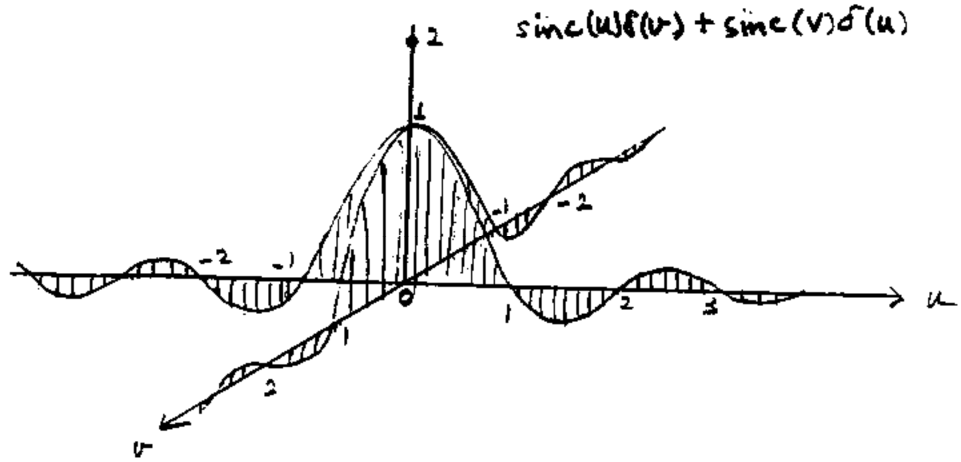
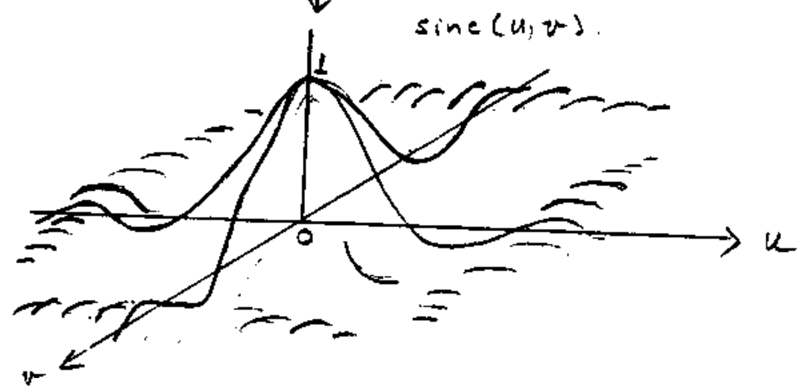
1a) i) $f(x,y) = \mathrm{rect}(x) + \mathrm{rect}(y) - \mathrm{rect}(x,y)$.

ii) $F(u,v) = \mathrm{sinc}(u)\delta(v) + \mathrm{sinc}(v)\delta(u) - \mathrm{sinc}(u,v)$.

iii)



minus

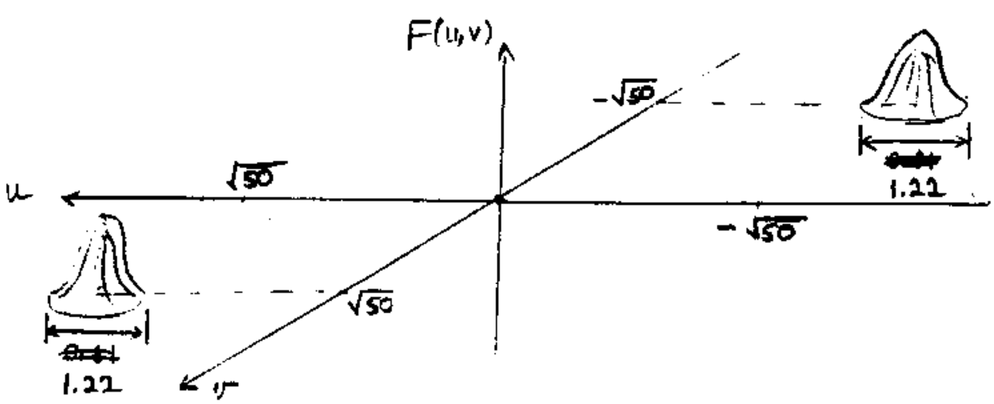


b) i) $f(x,y) = \mathrm{circ}(x/2, y/2) \cdot 1/2 \cos[2\pi\sqrt{50}(x+y)]$

ii) $F(u,v) = 4\,\mathrm{jinc}(2u,2v) * \frac{1}{4}[\delta(u-\sqrt{50}, v-\sqrt{50}) + \delta(u+\sqrt{50}, v+\sqrt{50})]$

