Reduced-Rank Linear Equalization for the Forward Link in High-Speed CDMA

Samina Chowdhury

PhD Preliminary Examination,

Electrical and Computer Engineering,

Purdue University.

Major Professor : Michael D. Zoltowski

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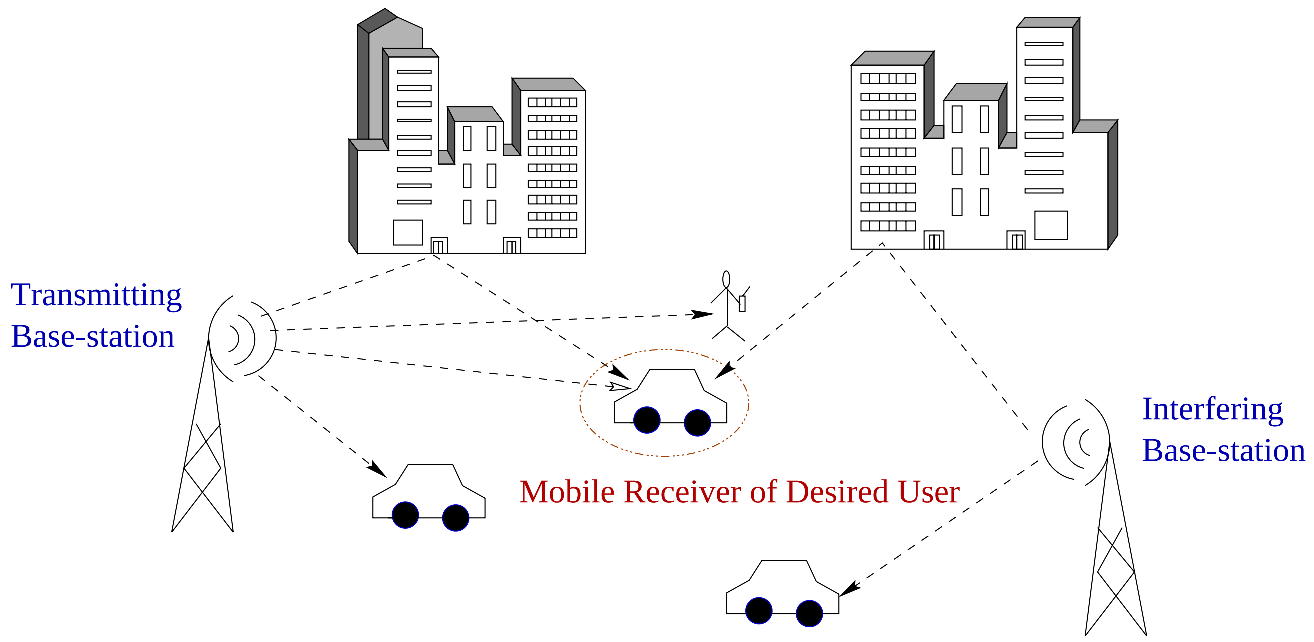
Outline

- CDMA Forward Link
  - Rake Receivers
- Chip-level MMSE Equalizers
  - Single and Multiple Base-station Scenario
- The Multi-Stage Nested Wiener Filter
- Simulation Results
  - Known Channels
  - Training-based Adaptation
- Structured Equalizers in Sparse Multipath
- Conclusions and Future Work

High-speed CDMA Forward Link



Down-link – Bottleneck in future cellular/PCS systems

- Internet usage is download oriented
- Number of users and data rate growing very fast

Data and Channel Model

Base-station transmits "synchronous sum signal" at chip rate

$$s[n] = c_{bs}[n] \sum_{j=1}^{N_u} \sum_{m=0}^{N_s-1} b_j[m] c_j[n - N_c m]$$

Base-station long code

j-th user's symbols

j-th user's channel code

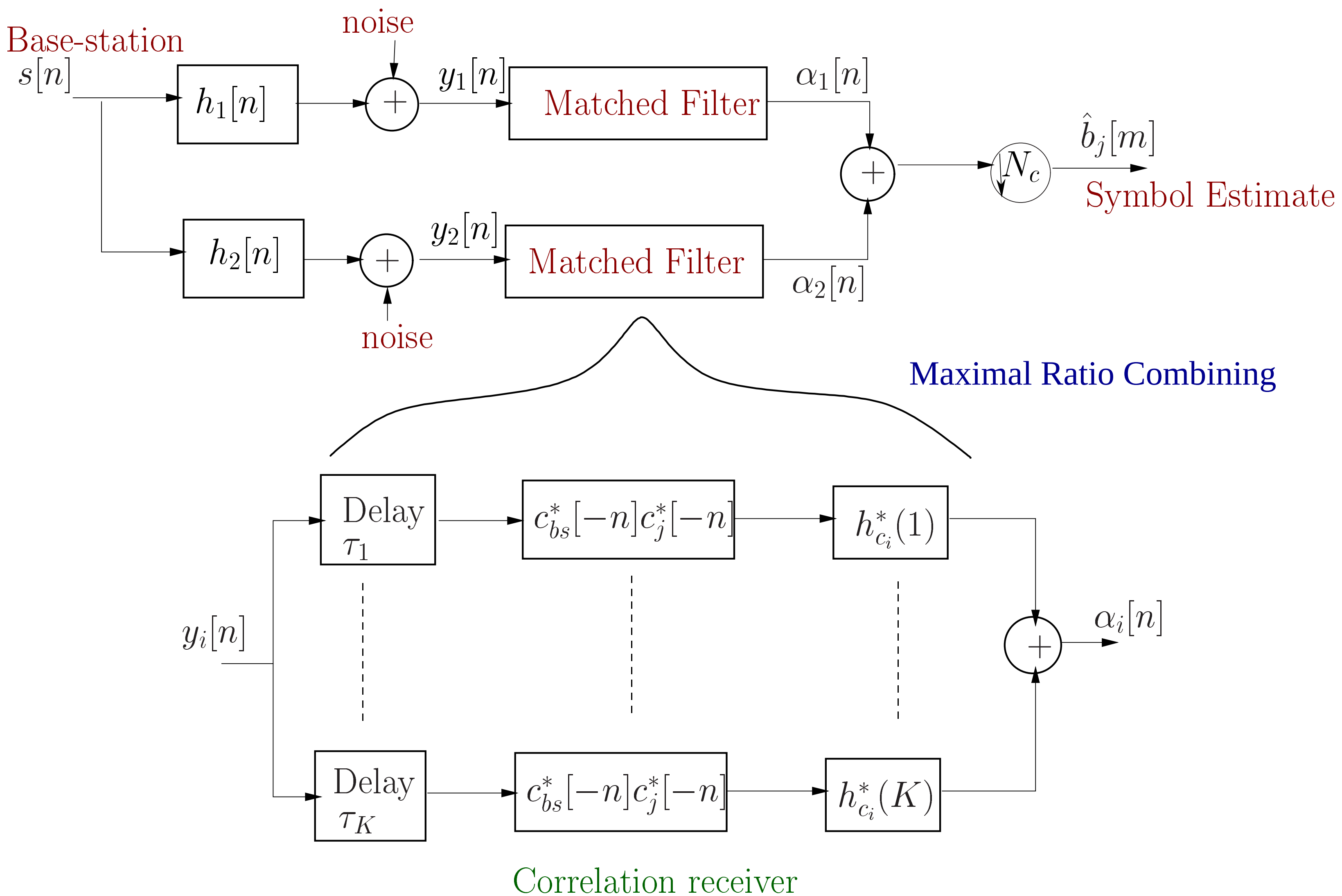
Channel between the transmitter and i-th antenna at the receiver :

$$h_i(t) = \sum_{k=0}^{N_m-1} h_{c_i}(k) p_{rc}(t - \tau_k) \quad i = 1, 2.$$

Complex Gains

Chip pulse shaping waveform

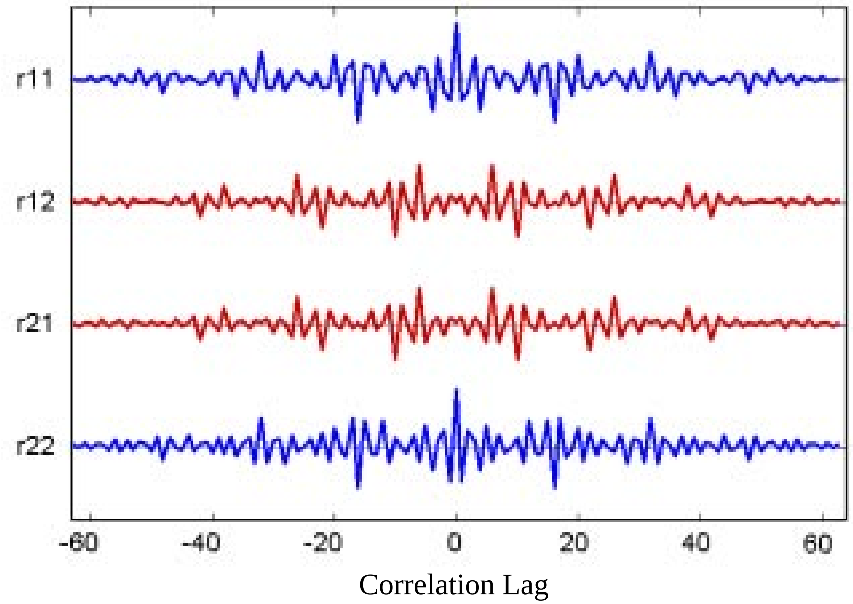
Conventional Receiver : RAKE



CDMA Downlink : Problems

- *Orthogonal* Walsh-Hadamard codes used to spread data symbols – perfect separation of desired signal in *flat fading* scenario.

