EE438 Signals and Systems – Spring 2001

Homework 8

Due Date: Wednesday, 25 April 2001

1. The $M \times N$ point 2D DFT $X_{MN}(k,\ell)$ of an $M \times N$ 2D signal $x(m,n)$ is defined as follows

$$X_{MN}(k,\ell) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m,n) e^{-j2\pi\left(\frac{mk}{M} + \frac{n\ell}{N}\right)}$$

Calculate the 10×12 point DFT for the following 2D signals:

a. $x(m,n) = \begin{cases} 1, & (m,n) = (3,4),\ (-3,-4) \\ 0, & \text{else} \end{cases}$

b. $x(m,n) = \begin{cases} 1, & 0 \le m \le 4 \\ 0, & 5 \le m \le 9 \end{cases}$

c. $x(m,n) = \cos[2\pi(m/10 - n/6)]$

2. You have a subroutine that will calculate an L point 1D DFT for any L which is a power of 2.

a. Show how you would use this subroutine to calculate the 2D DFT of an $M \times N$ point image where M and N are both power of 2.

b. Determine approximately how many complex operations (CO's) would be required (1 complex operation = 1 complex addition and 1 complex multiplication) to perform the $M \times N$ point DFT assuming that your subroutine requires $L \log_2(L)$ CO's to perform a L point 1D DFT.

3. Consider the image $g_a(x,y) = \text{jinc}(x - 2.5,\ y - 2.5)$ where the subscript denotes analog.

a. Calculate and sketch its 2D CSFT $G_a(u,v)$.

The image is sampled at interval 0.5 in both the x and y directions to yield $g(m,n) = g_a(0.5m,\ 0.5n)$.

b. Based on your answer to part a, calculate and sketch the 2D DSFT $G(e^{j\mu}, e^{j\nu})$.

c. Based on your answer to part b, calculate and sketch the 10×10 2D DFT of $g(m,n)$ $0 \le m \le 9, 0 \le n \le 9$. You may assume that $g(m,n) = 0$ outside this range.

Note: Your sketches need only show magnitude.

Hint: You should only do one actual Fourier transform, and that is for part a.

4. Suppose the $M \times N$ image $g(m,n)$ has 2D DFT $G_{MN}(k,\ell)$. Define a new image $f(m,n) = (-1)^{m+n} g(m,n)$.

   a. Find the 2D DFT $F_{MN}(k,\ell)$ in terms of $G_{MN}(k,\ell)$.

   b. Suppose $g(m,n)$ is as given in part c) of Problem 3. Sketch $F_{10,10}(k,\ell)$.

5. The unit sample response of a 2-D FIR digital filter is given by

$$h(m,n) = \begin{cases} 1 & , \quad (m,n) = (0,0) \\ 1/2 & , \quad (m,n) = (-1,0) \text{and} (1,0) \\ -1/2 & , \quad (m,n) = (0,-1) \text{and} (0,1) \\ 0 & , \quad \text{else} \end{cases}$$

   a. Find a difference equation that can be used to implement this filter.

   b. Find the filtered output corresponding to the input image

$$\begin{array}{ccccccccc} & & & \vdots & & & \vdots & & \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ \cdots & 0 & 1 & 1 & 1 & 1 & 1 & 0 & \cdots \\ & 0 & 1 & 1 & 1 & 1 & 1 & 0 & \\ & 0 & 1 & 1 & 1 & 1 & 1 & 0 & \\ & 0 & 1 & 1 & 1 & 1 & 1 & 0 & \\ \cdots & 0 & 1 & 1 & 1 & 1 & 1 & 0 & \cdots \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ & & & \vdots & & & \vdots & & \end{array}$$

   c. Find a simple expression for the frequency response $H(e^{j\mu}, e^{j\nu})$ of this filter.

   d. Discuss the relation between your answer to part b) and the filter frequency response.