition to the fitness, figure 9(b) and (d), along with Tables 3 and 4, presents the average errors for the calibration and prediction years. The average error represents the combined effects of Type I and II error types according to their dominance in the townships than with the townships near the city centre. This suggests that well-developed townships yield fewer modelling errors. For a majority (~80%) of the townships, the errors are less than 30% in both the simulation and prediction scenarios, and both have a similar range of error variation.

The next evaluation is to study the components of errors to discover any useful patterns related to our modelling. The distribution of the modelling errors with respect to the urbanization level is shown in figure 10. It can be seen clearly that the error domination for a specific township is related to its urbanization level. Type I errors dominate townships with a lower urbanization level, while Type II errors dominate those with a higher urbanization level. For example, townships 1–5 in the

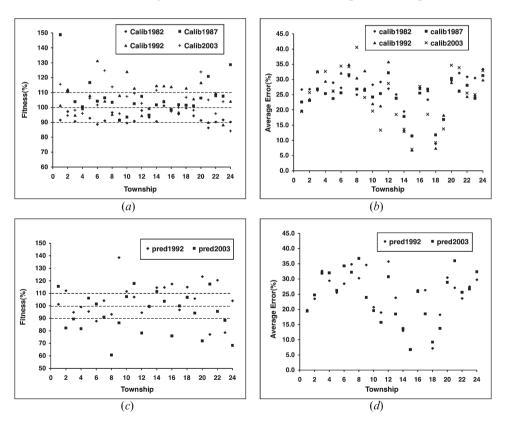


Figure 9. Fitness (left) and average error (right) of calibrated (top) and predicted (bottom) years.

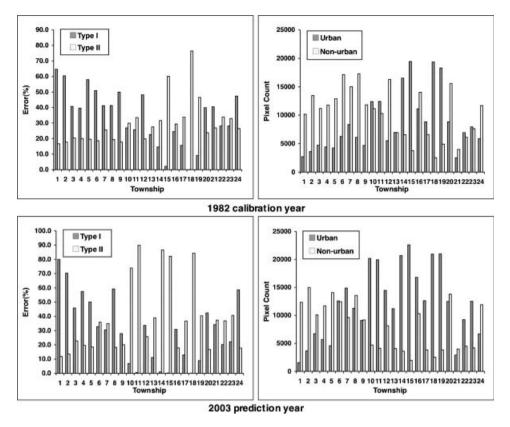


Figure 10. Modelling errors (left) vs. urban development level (right).

calibration year 1982 are mostly composed of non-urban pixels. This results in most modelling errors being Type I, and Type II errors are a small percentage. This is due to the fact that the Type II error count is divided by a large number of non-urban pixels. Another example is township 15 in the middle of the city, which has a high percentage (82.2%) of Type II errors and only 0.22% of Type I errors for the prediction year 2003. In general, Type I errors are more dominant in townships far from the city, while Type II is more dominant in closer townships. The weighted average of the errors is a more comprehensive indicator, since it takes into account not only the error magnitude but also the urbanization level. It balances the two types of errors and is less sensitive to the absolute error count.

This paragraph further compares the modelling results with the ground data in terms of spatial connectivity and smoothness. For this purpose, we examine a window of  $20 \times 20$  pixels ( $1200 \text{ m} \times 1200 \text{ m}$ ) at three different locations off the city centre. As shown in figure 11, the simulated (1987) and predicted (1992) images and their corresponding ground data are selected. A clear observation is that the modelling results have a higher connectivity than the classified ground-data images. Ground data as a result of image classification are more discrete than the connected and continuous simulation results. Another important observation is that windows with close urban counts may still have different urban structures, which suggests that urban count alone is not sufficient to judge the modelling quality and that additional measures are needed to describe the urban structure. Type I and II errors

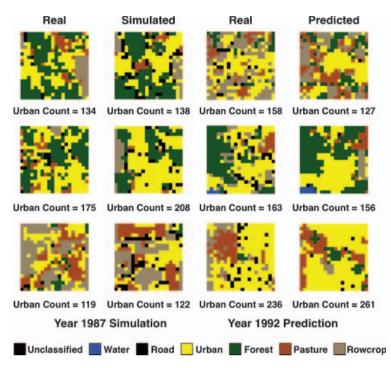


Figure 11. Ground data and modelling results in three selected windows for 1987 (left) and 1992 (right).

meet this need by measuring how the modelling results match the real data on a pixel-by-pixel basis. These two criteria along with the urban count (fitness) ensure that the modelling results will have not only the correct urbanization level but also the right urban structure.