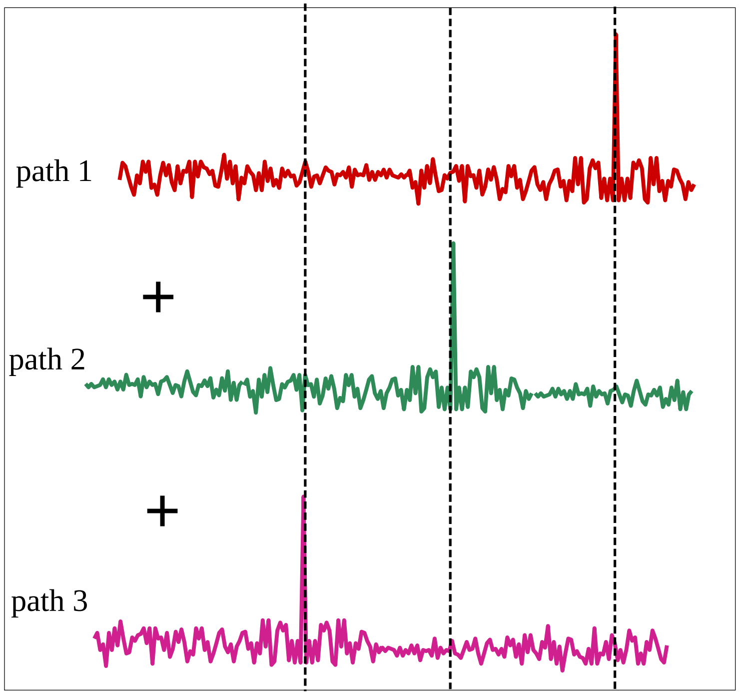
- Walsh-Hadamard codes have poor *auto-correlation* and *cross-correlation* properties at non-zero lag values



➢ *Frequency-selective fading* ➜

♦ Inter-Chip Interference (ICI)

♦ Multiple-Access Interference (MAI).



➢ Delay spread may induce Intersymbol Interference (ISI)

➢ RAKE receiver treats interference as white noise

➢ When many users are active, RAKE performance degrades

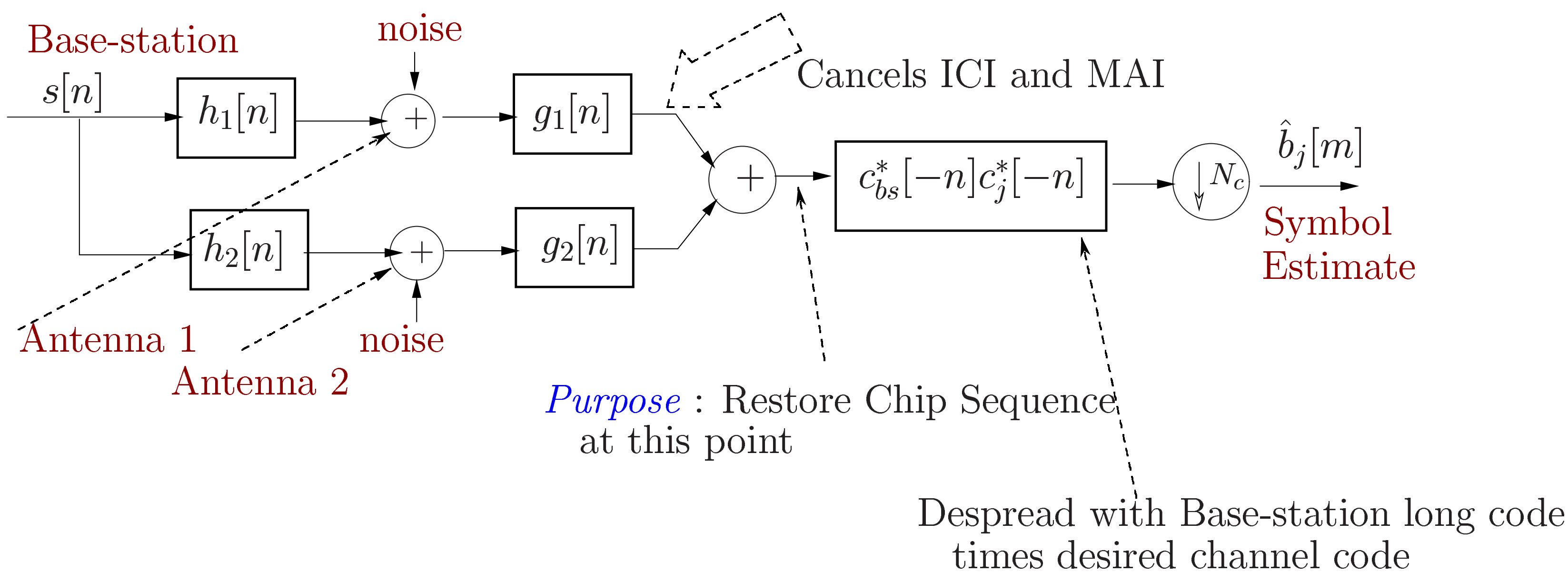
Downlink Specific *Linear* Equalizers

Features

- Equalizer restores "chip sequence", followed by despreading with channel code times long code
- The equalizer is independent of the user's channel code.
- Equalizer is unchanged over coherence time of downlink channel
- Multiple antennas at mobile receiver provide space-time diversity - increases cost and power consumption of the mobile unit
- Chip-level MMSE equalizer proposed independently by I. Ghauri and D. Slock, C. Frank and E. Visotsky and later by T. Krauss and M. D. Zoltowski.

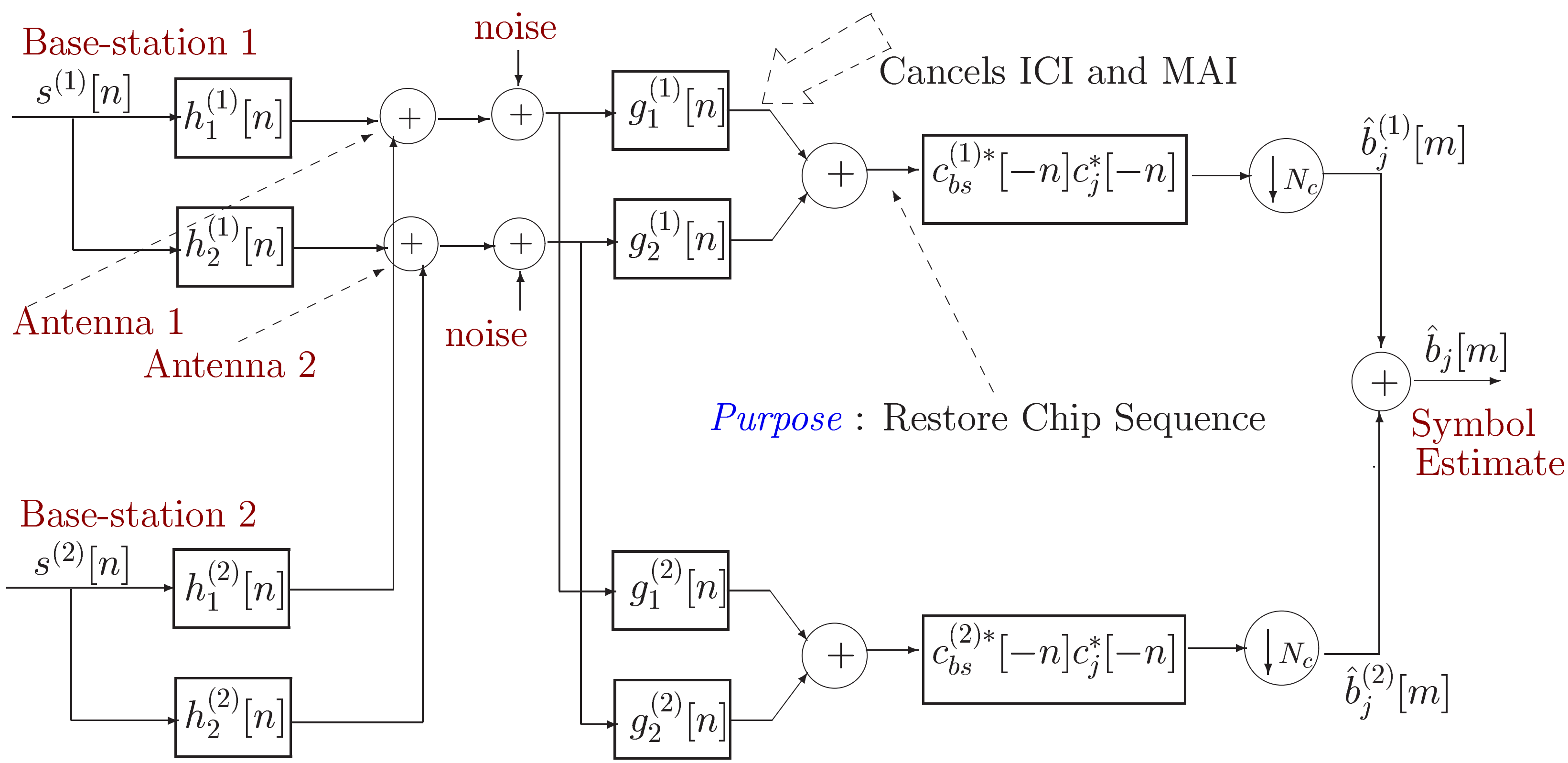
Multichannel MMSE Chip-level Equalizer :

One Transmitting Base-station



Multichannel MMSE Chip-level Equalizer :

Two Base-stations : " Soft Hand-off"
Two Equalizers in Parallel



Multi-channel Received Data

$$\begin{aligned}\mathbf{y}[n] &= \mathbf{H}^{(1)}\mathbf{s}^{(1)}[n] + \mathbf{H}^{(2)}\mathbf{s}^{(2)}[n] + \boldsymbol{\eta}[n] \\ &= \boldsymbol{\mathcal{H}}\mathbf{s}[n] + \boldsymbol{\eta}[n]\end{aligned}$$

$$\mathbf{s}^{(k)}[n] = \begin{bmatrix} s^{(k)}[n] \\ s^{(k)}[n-1] \\ \vdots \\ s^{(k)}[n-(N_g+L-2)] \end{bmatrix}$$

L = Channel length in chips,

N_g = Equalizer length in chips,

$\boldsymbol{\eta}$ is the noise vector,

$\mathbf{H}^{(k)}$ is the $2N_g \times (L + N_g - 1)$ channel convolution matrix,

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \mathbf{H}^{(1)} & \mathbf{H}^{(2)} \end{bmatrix} \qquad \mathbf{H}^{(k)} = \begin{bmatrix} \mathbf{H}_1^{(k)} \\ \mathbf{H}_2^{(k)} \end{bmatrix},$$

$$\mathbf{H}_i^{(k)} = \begin{bmatrix} h_i^{(k)}[0] & 0 & \cdots & 0 \\ h_i^{(k)}[1] & h_i^{(k)}[0] & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ h_i^{(k)}[L-1] & h_i^{(k)}[L-2] & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & \cdots & h_i^{(k)}[L-1] \end{bmatrix}^T$$

MMSE Criterion :

$$\underset{\text{over}\quad \mathbf{g}_c^{(\mathrm{k})}}{\text{minimize}}\; E\left[\|\mathbf{g}_c^{(k)H}\mathbf{y}[n] - s^{(k)}[n-D_c]\|^2\right]$$

where D_c is the combined equalizer and channel delay.

