Sol'n. to Prob. 1

(a) $x(n) = \cos\left(\underbrace{\tfrac{\pi}{2}}_{\frac{2\pi(4)}{16}} n\right)\{u(n) - u(n-8)\}$

$$\cos\left(\frac{2\pi(4)}{16}n\right) \xleftrightarrow[16]{DFT} 8\,\delta(k-4) + 8\,\delta(k-12) = X_{16}(k)$$

$k = 0, 1, \ldots, 15$

N = 16 pt. DFT of $h(n)$:

$$H_{16}(k) = \sum_{n=0}^{15} \left(\tfrac{1}{2}\right)^n e^{-j\frac{2\pi kn}{16}} = \sum_{n=0}^{15}\left(\tfrac{1}{2}e^{-j\frac{2\pi k}{16}}\right)^n$$

$$= \frac{1 - \frac{1}{2^{16}} e^{-j2\pi k}}{1 - \frac{1}{2}e^{-j\frac{2\pi k}{16}}} = \frac{1}{1 - \frac{1}{2}e^{-j\frac{\pi k}{8}}}$$

$k = 0, 1, \ldots, 15$

where: $1 - \frac{1}{2^{16}} \approx 1$

(b)

$$Y_{16}(k) = H_{16}(k)\,X_{16}(k)$$

$$= 8\,\frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{2}}}\,\delta(k-4) + 8\,\frac{1}{1-\frac{1}{2}e^{-j\pi\frac{3}{2}}}\,\delta(k-12)$$

$$= \frac{16}{2+j}\,\delta(k-4) + \frac{16}{2-j}\,\delta(k-12)$$

$$= \frac{16}{\sqrt{5}}\,e^{-j26.6^\circ}\,\delta(k-4) + \frac{16}{\sqrt{5}}\,e^{+j26.6^\circ}\,\delta(k-12)$$

$$y_8(n) = \frac{2}{\sqrt{5}}\cos\left(\frac{\pi}{2}n - 26.6^\circ\right)\{u(n) - u(n-15)\}$$

Sol'n. to Prob. 1 (cont.)

(c) $x(n)$ is of length $L=16$
$h(n)$ is of length $M=16$
} linear convolution $y(n) = x(n) * h(n)$ is of length $M+L-1 = 31$

time-domain aliasing: $$y_{16}(n) = \sum_{\ell=-\infty}^{\infty} y(n-\ell 16)\{u(n)-u(n-16)\}$$

$$= y(n) + y(n+16) \qquad n=0,1,\ldots,15$$

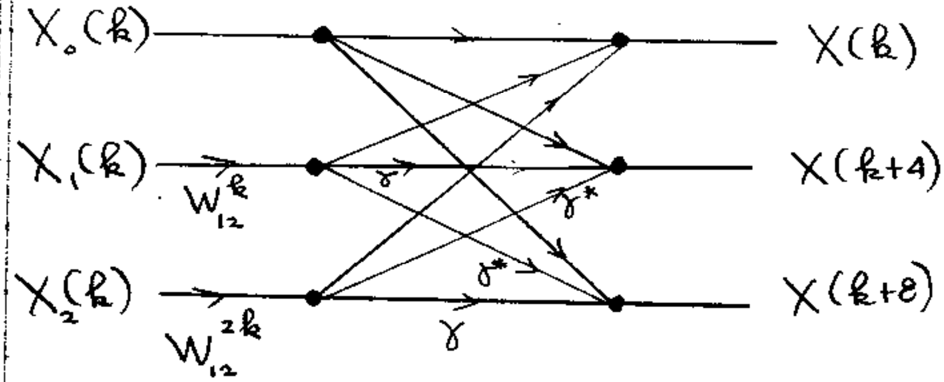
since $y(n)$ is only nonzero for $n=0,1,\ldots,30$

thus: $y_{16}(15) = y(15) + y(31) = y(15)$ } only unaliased point

all other 15 points are aliased: $y_{16}(n) \neq y(n)$ for $n=0,1,\ldots,14$

Solution to Problem 2

- the computation involved in computing $X(k)$, $X(k+4)$, and $X(k+8)$ from $X_0(k)$, $X_1(k)$, and $X_2(k)$ may be represented in terms of a 3 pt. DFT as



$k=0,1,2,3$

- where: $W_{12} = e^{-j\frac{2\pi}{12}}$ and $\gamma = e^{-j\frac{2\pi}{3}}$
- note: the 3 pt. butterfly involves 6 complex mults. for each k.

