The Capacity of Ad hoc Networks under a Realistic Link Layer Model

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Presenting on behalf of:
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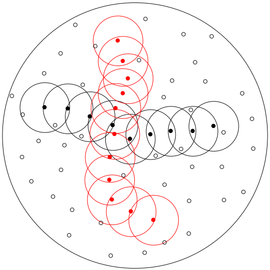
Outline

1. Motivation
2. Capacity under a Realistic Link Layer Model
   - Model
   - Capacity Analysis

Capacity [Gupta, IEEE Trans. Info Theory, 2000]

- n static nodes deployed randomly and uniformly over fixed area
- Random source-destination pairs
- Limited transmit power $\Rightarrow$ multi-hopping
- Observation: Relaying load lowers network capacity

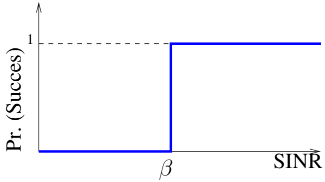


Main Result [Gupta, IEEE Trans. Info Theory, 2000]

Per node capacity of a random ad hoc network is $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$, and this bound can be achieved.

Motivation

- Gupta-Kumar:
  - Scheduling $\Rightarrow$ SINR $\geq \beta$
  - Assume PER is 0 if SINR $\geq \beta$
- For a given modulation and coding scheme:
  - PER $\rightarrow$ 0 continuously and asymptotically as SINR $\rightarrow \infty$
  - Finite SINR $\Rightarrow$ PER $\neq 0$
- Even with SINR $\geq \beta$, PER $\neq 0$ on each link!
- **Question:** Are the capacity results affected?



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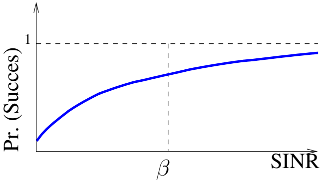

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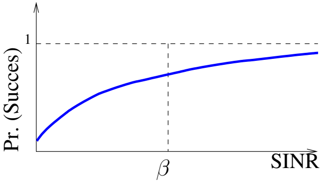

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Network with Lossy Links

- **Assume:** Each link in the network is lossy with probability $q = 1 - p > 0$
- Number of hops from source to destination increases as network size increases, $h_n \rightarrow \infty$
- End-to-end throughput is
  $$\Theta\left(\frac{1}{\sqrt{n \log n}}\right) \cdot \Theta\left(p^{h_n}\right)$$
- **Will show:**
  - PER accumulates over multiple hops, capacity degrades
- **Main Finding:**
  - Scheduling policy and/or retransmission strategy and/or coding should scale with network size

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Outline

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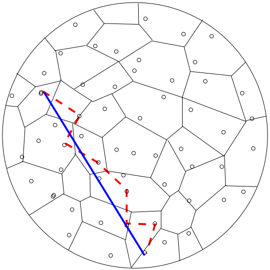
Settings

Gupta-Kumar [Gupta, IEEE Trans. Info Theory, 2000]

- Voronoi tessellation: Uniform cell size, ρ_n
- Routing: Cellular, straight-line
- Connectivity: Cell size, $\rho_n = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$
- Scheduling: Ensures SINR $\geq \beta$, each cell transmits once every $K = K(\beta)$ slots
- Retransmissions: None
- Modulation, coding: Do not scale with n



Realistic Link Layer

- SINR to PER mapping is continuous

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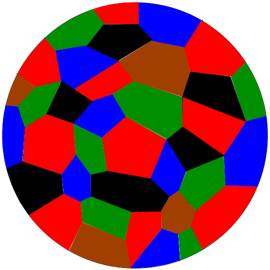

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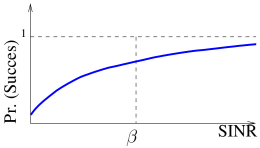

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Proof Plan

Will show: Lossy links can degrade network capacity

For each connection, for a fixed fraction of hops:

1. **Signal Strength $\leq$:** Transmitter-receiver separation $\geq t\rho_n$
2. **Interference $\geq$:** Another node transmitting simultaneously within a distance of $(M+8)\rho_n$ of the receiver

- SINR upper bounded by a fixed constant, $\left(\frac{t}{M+8}\right)^{-\alpha}$
- PER lower bounded by $1-p>0$
- $\Rightarrow O\left(\frac{1}{n}\right)$ instead of $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$ per node throughput!