Solution :

$$\mathbf{g}_c^{(k)} = \{\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H + \mathcal{R}_{\eta\eta}\}^{-1}\mathbf{H}^{(k)}\boldsymbol{\delta}_{D_c}$$

where $\mathcal{R}_{\eta\eta} = E\left[\boldsymbol{\eta}^H[n]\boldsymbol{\eta}[n]\right]$

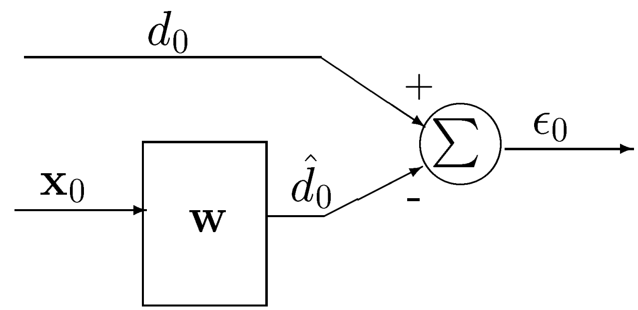
$$\boldsymbol{\delta}_{D_c} = \begin{bmatrix} 0 & \dots & 0 & 1 & 0 & \dots & 0 \end{bmatrix}^T$$

$(D_c + 1)$-th position

Assuming long code to be I.I.D.

The MMSE solution is in the form of the classical Wiener filter

$$\mathbf{w} = \mathbf{R}_{xx}^{-1}\mathbf{r}_{dx}$$



Drawbacks

- Direct computation of the MMSE equalizer requires estimate of $\mathbf{R}_{xx}^{-1}$.
- Equalizer may have to be many chips in length — slow convergence in adaptive implementations

Proposed Solution :

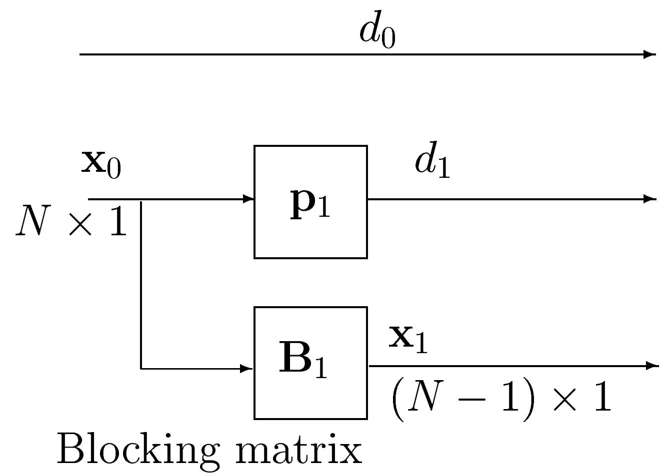
- Implement a low-complexity reduced-rank approximation of the full-rank chip-level MMSE equalizer.
- Compare performace of the reduced-rank equalizer to the full-rank Wiener filter and other reduced-rank equalizers.
- Evaluate convergence properties in stationary and non-stationary environment.

Multi-Stage Nested Wiener Filter

First formulated by J.S. Goldstein and I.S. Reed

[1] J. S. Goldstein, I. S. Reed and L. L. Scharf. "A Multistage Representation of the Wiener Filter based on Orthogonal Projections". *IEEE Trans. Information Theory*, Nov. 1998.

First Stage : Decomposition

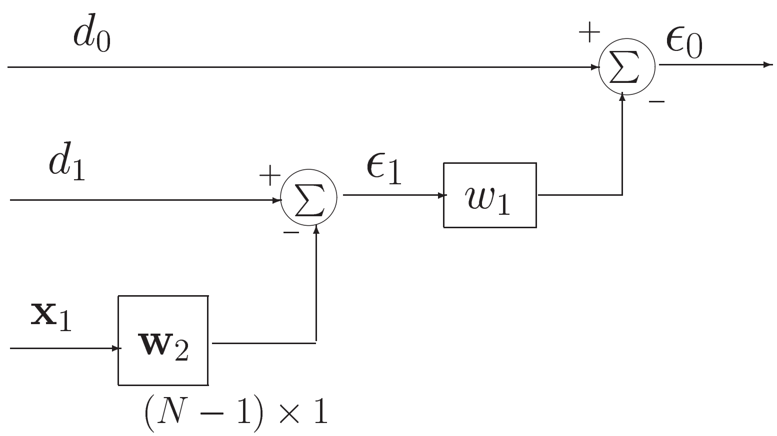


Objective : $\max_{\mathbf{p}_1} \; E[Re\{\mathbf{p}_1^H \mathbf{x_0} \, d_0^*\}]$ subject to $\mathbf{p}_1^H \mathbf{p}_1 = 1.$

Solution : $\mathbf{p}_1 = \frac{\mathbf{r}_{dx}}{||\mathbf{r}_{dx}||}$ where $\mathbf{r}_{dx} = E[\mathbf{x_0} \, d_0^*]$

- $\mathbf{B}_1$ is $(N-1) \times N$ matrix such that $\mathbf{B}_1 \mathbf{p}_1 = 0.$
- After one stage $\mathbf{R}_{x_1} = \mathbf{B}_1 \mathbf{R}_{xx} \mathbf{B}_1^H$, $\mathbf{r}_{x_1 d_1} = \mathbf{B}_1 \mathbf{R}_{xx} \mathbf{p}_1.$

First Stage : Nested Recursion



Objective : Find MMSE estimate of d_1 from $\mathbf{x}_1$

Solution : $\mathbf{w}_2 = \mathbf{R}_{x_1}^{-1}\mathbf{r}_{x_1 d_1}$

- Error Residue from the First Stage is

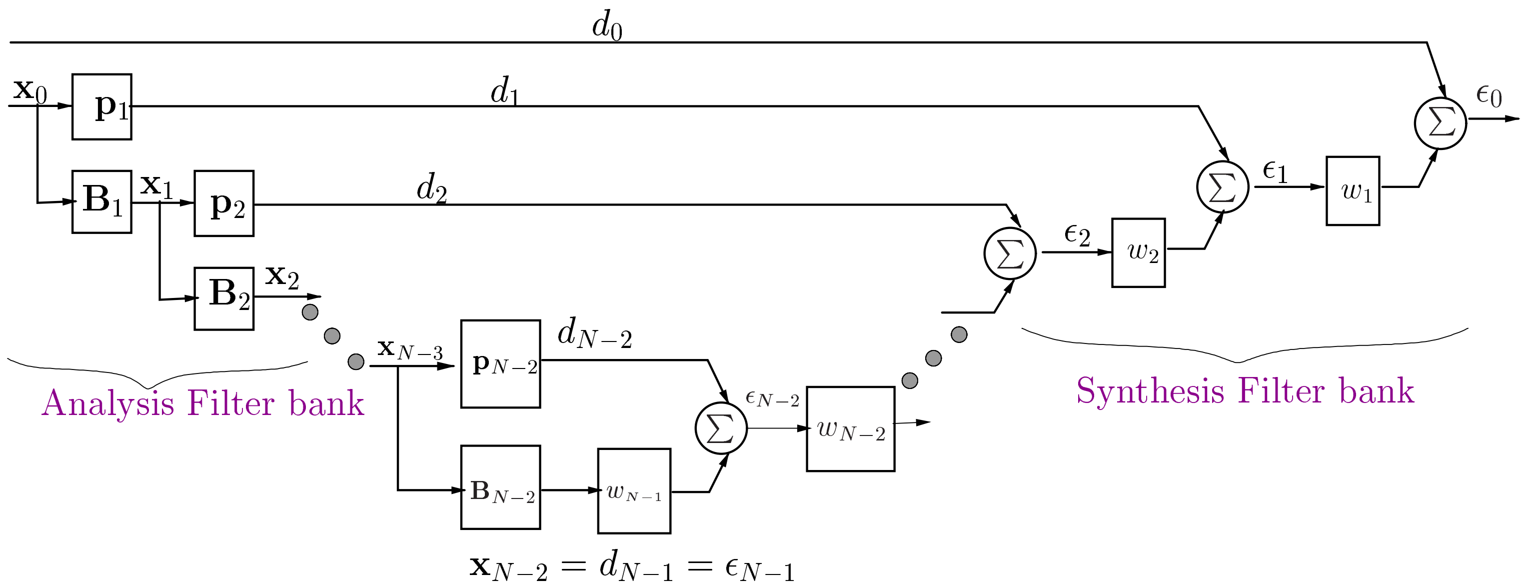
$$\epsilon_1 = d_0 - \mathbf{w}_2^H \mathbf{x}_1 = d_0 - \mathbf{w}_2^H \mathbf{B}_1 \mathbf{x_0}$$

- MMSE Estimate of d_0 from ϵ_1 is $w_1^* \epsilon_1$, where

$$w_1 = \epsilon_1^{-1} \mathbf{p}_1^H \mathbf{r}_{dx} = \|\mathbf{r}_{dx}\|/\epsilon_1$$

- $E[|\epsilon_0|^2]$ is identical to the Mean-Square Error of a N-dimensional Wiener filter

Multi-Stage Nested Wiener Filter : Properties



- Forward Recursion :

$$\mathbf{p}_k = \frac{E[\mathbf{x}_{k-1}\, d_{k-1}^*]}{||E[\mathbf{x}_{k-1}\, d_{k-1}^*]||} \qquad \mathbf{B}_k = null(\mathbf{p}_k) \qquad k = 1, \ldots N-2$$

- Backward Recursion :

$$w_k = E[\epsilon_k d_{k-1}^*]/E[|\epsilon_k|^2] \quad \epsilon_{k-1} = d_{k-1} - w_k^* \epsilon_k \quad k = N-1, \ldots 1$$

Multi-Stage Nested Wiener Filter Properties : Contd.