(a) from class we know that to compute an N pt. DFT via a decimation-in-time radix 2 FFT requires a total of $\frac{N}{2}\log_2 N$ complex mults.

answer: for $N=4 \Rightarrow \frac{4}{2}\log_2 4 = 4$ complex mults

(b) When an N pt. DFT is factored as $N = ML$ and the Divide and Conquer Approach is applied, we know from class (and the Demo DivideConquer.m) that the no. of complex multiplies is $ML^2 + N + LM^2$. Here $M=3$ and $L=4$. However, we will compute the 4-pt. DFT in terms of a radix 2 FFT. The formula is thus modified as

$$M\left(\frac{L}{2}\log_2(L)\right) + N + LM^2 = 3(4) + 12 + 4(3)^2 = 60$$

(c) direct computation of a 12 pt. DFT requires

$12^2 = 144$ complex mults.

- $\frac{60}{144} = 41.7\% \Rightarrow 58.3\%$ reduction in complex mults.

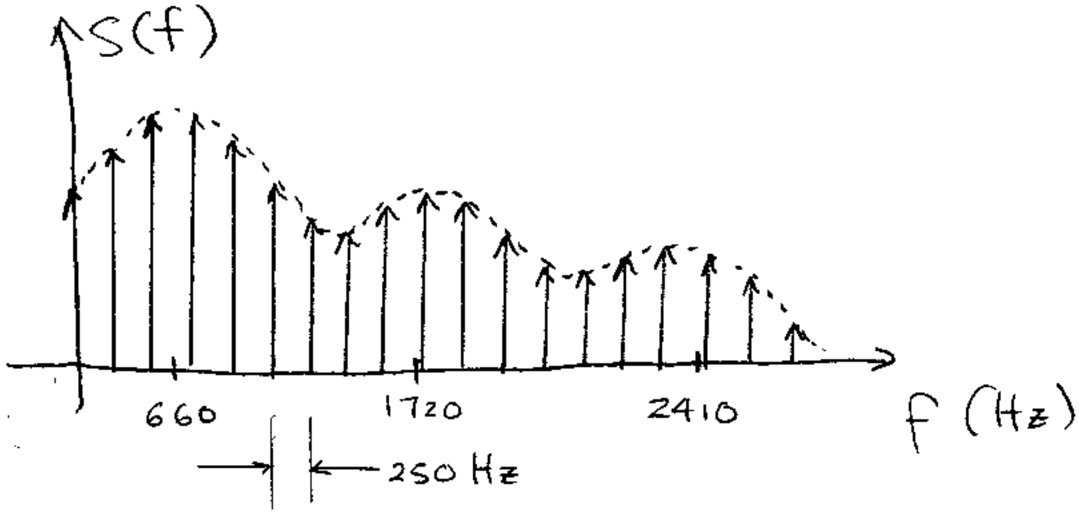
Sol'n. to Problem 3

(a) Pitch period: $\frac{40}{F_s} = \frac{40}{10^4/\text{sec}} = 4$ ms

(b) Pitch frequency $= \frac{1}{4\times10^{-3}\text{s}} = 250$ Hz $\Rightarrow$ female

(c) $F_1 = (.066)\times10^4 = 660$ Hz
$F_2 = (.172)\times10^4 = 1720$ Hz
$F_3 = (.241)\times10^4 = 2410$ Hz

(d) consulting the table of average formants for the vowels, the most likely voiced phoneme is AE (or ae) -- the a sound in bat



$\Rightarrow$ since periodic forever, get spectral lines, i.e., Dirac Delta functions at every integer multiple of the pitch frequency