ECE 438 Assignment No. 10 Summer 2004

1. This problem investigates least-squares linear prediction, using the covariance method for determining the optimal coefficients. Consider the signal

$$x(n) = \left\{2^{-n} + 4^{-n}\right\}u(n)$$

a. Calculate an expression for the covariance function of x(n) in terms of k, l, and P, for $0 \le k \le P$ and $0 \le l \le P$.

$$\phi(k,l) = \sum_{n=P}^{\infty} x(n-k)x(n-l)$$

b. Set up the appropriate equations for determining the optimal coefficients for the second-order predictor (P=2) shown below. Then solve for the coefficients α_1 and α_2. (You may use a calculator/Matlab to determine the covariance values and to solve the system, but be sure to show the equations you are solving.)

$$\hat{x}(n) = \alpha_1 x(n-1) + \alpha_2 x(n-2)$$

c. Determine the numerical value of the error for this predictor (given below, P=2).

$$E = \sum_{n=P}^{\infty} \{x(n) - \hat{x}(n)\}^2$$

d. Repeat parts (b) and (c) for a first-order predictor (P=1). Compare the resulting error to the second-order predictor.

e. Determine the optimal coefficients for a third-order predictor.
Hint: This is very easy if you did parts (b) and (c) correctly.

2. This problem uses the autocorrelation method for determining the optimal LPC coefficients. Consider the signal

$$x(n) = \left\{2^{-n} + 4^{-n}\right\}u(n)$$

a. Calculate an expression for the autocorrelation function for x(n).

$$R(m) = \sum_{n=-\infty}^{\infty} x(n)x(n-m)$$

b. Set up the appropriate equations for determining the optimal coefficients for the second-order predictor (P=2) shown below. Then solve for the coefficients α_1 and α_2. (You may use a calculator/Matlab to determine the autocorrelation values and to solve the system, but be sure to show the equations you are solving.)

$$\hat{x}(n) = \alpha_1 x(n-1) + \alpha_2 x(n-2)$$

c. Determine the numerical value of the error for this predictor (given below). NOTE that this expression is different from problem (1).

$$E = \sum_{n=-\infty}^{\infty} \{x(n) - \hat{x}(n)\}^2$$

d. Compare the coefficients and error to the P=2 results from the covariance method of problem (1).

3. Define the complex cepstrum $\hat{x}(n)$ of a DT signal x(n) as the inverse DTFT of the log of the DTFT of x(n):

$$\hat{x}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left[X(e^{j\omega})\right] e^{j\omega n} d\omega$$

Consider an all-pole model of the vocal tract given by the following transfer function:

$$V(z) = \prod_{k=1}^{Q} \frac{1}{(1-p_k z^{-1})(1-p_k^* z^{-1})}$$

where $p_k = r_k e^{j\theta_k}$ and $0 < r_k < 1$, for k=1,...,Q. Let v(n) be the inverse Z-transform of V(z) (i.e. v(n) is the impulse response of the vocal tract). Show that the cepstrum of v(n) may be expressed as

$$\hat{v}(n) = 2\sum_{k=1}^{Q} \frac{(r_k)^n}{n} \cos(\theta_k n) u(n-1)$$

Hint: You may want to use the following DTFT relationship:

$$n\,x(n) \overset{DTFT}{\Leftrightarrow} j\frac{d}{d\omega}X(e^{j\omega})$$