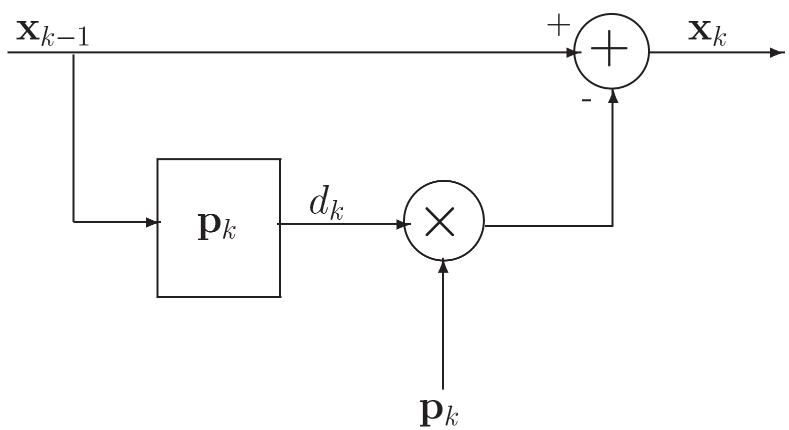
- The pyramidal decomposition decorrelates $\mathbf{x}_0$ at lags greater than one, resulting in an output $\tilde{\mathbf{d}}_N = \begin{bmatrix} d_1 & d_2 & \cdots & d_N \end{bmatrix}^T$ characterized by a *tridiagonal* covariance matrix
- The nested scalar Wiener filters operate on $\tilde{\mathbf{d}}_N$ to form an *uncorrelated* error vector $\tilde{\epsilon}_N = \begin{bmatrix} \epsilon_1 & \epsilon_2 & \cdots & \epsilon_N \end{bmatrix}^T$
- Choosing $\mathbf{B}_k = I - \mathbf{p}_k \mathbf{p}_k^H$ results in filters $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_k, \ldots$ which are mutually orthogonal, and of the same length N

Rank Reduction with the Multi-Stage Nested Wiener Filter

- Rank Reduction is achieved by terminating the orthogonal decomposition at some stage $D < N - 1$
- $M_D = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \dots & \mathbf{p}_D \end{bmatrix}$ forms an orthonormal basis for $\mathbf{w}_D$
- $\mathbf{w}_D$ lies in Krylov subspace spanned by $T_D = \begin{bmatrix} \mathbf{r}_{dx} & \mathbf{R}_{xx}\mathbf{r}_{dx} & \mathbf{R}_{xx}^2\mathbf{r}_{dx} & \dots & \mathbf{R}_{xx}^{D-1}\mathbf{r}_{dx} \end{bmatrix}$
- As the number of stages increase, the process $\mathbf{x}_k$ tends to become white, and the filter $\mathbf{p}_{k+1}$ goes to zero — the optimal MSE is then achieved at that stage

Low-complexity Implementation of the MSNWF

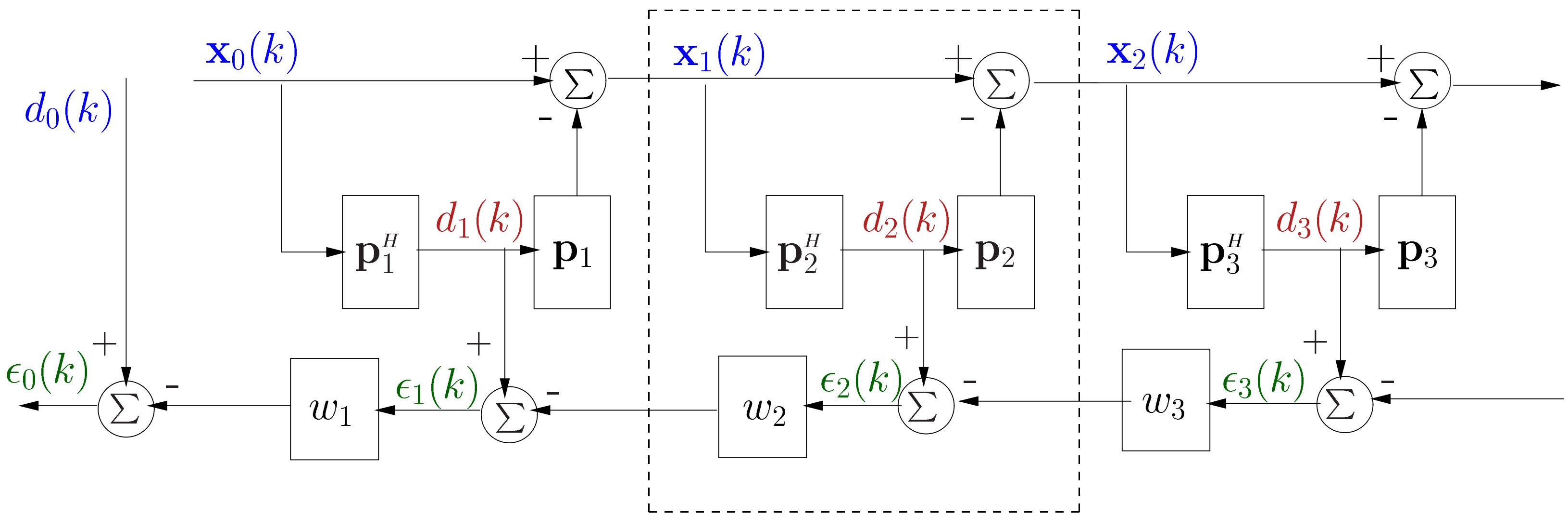
The ‘blocking matrix’ is a very useful concept to develop the MSNWF but can be replaced as follows



$$
\begin{aligned}
d_k &= \mathbf{p}_k^H \mathbf{x}_{k-1} \\
\mathbf{x}_k &= \mathbf{B}_k^H \mathbf{x}_{k-1} = [I - \mathbf{p}_k \mathbf{p}_k^H] \mathbf{x}_{k-1} \\
&= \mathbf{x}_{k-1} - d_k \mathbf{p}_k
\end{aligned}
$$

Equivalent Analysis Filterbank

MSNWF as a Lattice Filter



Modular Structure

$$d_i(k) = \mathbf{p}_i^H \mathbf{x}_{i-1}(k)$$

$$\mathbf{x}_i(k) = \mathbf{x}_{i-1}(k) - \mathbf{p}_i d_i(k)$$

$$w_i(k) = \|\mathbf{p}_i(k)\|/\epsilon_i^2(k)$$

$$\epsilon_{i-1}(k) = d_{i-1}(k) - w_i \epsilon_i(k)$$

The filter can be truncated at any stage $D \leq N-1$.

$\epsilon_D(k) = d_D(k)$

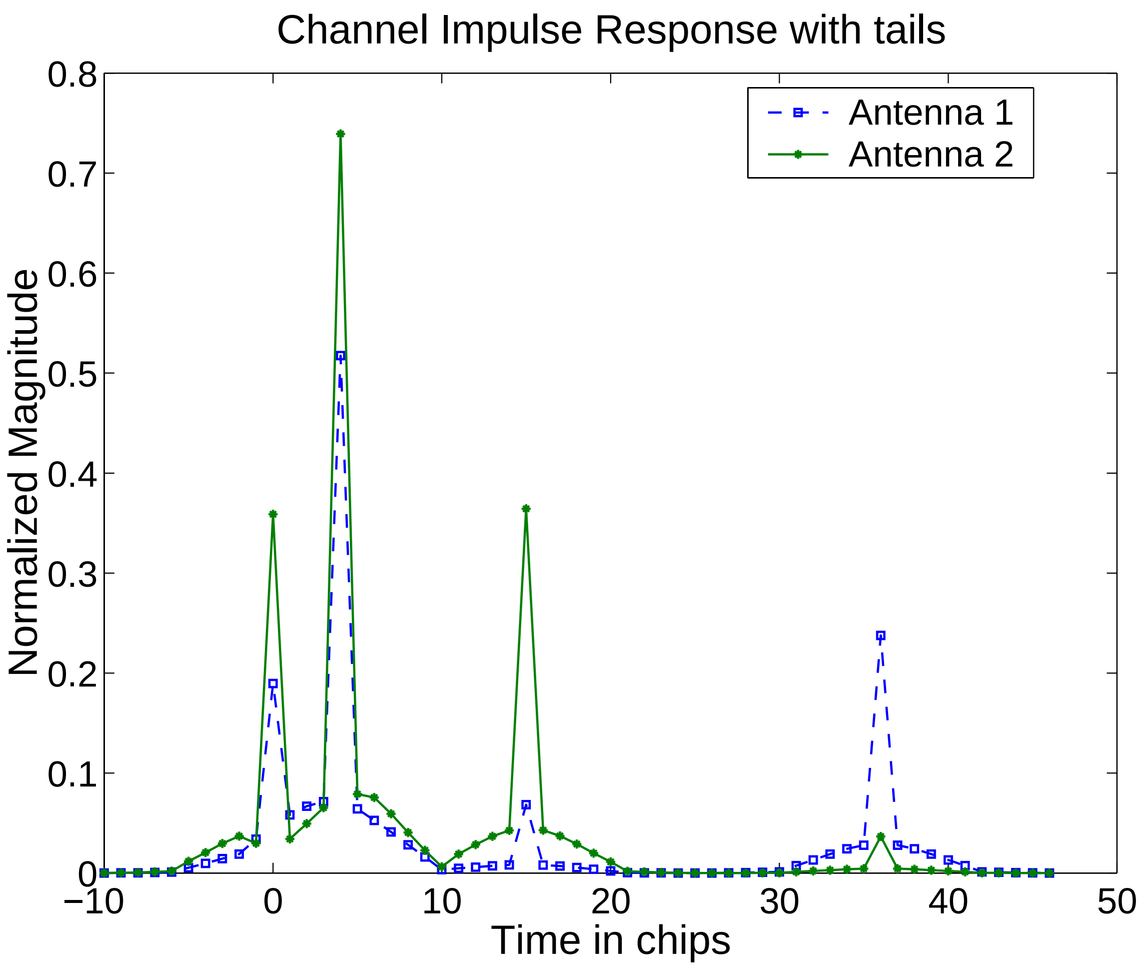
Simulation Parameters for CDMA Downlink

- Chip-rate 3.6864 MHz ($T_c = 0.27\mu s$)
- Spreading factor = 64
- BPSK Data Symbols
- Walsh-Hadamard Channel Codes
- QPSK scrambling code, length 32768.
- Square-root raised cosine chip waveform, $\beta = 0.22$
- Receiver uses chip-matched filtering
- 4 equal power multipaths, randomly in between 0 and 10 μs ($\approx$ 37 chips)
- Arrival times at 2 antennas the same, with independent fading
- Saturated system - 64 *equal power* users
- Equalizer length 57 chips
- Delay D_c chosen so as to minimize MSE

