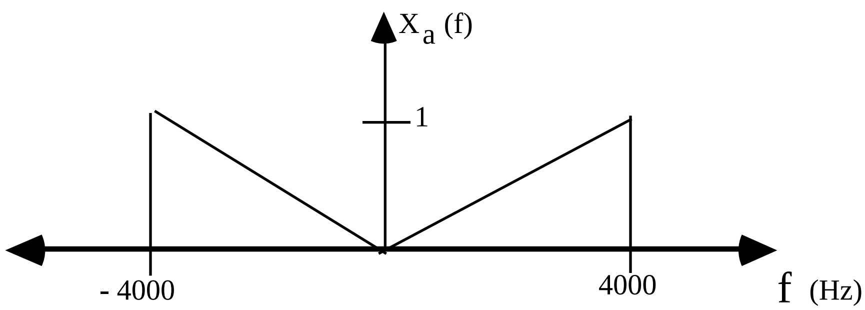
EE438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS

Assignment #3 - Spring 2001

due: 5 February 2001

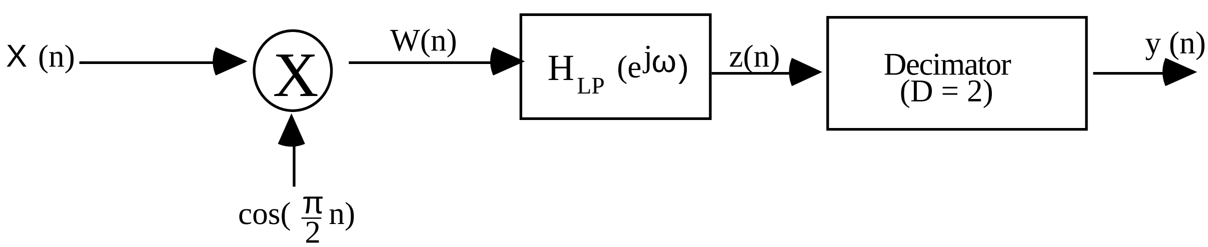
1. An analog signal, $x_a(t)$, with CTFT



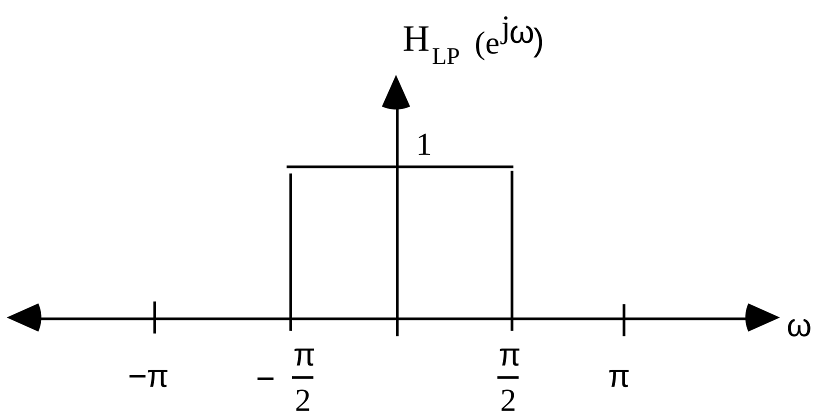
is sampled at a rate of $f_s = 16$ KHz to obtain the discrete-time signal, x(n).

a. Sketch the magnitude of the DTFT of x(n), $X(e^{j\omega})$, over the interval $-\pi \le \omega \le \pi$. Label the axes with as much detail as possible.

b. Suppose that x(n) is input to the following discrete-time system



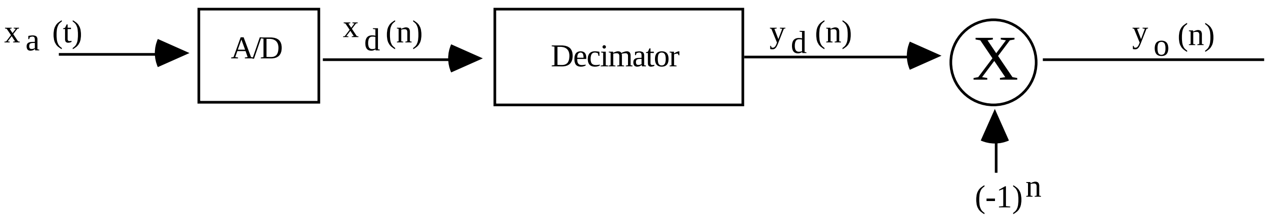
where the decimator input-output relationship is y(n) = z(2n) and the frequency response of the lowpass filter is