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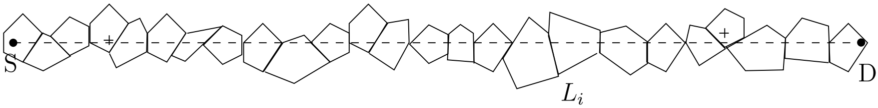
Step 1: Signal Strength



Lemma

Fix t such that $0 < t < 1$. For connection i of length L_i, let h_i be the number of hops that are longer than $t\rho_n$. Then,

$$h_i \geq \left(\frac{1 - \frac{16t}{\pi}}{8 - t} \right) \cdot \frac{L_i}{\rho_n}$$

Proof Idea

Uniform cell size $\Rightarrow$ Not all hops are arbitrarily short

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Step 2: Interference

contd.

Lemma

Fix $M > 9$. Let N_i be the number of hops of connection i such that there is no simultaneous interfering transmission within a circle of radius $(M+8)\rho_n$ around the receivers of those hops (interference-free cells). Then,

$$N_i \leq \frac{2K}{M}\frac{L_i}{\rho_n}.$$

Proof Idea

At most K colors $\Rightarrow$ Co-channel interferer in the M-neighborhood

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Summary

- For at least $\phi \frac{L_i}{\rho_n}$ hops, $0 < \phi < \frac{1}{8}$:
  - Upper bound on signal strength, $(t\rho_n)^{-\alpha}$
  - Lower bound on interference, $((M+8)\rho_n)^{-\alpha}$
  - $\Rightarrow$ Upper bound on SINR, $\left(\frac{t}{M+8}\right)^{-\alpha}$
  - $\Rightarrow$ Lower bound on PER $1 - p > 0$
- Throughput is $O\left(\frac{1}{n}\right)$ and not $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$, for the scheduling and routing scheme proposed by Gupta and Kumar
- Results hold for straight line as well as arbitrary routing
- We can do better!!

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Throughput Improvement via Scheduling

Key Factor for Throughput Degradation

Interferer within $(M + 8)\rho_n \Rightarrow$ upper bound on SINR

- Use reduced spatial reuse, i.e., K_n colors instead of K colors, and $K_n \to \infty$
- Link quality improves $\Rightarrow$ End-to-end PER guaranteed
- But spatial re-use reduces as scheduling scales, throughput is $\Theta\left(\frac{1}{K_n\sqrt{n \log n}}\right)$
- Can also show that the above bound is achievable if $K_n = \Omega\left(\sqrt{\log n}\right)$, capacity then is $\Theta\left(\frac{1}{\sqrt{n} \cdot \log n}\right)$ More on K_n...

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Summary

- Network capacity under a realistic link layer model
- Aggressive Scheduling and Dumb Retransmission $\Rightarrow O(\frac{1}{n})$
- Conservative Scheduling $\Rightarrow \Theta\left(\frac{1}{K_n\sqrt{n\log n}}\right)$
- Since lossy links and long paths are the major limiting factors, results can be generalized to any other large multi-hop network (Ongoing Work)
- Extension under Fading and Shadowing (Ongoing Work)
- Could also scale number of retransmission attempts and coding scheme (Ongoing Work)

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THANK YOU!

What should K_n be?

Want $P(\{\text{End-to-end Packet loss}\}) < \epsilon$

$$P(\{\text{End-to-end Packet loss}\}) = P\left(\bigcup_i \{\text{Packet lost over link } i\}\right)$$

$$P\left(\bigcup_i \{\text{Packet lost over link } i\}\right) \leq \sum_i P(\{\text{Packet lost over link } i\})$$
$$\leq \frac{16}{\pi} \cdot \frac{L_i}{\rho_n} \cdot P(\{\text{Packet lost over link } i\})$$

Need to ensure that:

$$P(\{\text{Packet lost over link } i\}) < c \cdot \rho_n \cdot \epsilon$$

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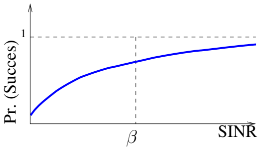
What should K_n be?
contd.

$$P(\{\text{Packet lost over link } i\}) < c \cdot \rho_n \cdot \epsilon$$
$$\Rightarrow \text{SINR}_i \geq f^{-1}(c\epsilon\rho_n)$$



- Can show that SINR grows linearly with K_n

$$\Rightarrow K_n = \Omega\left(f^{-1}(c\epsilon\rho_n)\right)$$

- Interference is sum of large number of random signals
- Central Limit Theorem $\Rightarrow$ Gaussian distribution for PER

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What should K_n be?
contd.

$$P(\gamma) = \text{erfc}(\gamma)$$

$$\text{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$$

$$\Rightarrow K_n = \Omega\left(\sqrt{\log n - \log\log n}\right)$$

Throughput of the order of $\Theta\left(\frac{1}{\sqrt{n} \cdot \log n}\right)$ Return

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