

1. Show the following relations are true:

a. $\cos(a) = \frac{1}{2}(e^{ja} + e^{-ja})$ $\sin(a) = \frac{1}{2j}(e^{ja} - e^{-ja})$ (Use Euler's rule)

b. $(z_1 + z_2)^* = z_1^* + z_2^*$ $(z_1 z_2)^* = z_1^* z_2^*$ $z z^* = |z|^2$

2. Perform the convolution of the following pair of signals:

$$x_1(n) = \left(\frac{1}{2}\right)^n u(n) \quad \text{and} \quad x_2(n) = \left(\frac{1}{3}\right)^n u(n)$$

3. Consider the following system:

$$y[n] = x[n] + a y[n-1] \quad \text{where } a \text{ is some complex constant.}$$

- i. Find the impulse response.
- ii. For which values of a is the system stable? Justify your answer.
- iii. Find an expression for the frequency response (you don't need to simplify).
(Hint: See the example in the lecture notes, p.16+, section 1.2.3.)
- iv. Find an expression of the *magnitude* of the frequency response (simplify this one as much as possible).