$= \mathrm{jinc}[2(u-\sqrt{50}), 2(v-\sqrt{50})] + \mathrm{jinc}[2(u+\sqrt{50}), 2(v+\sqrt{50})]$

iii)



1c. i) $f(x,y) = \text{rect}\left(\frac{x}{a}, \frac{y}{b}\right) * \left\{\sum_i \sum_j \left[\delta(x - iX, y - jY) + \delta(x - iX/2, y - jY/2)\right]\right\}$

$$= \text{rep}_{XY}\left[\text{rect}\left(\frac{x}{a}, \frac{y}{b}\right)\right] + \text{rep}_{XY}\left[\text{rect}\left(\frac{x - X/2}{a}, \frac{y - Y/2}{b}\right)\right]$$

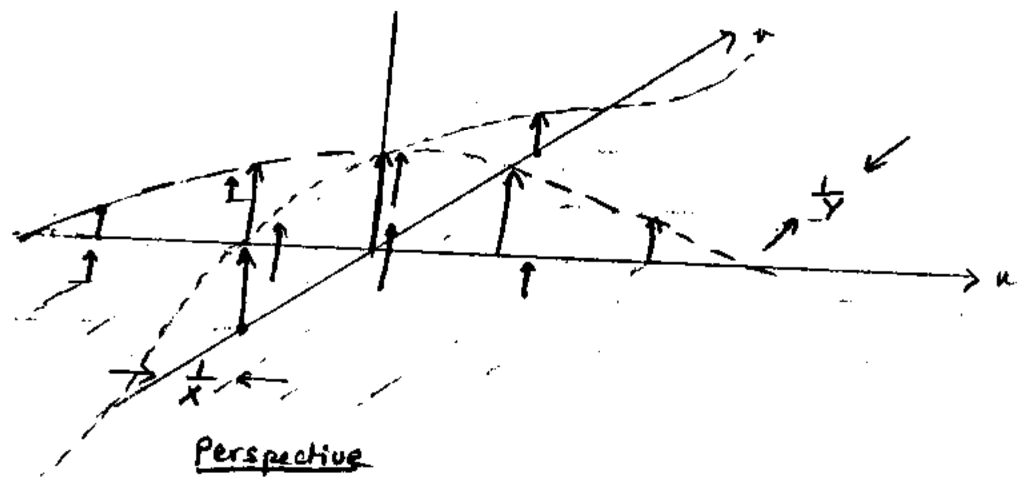
ii) $F(u,v) = \frac{ab}{XY}\sum_m \sum_n \text{sinc}(au, bv)\, \delta\left(u - \frac{m}{X}, v - \frac{n}{Y}\right)\left[1 + e^{-j2\pi(uX/2 + vY/2)}\right]$

$$= \frac{ab}{XY}\sum_m \sum_n \text{sinc}\left(\frac{am}{X}, \frac{bn}{Y}\right)\left[1 + e^{-j\pi(m+n)}\delta\left(u - \frac{m}{X}, v - \frac{n}{Y}\right)\right]$$

Note that $1 + e^{-j\pi(m+n)} = \begin{cases} 2 & , m+n \text{ even} \\ 0 & , m+n \text{ odd} \end{cases}$