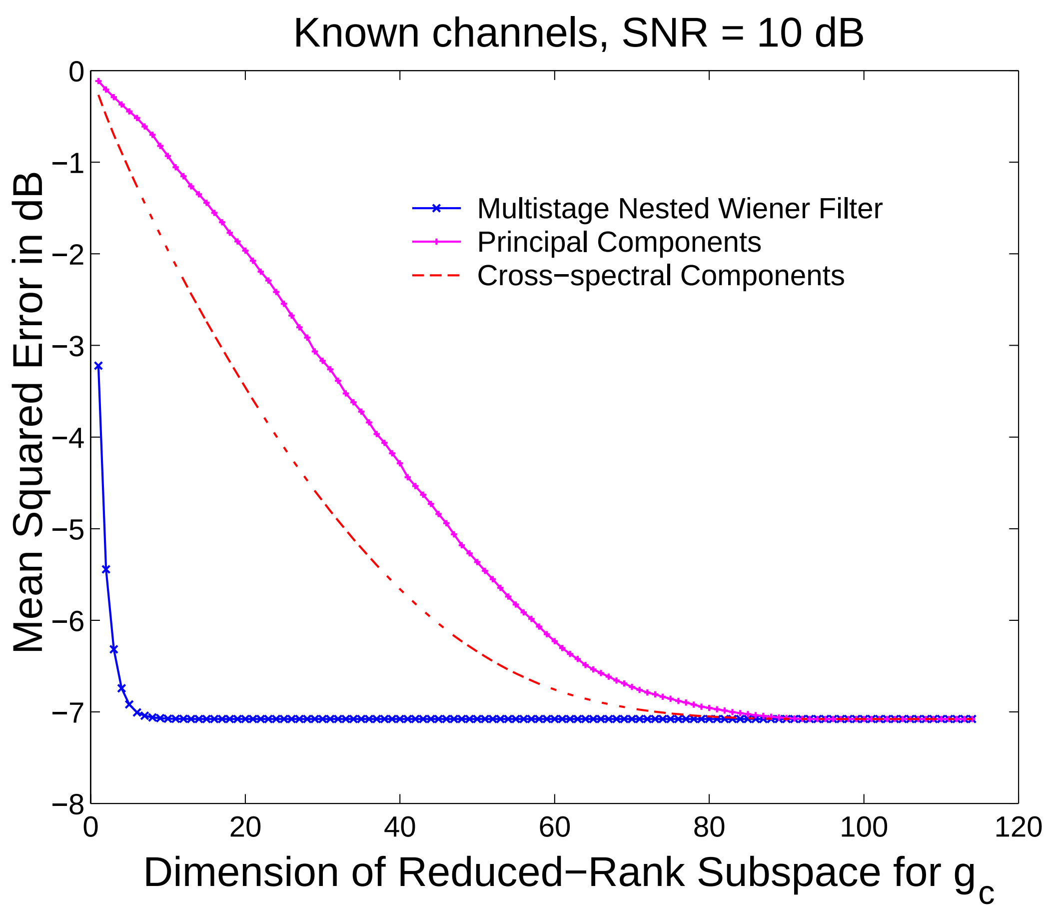
Simulation Parameters : Two Base-stations

- Equal power received signals from two base-stations
- 4 equal power multipath arrivals from 2nd base-station, with random delays and a maximum delay spread of 10 μs
- Received signal sampled at twice chip rate to get $y_{1i}[n] = y_i[nT_c]$ and $y_{2i}[n] = y_i[nT_c + T_c/2]$
- Soft Hand-off – Desired signal transmitted from both base-stations, receiver designs equalizers for both and combines the two outputs

Typical Channel Impulse Response

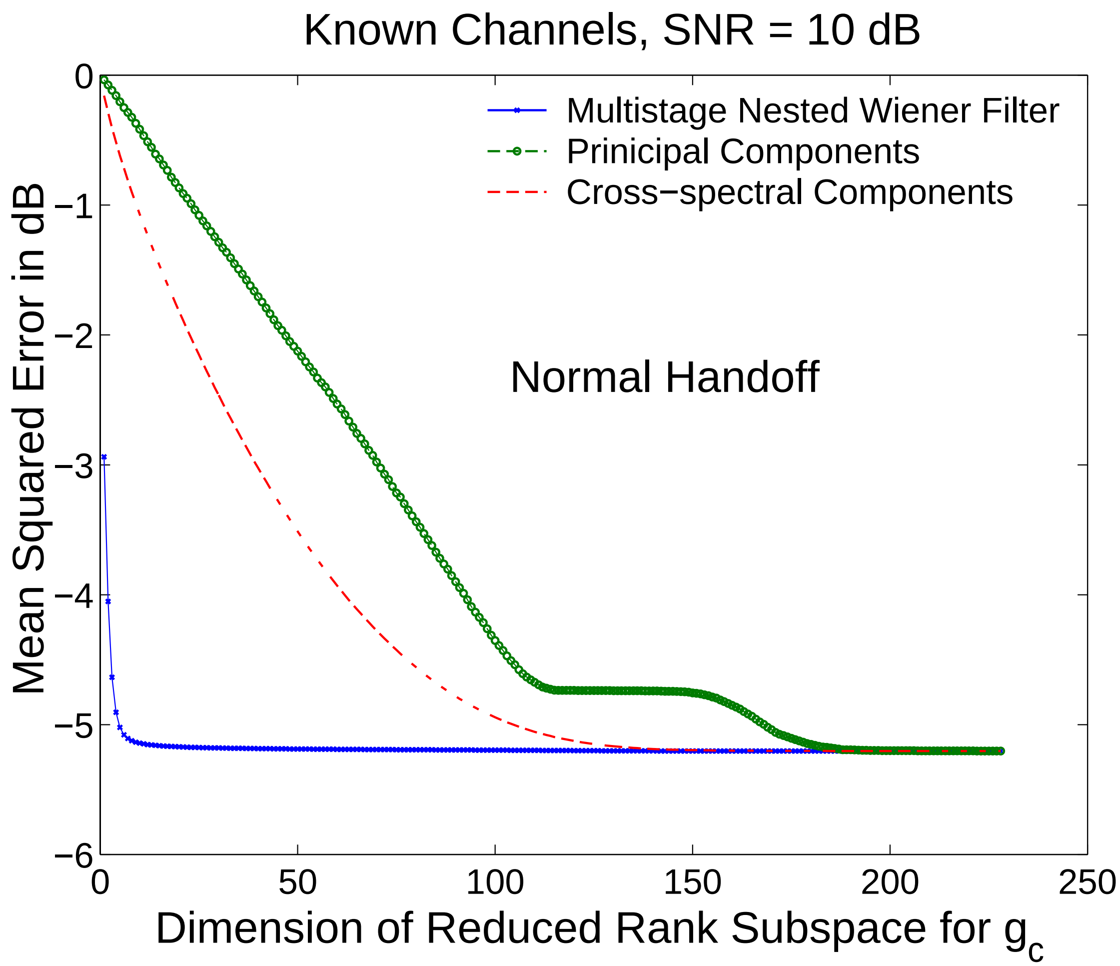


Mean-Square Error vs. Dimension of Subspace for Different Reduced-Rank methods : One Base-station