The modelling results over the entire study area are shown in figure 12 for the calibration years (1982 and 1987) and figure 13 for the prediction years (1992 and 2003). The simulated images are produced using the corresponding ground data as a reference for rule calibration, while the prediction results are produced without calibration at the destination year. Once the calibrated rules are obtained for 1987, the same rules are used to predict 1992. The 2003 image is predicted from the calibrated rules for 1992. It is seen that the prediction in general provides results that are as good as the simulated results in terms of the urbanization pattern. The results validate that the fuzzy inference guided cellular automata is able to adapt to the urban dynamic changes spatially (township effect) and temporally (time-variable). This supports the understanding that the urbanization process is dynamic in space, and the transition rules need to vary spatially in the study area. Rule calibration over time reduces the accumulation and propagation of simulation errors over the prediction period. The results prove that multi-temporal satellite images are effective ground data to serve this purpose. Moreover, the close structure match between the modelled and real data is mainly due to the use of Type I and II error measures. Minimizing such errors ensures that the modelling process reproduces a realistic urbanization pattern. Finally, figure 14 presents the prediction results for years 2010 and 2020. Future prediction years assume that the same urban pattern calibrated at

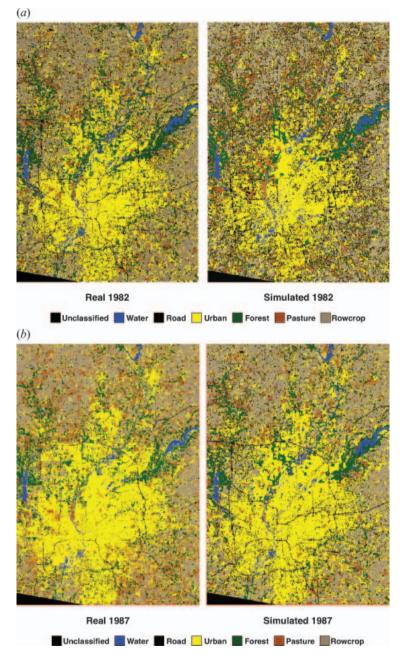


Figure 12. (a) Indianapolis 1982 calibration results. (b) Indianapolis 1987 calibration results.

2003 will be followed with the same urbanization rate. Smooth results can be seen for future prediction with accelerated directional urbanization toward the northeast, north, and east directions, which follows the pattern in the historical data from 1973 to 2003.

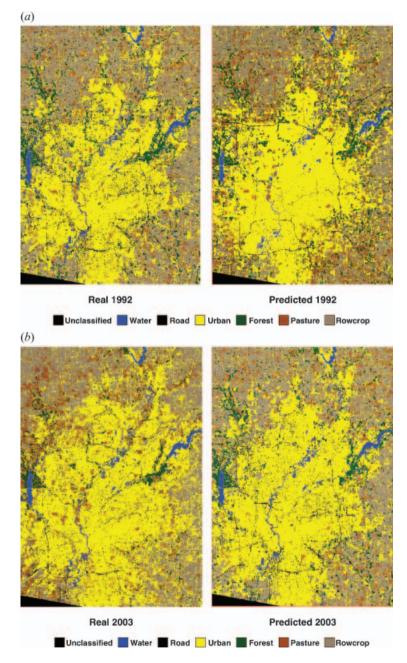


Figure 13. (a) Indianapolis 1992 prediction results. (b) Indianapolis 2003 prediction results.

