

1. We defined the short-time discrete-time Fourier transform (stDTFT) as the following:

$$X_m(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)w(n-m)e^{-j\omega n}.$$

Let $x(n)$ and $w(n)$ have DTFTs $X(e^{j\omega})$ and $W(e^{j\omega})$. Show that the following relationships are true. Clearly show all the steps.

a. Shifting property -- If $v(n) = x(n-n_0)$, then $V_m(e^{j\omega}) = X_{m-n_0}(e^{j\omega})e^{-j\omega n_0}$.

b. $X_m(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j(\omega-u)})W(e^{ju})e^{-jum} du$

c. Inverse stDTFT -- $x(n) = \frac{1}{2\pi w(0)} \int_{-\pi}^{\pi} X_n(e^{j\omega})e^{j\omega n} d\omega$

2. A vowel is spoken by someone. The pitch period is 5 milliseconds and the first three formant frequencies are 500 Hz, 1.5 kHz, and 2.5 kHz. The waveform is sampled at a rate of 20 kHz, and the stDTFT defined above is used to create a spectrogram. Sketch roughly what the spectrogram display of $|X_m(e^{j\omega})|$ would look like as a function of ω (rad/sample) and m (discrete time index) if the window $w(n)$ has the lengths below. Be sure to label all relevant quantities in the figures.
- $N = 50$ points
 - $N = 600$ points
 - For both (a) and (b), state whether the spectrogram is wideband or narrowband.

3. A *Hanning* window of length M is defined by the following:

$$w_{\text{hann}}(n) = \left\{ \frac{1}{2} + \frac{1}{2} \cos \left[\frac{2\pi}{M} \left(n - \frac{M-1}{2} \right) \right] \right\} \{u(n) - u(n-M)\}$$

- a. Derive $W_{\text{hann}}(e^{j\omega})$, the DTFT of $w_{\text{hann}}(n)$.
Hint: You may want to use results that were derived in class.
- b. Sketch $w_{\text{hann}}(n)$ and $|W_{\text{hann}}(e^{j\omega})|$. For the DTFT, you don't have to compute the peak sidelobe amplitude, but do indicate the mainlobe width and the location of the nulls.