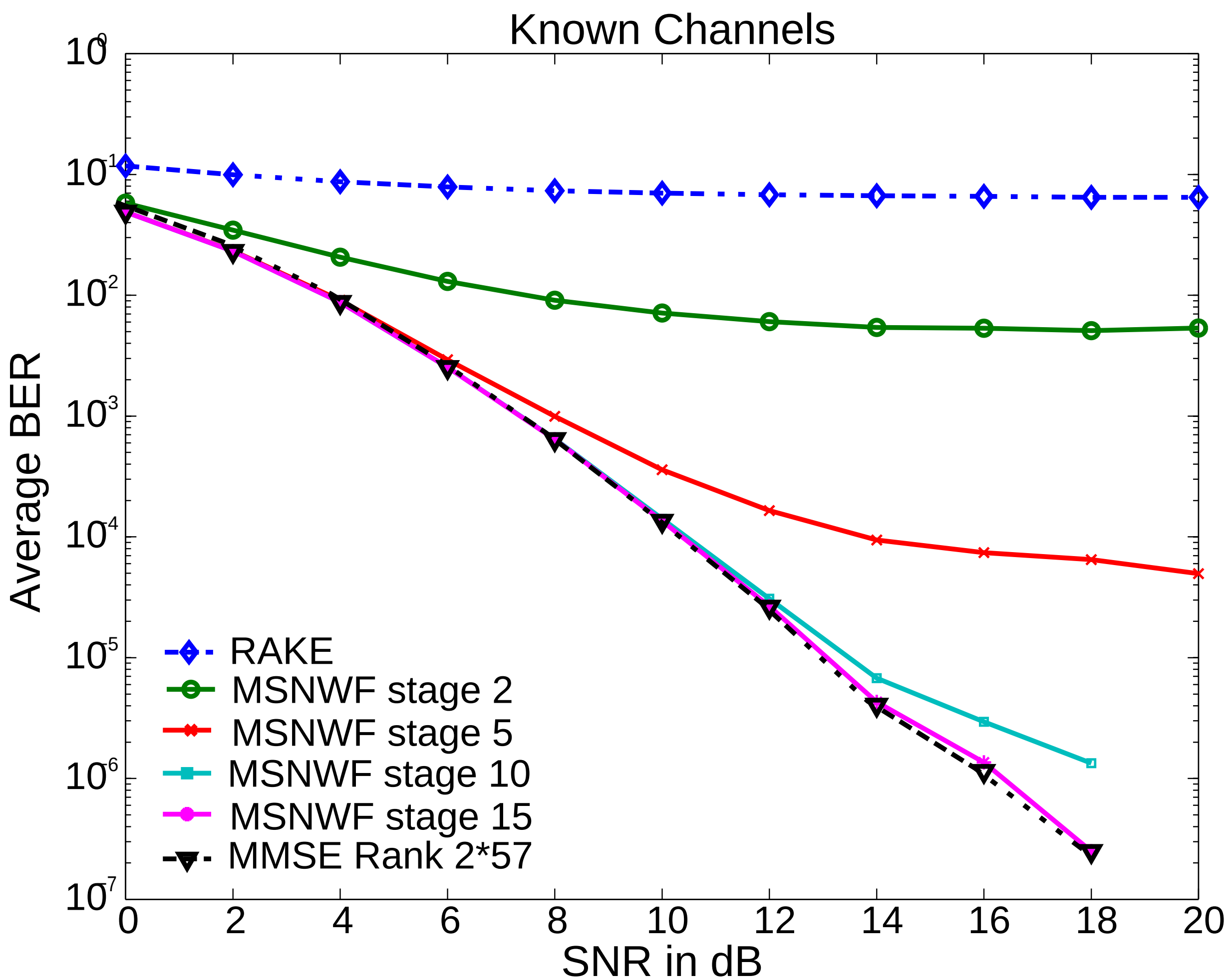


Assuming perfect channel estimation

Mean-Square Error vs. Dimension of Subspace for Different Reduced-Rank methods : Two Base-stations

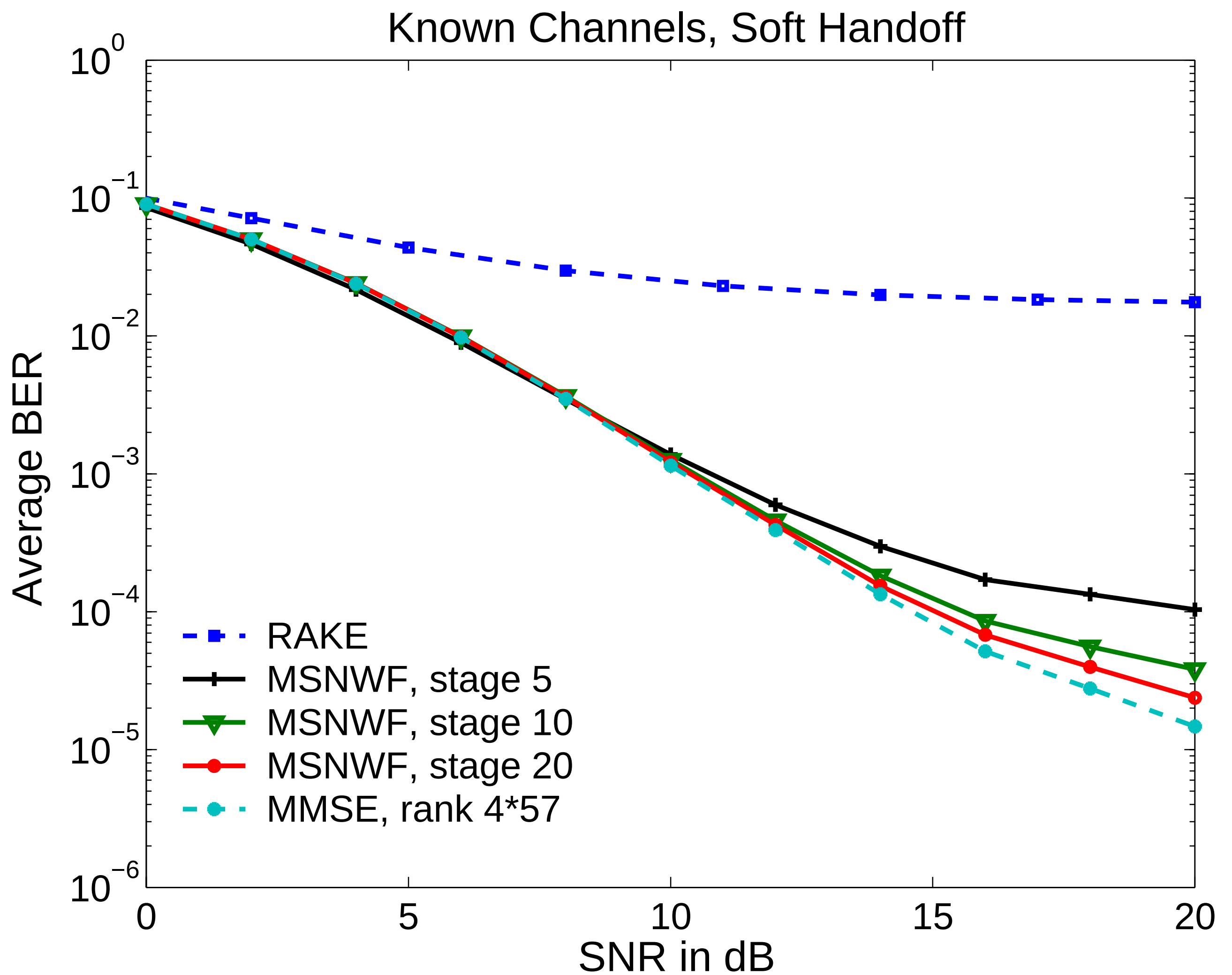


BER vs. SNR for CDMA Downlink : One Base-station



Dimension of Full-rank Filter is 2×57

BER vs. SNR for CDMA Downlink : Two Base-stations



Dimension of Full-rank Filter is 4×57

Training based Adaptive MSNWF

- Cannot train the equalizer on the chip-rate 'sum signal' as the mobile does not know all the active channel codes and data symbols
- Instead, we use the pilot channel of CDMA downlink, which has a known code and known symbols
- We employ 'block-adaptive' lattice-type MSNWF with Initialization :

$$\mathbf{p}_1 = \sum_{i=1}^{N_t} \mathbf{x}[i] d_0^*[i] = \hat{\mathbf{r}}_{dx}$$

- We assume the channels are time invariant during the period of interest

- The symbol estimate is given by

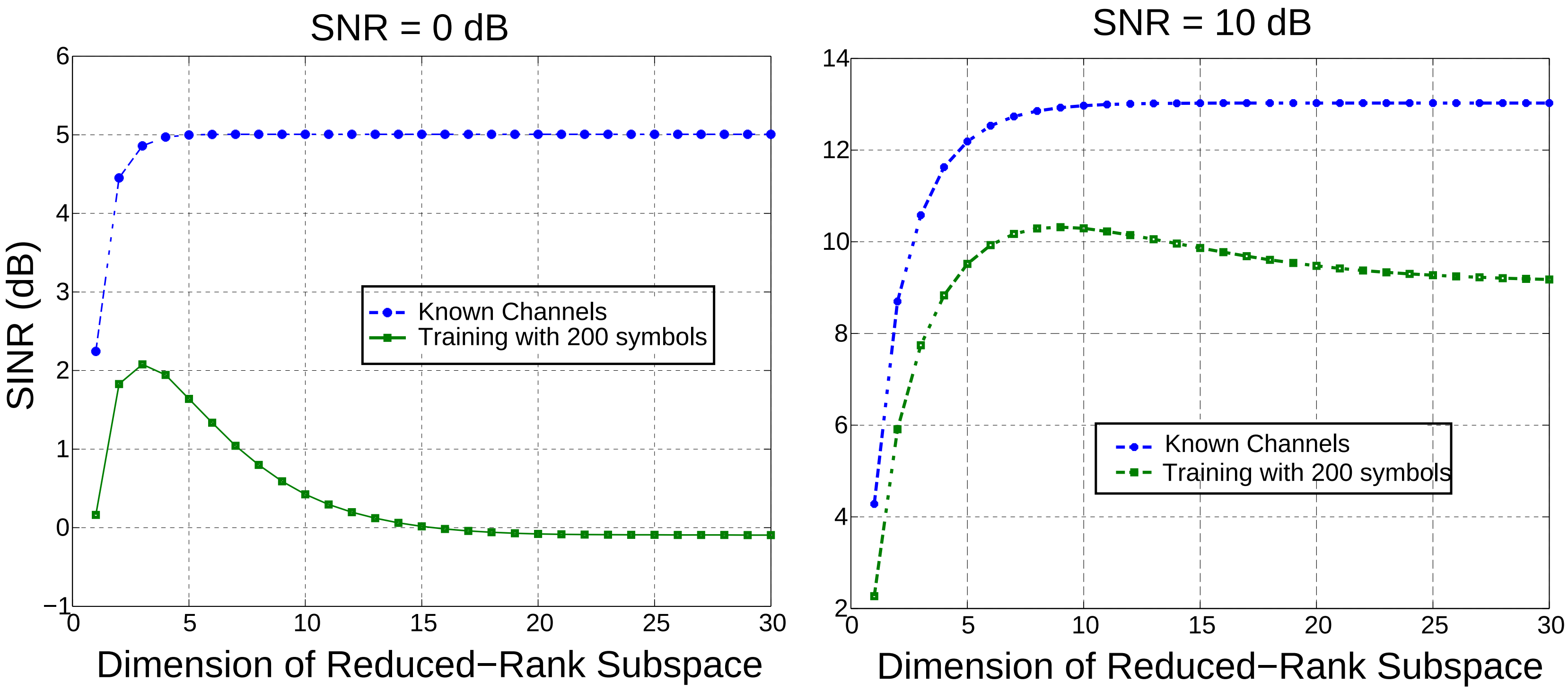
$$\hat{b}_1[m] = \sum_{i=0}^{N_c-1} \left\{ \mathbf{g}_c^H \mathbf{y}[n] \right\} c_{bs}^*[n+i] c_1^*[i]$$
$$\equiv \mathbf{g}_c^H \mathbf{C}_1^H[m] \tilde{\mathbf{y}}[m],$$