## 5. Conclusions

Coupling fuzzy inference with cellular automata has several distinct advantages. Fuzzy inference can take into consideration the linguistic knowledge about the urban development. In this way, concepts and semantic knowledge about urban development can be easily incorporated into the modelling process. The fuzzification step conceptualizes various input data, while the defuzzification step digitalizes

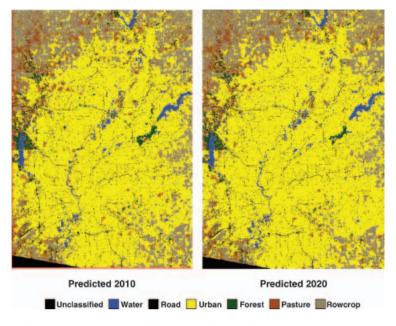


Figure 14. Future prediction of Indianapolis at year 2010 and 2020.

the fuzzy inference outcome. We regard this as very beneficial for urban planners and decision-makers, since they only need to provide rules or knowledge instead of exact mathematic expressions for geographic phenomena. Unlike the traditional crisp cellular automata, the fuzzy inference guided cellular automata method takes the development potential of a pixel into account to preserve the continuous spatial nature of the urban growth process. Moreover, this approach also simplifies the definition of the transition rules and provides good approximations for their parameters.

Calibration in fuzzy inference guided cellular automata modelling is meant to find the calibration coefficients, i.e. the corrections to the fuzzy inference output, within a small search space to best match the real urbanization level and pattern. We present a calibration strategy in both the spatial and temporal domains. The township-based calibration considers the spatial variations in the urban development and proves to be efficient in improving the prediction outcome. Calibration over time based on the historical satellite images detects the dynamic urban growth pattern so that the rules can be adapted accordingly, and the prediction errors will not accumulate for a long time. This is necessary, since some growth periods may experience excessive or slower growth rates than other periods.

The three quality measures, Fitness and Type I and II errors, are shown to be effective and comprehensive in identifying the best calibration results. The fitness measure ensures that the correct urban development level is achieved, while the Type I and II errors represent the mismatch between the simulated and the real urban patterns. Because of the nature of the pixel-by-pixel evaluation, Type I and Type II errors are shown to be critical in the quality evaluation. It is appropriate to retain the best modelling results as those with the fewest errors at a satisfactory fitness level (e.g.  $100 \pm 10\%$ ). Under the above quality measures, the majority (~3/4) of the townships achieve a fitness level of  $100 \pm 10\%$  in the calibration, while only about one-half of the townships meet the same fitness level in the prediction. Based on this

study, it is shown that 80% of the townships can achieve a fitness level of  $100\% \pm 20\%$  in urban prediction.

Our study observes that the Type I error dominates under-developed areas, while the Type II error dominates well-developed areas. However, the weighted average errors behave less dramatically across the townships, and in fact both the calibration and the prediction scenarios yield about the same average error of 25–30%. The study reveals that image classification contributes to such errors due to the discrete outcome of the pixel-based image classification and the local continuity of the cellular automata modelling. Consequently, the average errors are critical and tend to exaggerate the real inconsistency between the modelling results and the ground data, despite having a very similar pattern and structure.

This study also reveals a few topics to be further explored. The selection of the fuzzy membership functions and the fuzzy rules is rather subjective in this study, motivated mainly by a proof of concept study. However, a certain optimization is needed as a guide to better approximation for the cellular automata modelling. It would be interesting to study how the final modelling outcome is dependent on the fuzzy membership functions and the fuzzy rules. The searching space for the transition rule calibration in cellular automata may need to be tuned in an autonomous way during the calibration process to obtain effective as well as sufficiently accurate results. Object-based classification instead of the pixel-based classification might be more suitable for providing the ground data for the transition rule calibration and assessing the modelling outcome. Scale or resolution has always been a critical issue in urban modelling. The rules and methodology found at one scale need to be tested at another scale to evaluate their suitability and performance across different scales or resolutions. Moreover, the pixel size and the neighbourhood dimension in the transition rules should also be studied in terms of their effects on urban modelling and rule calibration.

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