Sketch the magnitude of the DTFT of y(n), $Y(e^{j\omega})$, over the interval $-\pi \le \omega \le \pi$. Label the axes with as much detail as possible.

2. Consider the system shown below with input

$$x_a(t) = \cos[2\pi(2000)t] + \cos[2\pi(6000)t]$$



Assume that the A/D converter acts as an ideal sampler, generating samples at a rate of $f_s = 10$ KHz to obtain the discrete-time signal, $x_d(n)$. The operation of the decimator is $y_d(n) = x_d(2n)$.

a. Determine the digital frequencies ω present in the signal $x_d(n)$. Each answer should lie in the interval $0 \le \omega \le \pi$.

b. Repeat for $x_d(n)$.

c. Repeat for $x_o(n)$.

3. The CT signal $x_a(t) = \cos(2\pi 4000t)$ is sampled at the rate $f_s = 10$kHz. We may represent the sampled signal as the CT signal $x_s(t) = \text{comb}_T[x_a(t)]$, or as the DT signal $x_d(n) = x_a(nT)$.

a. Sketch the signals $x_a(t), x_s(t)$, and $x_d(n)$.

b. Find and sketch the CTFT's $X_a(f)$ and $X_s(f)$.

c. Use the relation $X_d(e^{j\omega}) = X_s\left[\frac{f_s\omega}{2\pi}\right]$ for $|\omega| < \pi$ when f_s is greater than Nyquist rate to find the DTFT $X_d(e^{j\omega})$ of $x_d(n)$. *Hint:* The Dirac delta function has the following property

$$\delta(a\omega - b) = \frac{1}{|a|}\delta(\omega - \frac{b}{|a|})$$

d. Sketch $X_d(e^{j\omega})$

e. Show that you result is the same as that given in class under transform pairs for the DTFT, *i.e.* that $X_d(e^{j\omega}) = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)], \ |\omega| < \pi$.

4. Consider an analog signal modeled as a stationary random process (remember EE302??) with a first order probability density that is Gaussian with zero mean and variance σ^2. The signal is passed through a B-bit quantizer. Under the following assumptions

   i. the quantization noise may be modeled as a random variable uniformly distributed between $-\Delta/2$ and $\Delta/2$

   ii. the contribution of overload to quantization error may be neglected

   iii. the quantizer range is chosen so that the probability of overload is < .02

   determine an expression for the average signal power to average quantization noise power ratio in dB defined as

$$SNR_Q = 10 \log_{10} \frac{E\{x^2(t)\}}{E\{n_q^2(t)\}}$$

   in terms of B.
   Hint: Use condition (iii) and your EE302 knowledge to determine an expression for Δ in terms of σ^2 and B.

5. For the following signals, find the ZT if it exists, and sketch the ROC, indicating where poles and zeros occur.

   a. $x(n) = \delta(n+2) + 2\delta(n+1) + \delta(n)$

   b. $x(n) = \delta(n+1) + 2\delta(n) + \delta(n-1)$

   c. $x(n) = e^{j\omega_0 n}, \quad -\infty < n < \infty$

   d. $x(n) = e^{j\omega_0 n} u(n)$

   e. $x(n) = \sin(\omega_0 n) u(n)$

   f. $x(n) = 2^n u(-n) + 4^n u(n)$

   g. $x(n) = -3^n u(-n-1) + e^{j\omega_0 n} u(n)$

   h. $x(n) = n\, u(n)$

   Hint: For all Parts except a and b, you should express signals in terms of signals to which known transform pairs or properties may be applied. You should not have to evaluate any of these transforms directly.