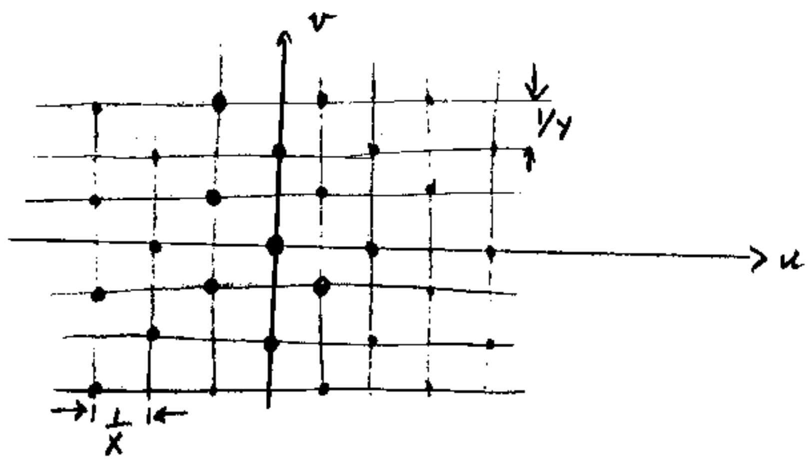
$$\Longrightarrow F(u,v) = \frac{2ab}{XY}\sum_m \sum_{\substack{n \\ m+n \text{ even}}} \text{sinc}\left(\frac{am}{X}, \frac{bn}{Y}\right)\delta\left(u - \frac{m}{X}, v - \frac{n}{Y}\right).$$

iii)

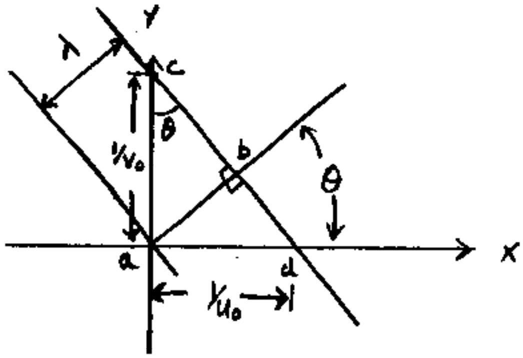


Perspective

Top View →



2.



a) From triangle abd, $\lambda = \frac{1}{u_0}\cos\theta \rightarrow u_0 = \frac{1}{\lambda}\cos\theta$

From triangle abc, $\lambda = \frac{1}{v_0}\sin\theta \Rightarrow v_0 = \frac{1}{\lambda}\sin\theta$.

Squaring and adding : $u_0^2 + v_0^2 = \frac{1}{\lambda^2}\left[\cos^2\theta + \sin^2\theta\right] = 1/\lambda^2$

$$\Rightarrow \lambda = \left[u_0^2 + v_0^2\right]^{-1/2}.$$

b) From triangle acd $\tan\theta = \dfrac{1/u_0}{1/v_0} = v_0/u_0$

$$\Rightarrow \theta = \tan^{-1}(v_0/u_0)$$

3. a). $$F(u,v) = \iint_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$F(-u,-v) = \iint_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(-ux-vy)}\, dx\, dy$$

$$= \iint_{-\infty}^{\infty} f(x,y)\, e^{j2\pi(ux+vy)}\, dx\, dy$$

$$= \left[\iint_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy\right]^*$$

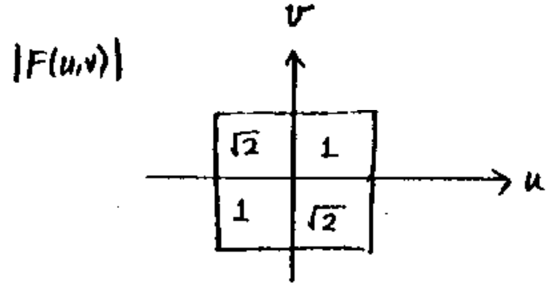
since f is real, $f(x,y) = f^*(x,y)$

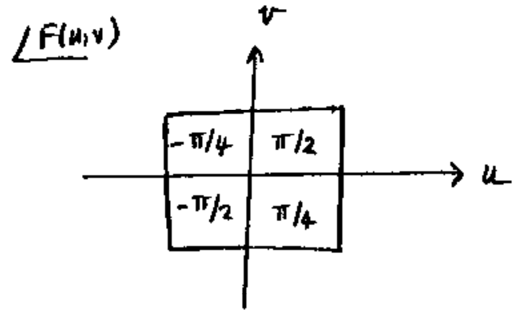
$$= F^*(u,v)$$

$$\Rightarrow |F(u,v)| = |F(-u,-v)|$$

and $\angle F(u,v) = -\angle F(-u,-v)$

4a)