where $n = mN_c + D_c$,

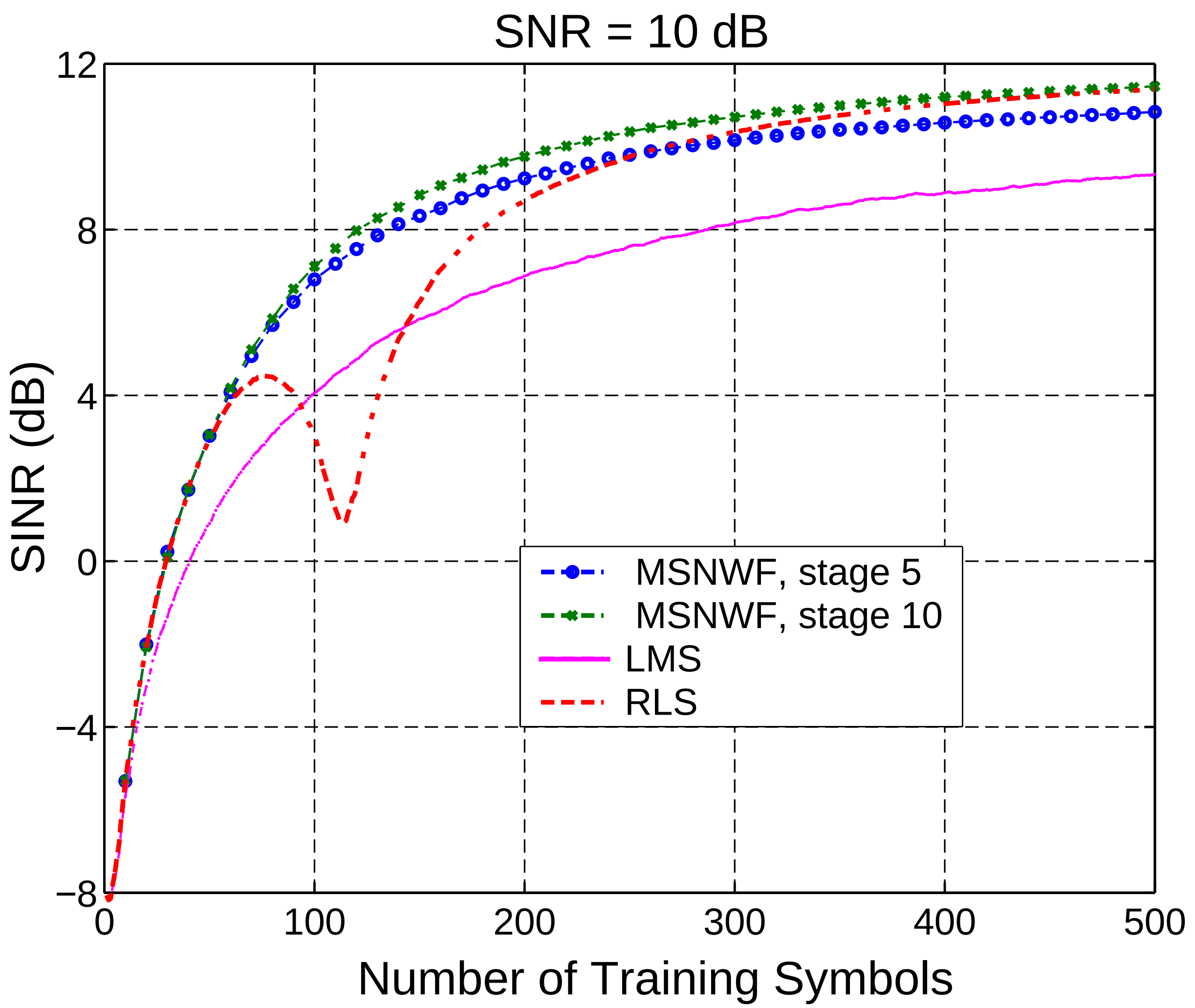
$$\tilde{\mathbf{y}}[m] = \begin{bmatrix} y[n+N_c-1] & \dots & y[n] & \dots & y[n-N_g+1] \end{bmatrix}^T$$

$$\mathbf{C}_1[m] = \begin{bmatrix} c_{bs}[mN_c+N_c-1]c_1[N_c-1] & 0 & \dots \\ \vdots & \ddots & \ddots \\ c_{bs}[mN_c]c_1[0] & \dots & \dots \\ \ddots & \ddots & \ddots \\ 0 & \dots & c_{bs}[mN_c]c_1[0] \end{bmatrix}$$

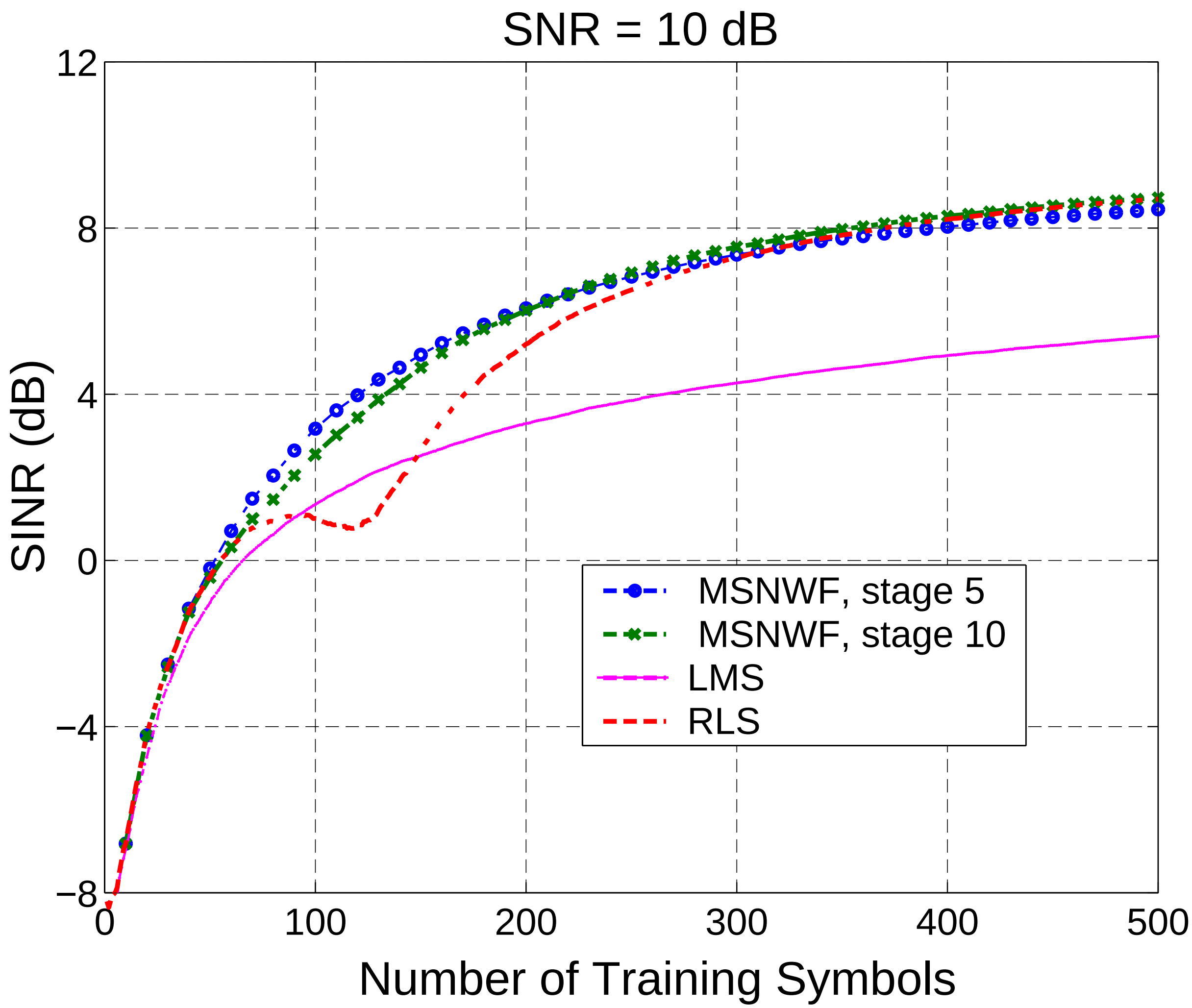
SINR vs. Dimension of Reduced-Rank Subspace for MSNWF, One Base-station



