

ASSIGNMENT #5
Due Monday, March 5, 2001 (due in class)

1. Recall that the N pt. DFT of an N length sequence $x(n)$ is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{k}{N}n} \quad k = 0, 1, \dots, N-1$$

For the sake of simplicity, we have dropped the subscript N used in class.

- (a) Prove Parseval's relationship for the DFT:

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

- (b) Prove the duality property of the DFT. That is, if

$$x(n) \xrightarrow{\text{DFT}} X(k)$$

prove that

$$X(n) \xrightarrow{\text{DFT}} Nx(-k)$$

where on the right hand side (RHS) we have invoked the fact that the DFT views $x(n)$ as periodic with period N . In actuality, the RHS above should be $Nx(0)$ for $k = 0$ and $Nx(N-k)$ for $k = 1, \dots, N-1$.

2. *Development of the Decimation in Frequency Radix 2 FFT.* Consider $x(n)$ to be an N length sequence where N is a power of 2. The N pt. DFT of $x(n)$ is computed as

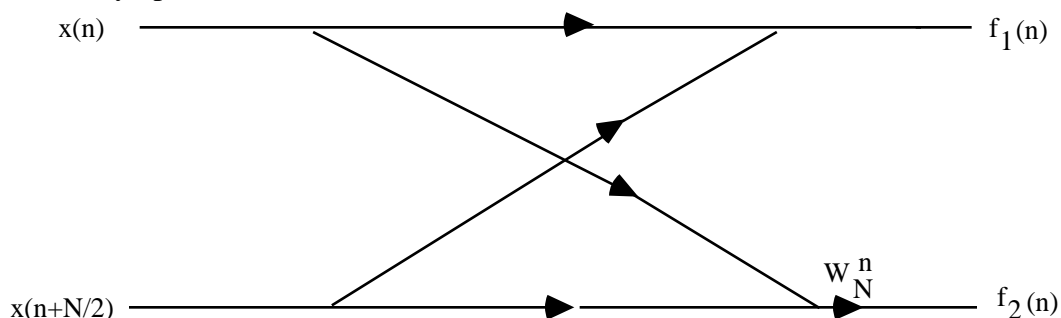
$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \quad k = 0, 1, \dots, N-1$$

where $W_N = e^{-j\frac{2\pi}{N}}$. Let $f_1(n)$ and $f_2(n)$ be sequences of length $N/2$ defined as

$$f_1(n) = x(n) + x(n + N/2) \quad n = 0, 1, \dots, (N/2) - 1$$

$$f_2(n) = \{x(n) - x(n + N/2)\} W_N^n \quad n = 0, 1, \dots, (N/2) - 1$$

Note that the computation of $f_1(n)$ and $f_2(n)$ from $x(n)$ and $x(n + N/2)$ may be symbolically expressed in terms of a butterfly operation defined as



(a) Show that

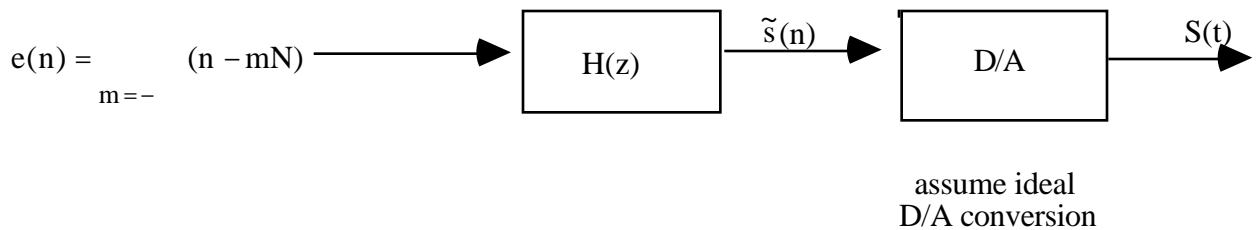
$$X(2k) = \sum_{n=0}^{(N/2)-1} f_1(n) W_{N/2}^{nk} \quad k = 0, 1, \dots, (N/2) - 1$$

$$X(2k + 1) = \sum_{n=0}^{(N/2)-1} f_2(n) W_{N/2}^{nk} \quad k = 0, 1, \dots, (N/2) - 1$$

i.e., the $N/2$ DFT values $X(k)$ for k even may be computed from the $N/2$ pt. DFT of $f_1(n)$ and the $N/2$ DFT values $X(k)$ for k odd may be computed from the $N/2$ pt. DFT of $f_2(n)$.

(b) For $N=8$, repeat this process two more times to obtain sequences of length 1. Using the butterfly operation shown above, draw a complete flow diagram for an $N = 8$ pt. Decimation in Frequency Radix 2 FFT. The input should values, $x(n)$, should be in natural order while the output DFT values, $S(k)$, should be in bit-reversed order.

3. The following system is used to digitally synthesize the phoneme IY. The format frequencies are 270 Hz, 2290 Hz, and 3010 Hz while the pitch frequency is 125 Hz.



The system operates at a data rate of 10 KHz and the transfer function is

$$H(z) = \frac{z^6}{[z^2 - 1.8\cos(\omega_1)z + .81][z^2 - 1.2\cos(\omega_2)z + .36][z^2 - 0.8\cos(\omega_3)z + .16]}$$

- Determine the appropriate value of N in the excitation $e(n)$.
- Determine the appropriate values of ω_1 , ω_2 , and ω_3 , in the denominator of the transfer function $H(z)$.
- Sketch approximately what the CTFT, $S(f)$, of $s(t)$ would look like.

Hint: Think about how the frequency response $H(e^{j\omega})$ depends on the angles at which the poles are located and their distances from the unit circle. Relate this to the CTFT of $s(t)$.

4. For the following problems, the short-time discrete-time Fourier transform STDFT is defined as

$$X_n(e^j) = \sum_{m=-} x(m)w(n - m)e^{-j m}$$

and the sequences $x(n)$ and $w(n)$ have DTFT's $X(e^j)$ and $W(e^j)$.

Show the following properties hold:

(a) Linearity -

if $v(n) = ax(n) + by(n)$, then $V_n(e^j) = aX_n(e^j) + bY_n(e^j)$

(b) Shifting property -

if $v(n) = x(n - n_0)$, then $V_n(e^j) = X_{n-n_0}(e^j)e^{-j n_0}$

(c) Show that the STDFT can be put in the form

$$x(n) = \frac{1}{2} \int_{-} W(e^j X(e^{j(+)})d$$

i.e., $X_n(e^j)$ is a smoothed spectral estimate of $X(e^j)$ at frequency . Clearly illustrate the steps involved in arriving at the expression above.

(d) Show that the inverse STDFT can be put in the form

$$x(n) = \frac{1}{2} \int_{-} X_n(e^j) e^{j n} d$$

5. A vowel sound is spoken by someone. The pitch period is 5 milliseconds. The formant frequencies are 500 Hz, 1.5 kHz, and 2.5 kHz. the waveform is sampled at 20 kHz producing the data sequence $x(n)$. The STDFT

$$X_n(e^j) = \sum_m x(m)w(n - m)e^{-j m}$$

is calculated. Sketch roughly what a spectrogram display of $|X_n(e^j)|$ would look like as a function of (rad/sample) and n (sample index) if the window $w(n)$ had length:

(a) $N = 50$ points

(b) $N = 1000$ points

(c) For each case above, state whether the spectrogram is wideband or narrowband.