b) $$F(u,v) = e^{j\pi/2}\,\mathrm{rect}(u-\tfrac{1}{2}, v-\tfrac{1}{2}) + e^{-j\pi/2}\,\mathrm{rect}(u+\tfrac{1}{2}, v+\tfrac{1}{2}) + e^{j\pi/4}\sqrt{2}\,\mathrm{rect}(u-\tfrac{1}{2}, v+\tfrac{1}{2}) + e^{-j\pi/4}\,\mathrm{rect}(u+\tfrac{1}{2}, v-\tfrac{1}{2})$$

$$\Rightarrow f(x,y) = \mathrm{sinc}(x,y)\left\{\left[e^{j\pi(x+y+1/2)} + e^{-j\pi(x+y+1/2)}\right] + \sqrt{2}\left[e^{j\pi(x-y+1/4)} + e^{-j\pi(x-y+1/4)}\right]\right\}$$

$$= \mathrm{sinc}(x,y)\left\{2\cos[\pi(x+y+1/2)] + 2\sqrt{2}\cos[\pi(x-y+1/4)]\right\}$$

c) $F(u,v)$ has the symmetry discussed in Problem 3 (called Hermitian symmetry). $\Rightarrow f(x,y)$ is real valued.

5. We know $\delta(x-x_0, y-y_0) \xrightarrow{\text{imaging system}} h(x-Mx_0, y-My_0)$. M = magnification.

Here $\delta(x-1, y-2) \xrightarrow{\text{Imaging system}} \mathrm{rect}(x+2)\,\delta(y+4)$

$\Rightarrow m = |M| = 2$ and $h(x,y) = \mathrm{rect}(x)\,\delta(y)$

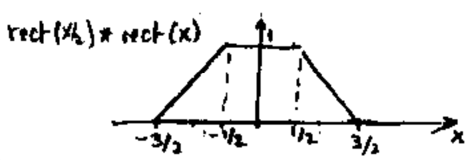
a) $\mathrm{rect}(x, y/4) = g(x,y) \longrightarrow \boxed{\text{Imaging system}} \longrightarrow f(x,y)$.

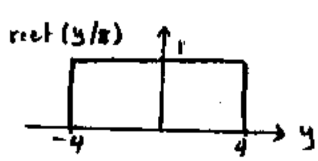
$$f(x,y) = \tfrac{1}{M^2} g(x/m, y/m) ** h(x,y) = \tfrac{1}{4}\,\mathrm{rect}(x/2, y/8) ** \mathrm{rect}(x)\delta(y)$$

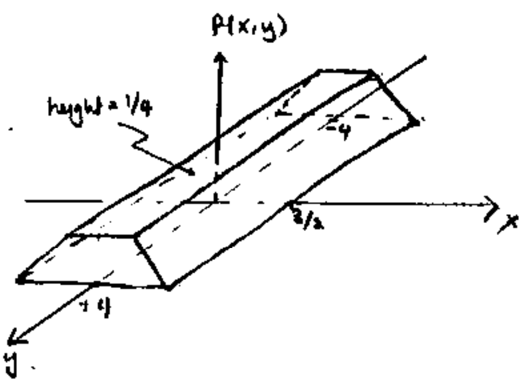
$$= \tfrac{1}{4}\left[\mathrm{rect}(x/2)\,\mathrm{rect}(y/8) ** \mathrm{rect}(x)\delta(y)\right] \quad \text{separability}$$

$$= \tfrac{1}{4}\left[\left(\mathrm{rect}(x/2) * \mathrm{rect}(x)\right)\cdot\left(\mathrm{rect}(y/8) * \delta(y)\right)\right]$$

$$= \tfrac{1}{4}\left[\left(\mathrm{rect}(x/2) * \mathrm{rect}(x)\right)\cdot \mathrm{rect}(y/8)\right].$$







b) $|M| = 2$

c) $h(x,y) = \mathrm{rect}(x)\,\delta(y)$

d) $H(u,v) = \mathcal{F}[h(x,y)] = \mathrm{sinc}(u)\cdot 1$
$= \mathrm{sinc}(u)$

6. $$G(u,v) = \iint_{-\infty}^{\infty} g(x,y)\, e^{-j2\pi(ux+vy)}\, dx\,dy \qquad (a)$$

a) We have:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad (1) \qquad \begin{cases} u = \rho\cos\phi \\ v = \rho\sin\phi \end{cases} \quad (2)$$

