EE 438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS

Assignment #2 - Spring 2001

due Monday, 29 January 2001 (in class)

1. Let a real-valued D-T signal x(n) be decomposed as $x(n) = x_e(n) + x_o(n)$, where $x_e(n)$ and $x_o(n)$ are the even and odd parts of x(n), respectively, defined as

$$x_e(n) = \frac{1}{2}\{x(n) + x(-n)\}$$

$$x_o(n) = \frac{1}{2}\{x(n) - x(-n)\}$$

Let $X_e(e^{j\omega})$ and $X_o(e^{j\omega})$ be the DTFT's of $x_e(n)$ and $x_o(n)$, respectively.

(a) Prove the following two relationships

$$X_e(e^{j\omega}) = \mathrm{Re}\{X(e^{j\omega})\}$$

$$X_o(e^{j\omega}) = j\mathrm{Im}\{X(e^{j\omega})\}$$

(b) Let $u_e(n)$ and $u_o(n)$ be the even and odd parts of the unit step sequence u(n). Determine the DTFT of $u_e(n)$, denoted $U_e(e^{j\omega})$, and determine the DTFT of $u_o(n)$, denoted $U_o(e^{j\omega})$.

(c) Combine $U_e(e^{j\omega})$ and $U_o(e^{j\omega})$ based on the results proved in (a) to obtain an expression for the DTFT of u(n), denoted $U(e^{j\omega})$. Show that

$$U(e^{j\omega}) = \frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k)$$

2. Consider the causal LTI system described by the difference equation

$$y(n) = 2\cos\omega_o y(n-1) - y(n-2) + A\sin\omega_o x(n)$$

where A and ω are just fixed constants with $0 < \omega_o < \pi$

(a) Determine an expression for the frequency response $H(e^{j\omega})$ of the system and provide a rough sketch of $|H(e^{j\omega})|$. Specify the value of $|H(e^{j\omega})|\ \omega = \omega_o$.

(b) Show that the impulse response of the system is $h(n) = A\sin[\omega_o(n+1)]u(n)$. You may do this by simply letting $x(n) = \delta(n)$ in the difference equation above and showing that $y(n) = A\sin[\omega_o(n+1)]$ for n = 0, 1, 2. This system is a digital sinusoidal oscillator.

3. Let x(n) and y(n) be two D-T signals and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their respective DTFT's.

(a) Defining v(n) as the product of x(n) and y(n), i.e., v(n) = x(n)y(n), show that the DTFT of v(n) may be determined from $X(e^{j\omega})$ and $Y(e^{j\omega})$ according to

$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\mu})Y(e^{j(\omega-\mu)})\, d\mu$$

(b) Using the result above determine the DTFT, $X(e^{j\omega})$, of the D-T signal

$$x(n) = \frac{\sin^2(\pi n/4)}{\pi^2 n^2}$$

4. Let x(n) and y(n) be two D-T signals and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their respective DTFT's

(a) By using the convolution theorem and other appropriate DTFT properties, determine and expression for the D-T signal whose DTFT is $X(e^{j\omega})Y^*(e^{j\omega})$.

(b) By using the result in part (a), show that

$$\sum_{n=-\infty}^{\infty} x(n)y^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$$

(c) By using the result in part (b), determine the numerical value of the sum

$$\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n/4)}{2\pi n}\,\frac{\sin(\pi n/6)}{5\pi n}$$

5.

(a) Let the D-T signal x(n) be obtained by sampling the C-T signal

$$x(t) = \text{sinc}^2(100t)$$

at a rate f_s samples/sec. Let $X(e^{j\omega})$ denote the DTFT of x(n).

(i) Plot $|X(e^{j\omega})|$ when $f_s = 800$ Hz.
(ii) Plot $|X(e^{j\omega})|$ when $f_s = 400$ Hz.
(iii) Plot $|X(e^{j\omega})|$ when $f_s = 100$ Hz.

(b) Repeat the above for

$$x(t) = \text{sinc}^2(100t)\cos(2\pi 200t)$$