

1. Consider the *notch filter* with the following transfer function:

$$H(z) = \frac{(z - e^{j\omega_0})(z - e^{-j\omega_0})}{(z - re^{j\omega_0})(z - re^{-j\omega_0})} G$$

where r and ω_0 are real constants such that $0 < r < 1$ and $0 < \omega_0 < \pi$, and G is a constant which specifies the *gain* of the filter.

- Draw a pole-zero diagram for $H(z)$.
 - Determine an expression for the gain G such that the filter has a DC response of 1 (i.e. $H(e^{j0})=1$). Simplify this into a real-valued expression.
 - Use Matlab to plot the magnitude response $|H(e^{j\omega})|$ for $-\pi < \omega < \pi$. Generate 3 plots using $r=\{0.90, 0.95, 0.99\}$ with $\omega_0=\pi/2$. Use the gain that you determined in part b.
 - Determine a linear constant-coefficient difference equation which implements this system. All the coefficients should be real-valued.
2. Compute the inverse Z-transform of the functions below. Show your work. You may use (only) the transform properties/pairs that were given in class.

a. $X(z) = \frac{z^{-4}}{4z - 1}$ where $ROC = \left\{ |z| > \frac{1}{4} \right\}$

b. $X(z) = \frac{1}{(z - a)^2}$ where $ROC = \{ |z| > |a| \}$

Hint for b: Differentiate $\frac{1}{z - a}$ and use the given properties.