To get the polar coordinate representation, we replace x, y, u, v with r, θ, ρ, ϕ in the RHS of (a) using (1) and (2). Further, we replace $dx\,dy$ by $r\,dr\,d\theta$ and change the limits of integration to span the entire 2-dim. space.

$$\begin{aligned} e^{-j2\pi(ux+vy)} &= e^{-j2\pi(\rho\cos\phi\, r\cos\theta + \rho\sin\phi\, r\sin\theta)} \\ &= e^{-j2\pi r\rho(\cos\phi\cos\theta + \sin\phi\sin\theta)} \\ &= e^{-j2\pi r\rho\cos(\phi-\theta)}. \end{aligned}$$

$$\Rightarrow \tilde{G}(\rho,\phi) = \int_0^{2\pi}\int_0^{\infty} \tilde{g}(r,\theta)\, e^{-j2\pi r\rho\cos(\phi-\theta)}\, r\,dr\,d\theta$$

b). $$\int_0^{2\pi}\int_0^{\infty} \tilde{g}(r,\theta+\theta_0)\, e^{-j2\pi r\rho\cos(\phi-\theta)}\, r\,dr\,d\theta = \mathcal{F}[\tilde{g}(r,\theta+\theta_0)]$$

$$= \int_{\theta_0}^{\theta_0+2\pi}\int_0^{\infty} \tilde{g}(r,\hat{\theta})\, e^{-j2\pi r\rho\cos(\phi-\hat{\theta}+\theta_0)}\, r\,dr\,d\hat{\theta} \qquad \hat{\theta} = \theta+\theta_0$$

$$= \int_0^{2\pi}\int_0^{\infty} \tilde{g}(r,\hat{\theta})\, e^{-j2\pi r\rho\cos[(\phi+\theta_0)-\hat{\theta}]}\, r\,dr\,d\hat{\theta}$$

$$= \tilde{G}(\rho, \phi+\theta_0).$$

since θ is periodic in 2π, it does not matter where we start the integration provided it covers 2π !

c). $$\tilde{G}(\rho,\phi) = \int_0^{2\pi}\int_0^{\infty} \tilde{g}_0(r)\, e^{-j2\pi r\rho\cos(\phi-\theta)}\, r\,dr\,d\theta$$

$$= \int_0^{\infty} r\,\tilde{g}_0(r)\left\{\int_0^{2\pi} e^{-j2\pi r\rho\cos(\theta-\phi)}\,d\theta\right\}dr \qquad \text{let } \hat{\theta} = \theta-\phi$$

$$= \int_0^{\infty} r\,\tilde{g}_0(r)\left\{\int_{-\phi}^{2\pi-\phi} e^{-j2\pi r\rho\cos\hat{\theta}}\,d\hat{\theta}\right\}dr$$

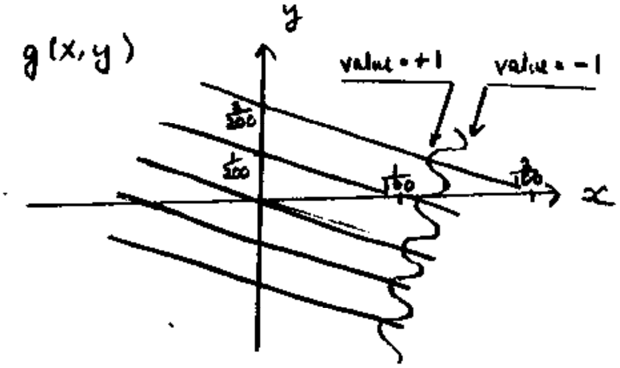
$$= \int_0^{\infty} r\,\tilde{g}_0(r)\underbrace{\left\{\int_0^{2\pi} e^{-j2\pi r\rho\cos\hat{\theta}}\,d\hat{\theta}\right\}}_{\text{No longer a function of }\phi}dr$$

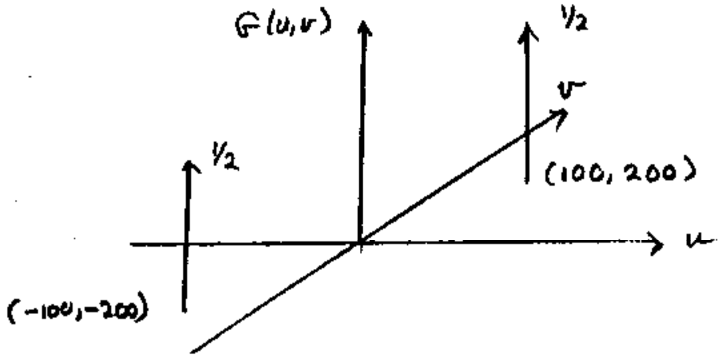
For the same reason given above, we may change limits of integration for $\hat{\theta}$.

$$= \tilde{G}_0(\rho).$$

7 $\quad g(x,y) = \cos[2\pi(100x + 200y)]$

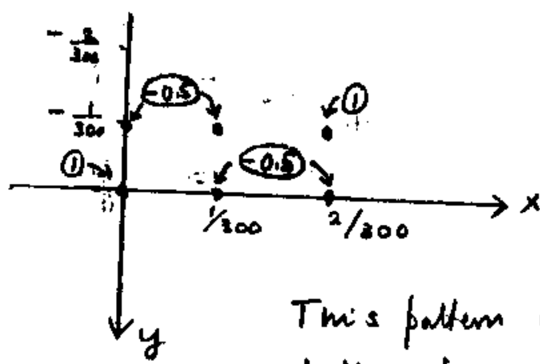
$$G(u,v) = \tfrac{1}{2}[\delta(u-100, v-200) + \delta(u+100, v+200)]$$





$$g_s(x,y) = \text{comb}_{\frac{1}{200},\frac{1}{300}}\{\cos[2\pi(100x + 200y)]\}$$

<u>Top View</u>

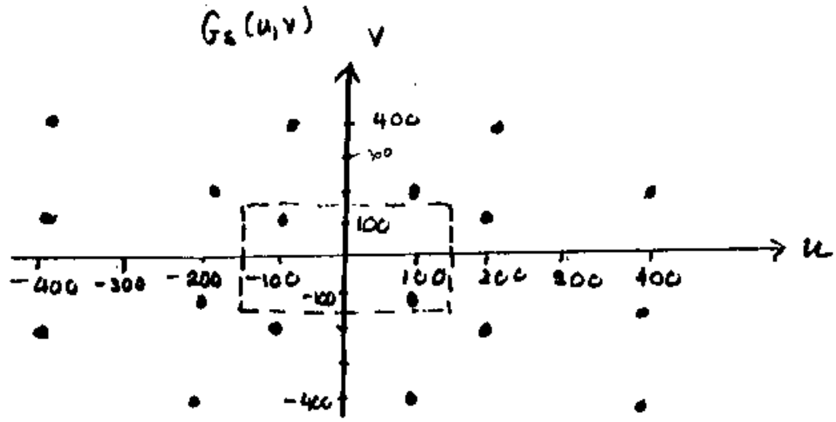


• → impulse

(−0.5) → "amplitude" of impulse.

This pattern is repeated with a spacing of 1/300 in x & y directions.

$$G_s(u,v) = (300)^2\, \text{rep}_{200,300}\left\{\tfrac{1}{2}[\delta(u-100, v-200) + \delta(u+100, v+200)]\right\}$$



• → impulses of 'height' $\frac{(300)^2}{2}$.

$G_r(u,v)$ is the part of $G_s(u,v)$ inside the dotted square, which represents a LPF with cutoff of 150 cycles/inch.

$$G_r(u,v) = \frac{(300)^2}{2}\left[\delta(u+100, v-100) + \delta(u-100, v+100)\right]$$

$$\Rightarrow g_r(x,y) = (300)^2 \cos\left[100x - 100y\right]$$