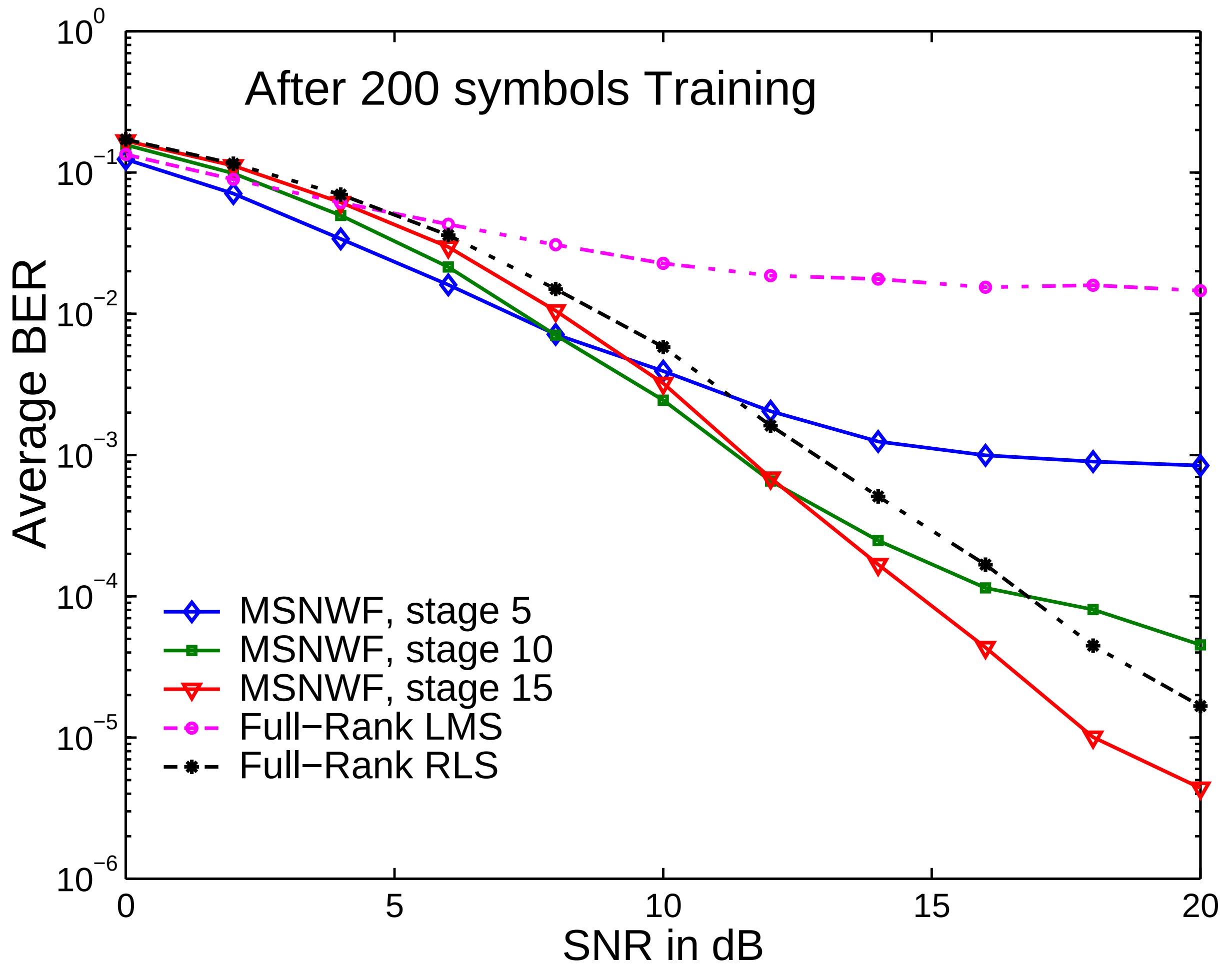
Output SINR vs. Time for Different Adaptive Equalizers One Base-station



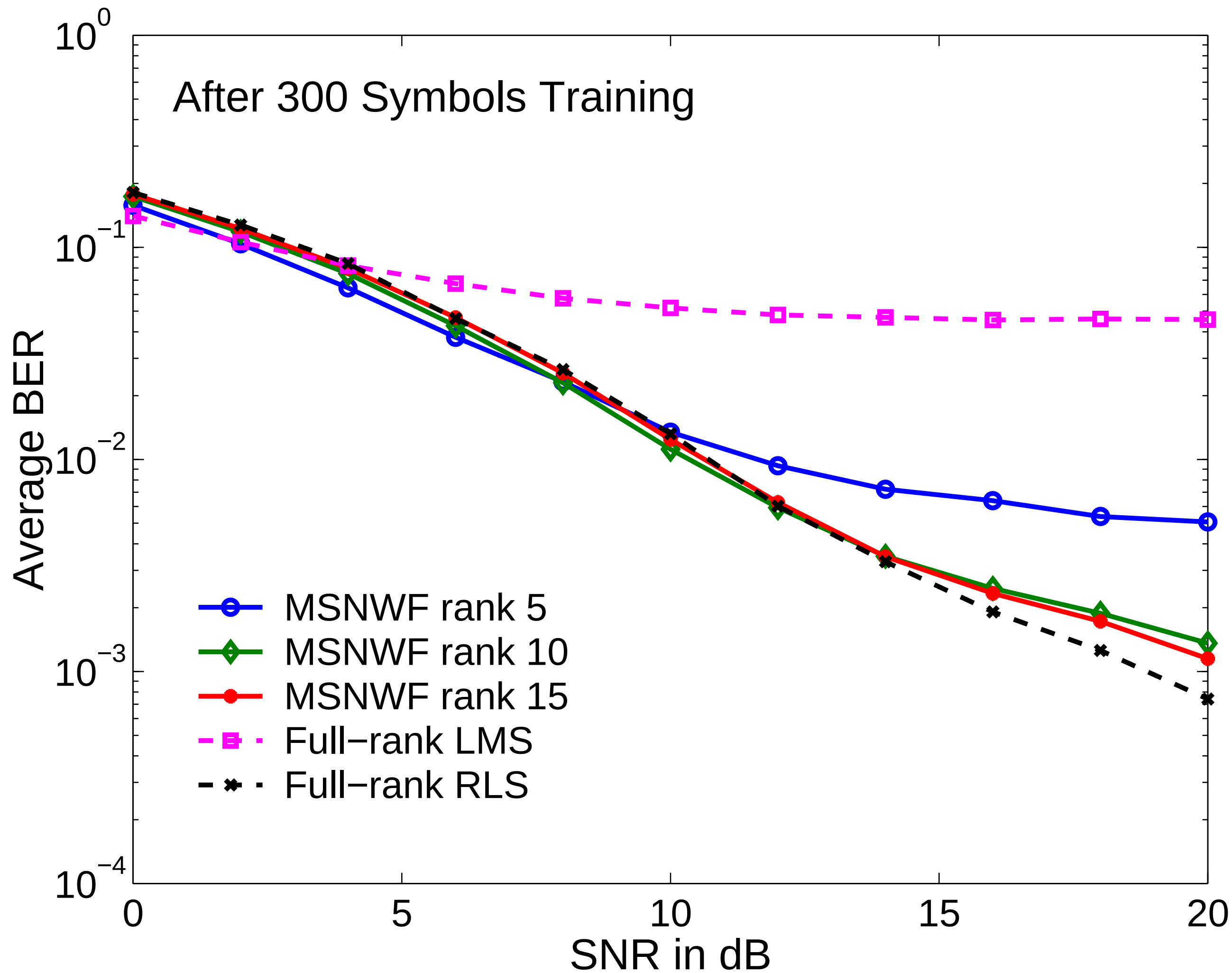
SINR vs. Time for Different Adaptive Equalizers, Two Base-stations, Soft Handoff



BER Performance of Adaptive Equalizers, One Base-station



BER Performance of Adaptive Equalizers, Two Base-stations, Soft Handoff



Future Work : Reduced-Rank Adaptive Equalization

- Simulate time-varying Rayleigh-faded channels
  - → Arrival times are fixed, but multipath gains vary
  - → Multipath delays change slowly
- Compare 'block-adaptive' vs. 'symbol-recursive' MSNWF algorithms as the Doppler spread is increased
- Devise a better training method that would not require correlation with long code over the effective delay spread

Complexity Issues

- In block-adaptive 'lattice' MSNWF, no need to store any $N \times N$ matrices
- a D-stage 'lattice' MSNWF has complexity in the order of $\mathcal{O}(NN_tD)$, where N_t is the block size
- The RLS algorithm requires $\mathcal{O}(N^2)$ computations for each iteration
- Symbol update MSNWF algorithm requires $\mathcal{O}(N^2D)$ computations per iteration
- We propose to do a thorough analysis of the computational complexity of the MSNWF.

Structured Equalizer in Sparse Multipath

Previously, we showed $\mathbf{r}_{dx} = \boldsymbol{\mathcal{H}}\boldsymbol{\delta}_{D_c} = \boldsymbol{\mathcal{H}}(:, D_c)$

If $N_g = L$ and $D_c = L - 1$

$$\mathbf{r}_{dx} = \begin{bmatrix} & \tilde{\mathbf{I}} \\ \tilde{\mathbf{I}} & \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} \text{ where } \tilde{\mathbf{I}} = \begin{bmatrix} 0 & \dots & 1 \\ & \ddots & \\ 1 & \dots & 0 \end{bmatrix}$$

Channel Model :

$$h_i(t) = \sum_{k=0}^{N_m - 1} h_{c_i}[k] p_{rc}(t - \tau_k)$$

Assume $\tau_k = KT_c$, $k = 1 \dots L_c$

$$\mathbf{h}_i = \mathbf{G}\,\mathbf{h}_{c_i}$$

where $\mathbf{G}$ is the convolution matrix associated with $p_{rc}(t)$, and $\mathbf{h}_{c_i}$ is vector of multipath coefficients.

$$\Rightarrow \mathbf{r}_{dx} = \begin{bmatrix} & \tilde{\mathbf{I}} \\ \tilde{\mathbf{I}} & \end{bmatrix} \begin{bmatrix} & \mathbf{G} \\ \mathbf{G} & \end{bmatrix} \begin{bmatrix} \mathbf{h}_{c_1} \\ \mathbf{h}_{c_2} \end{bmatrix}$$

$$= \begin{bmatrix} & \mathbf{G}_m \\ \mathbf{G}_m & \end{bmatrix} \begin{bmatrix} \mathbf{h}_{m_1} \\ \mathbf{h}_{m_2} \end{bmatrix} = \boldsymbol{\mathcal{G}}\, \mathbf{h}_m$$

where $\mathbf{G}_m$ contains only the L_c columns of $\tilde{\mathbf{I}}\mathbf{G}$ corresponding to τ_k and $\mathbf{h}_m$ is the corresponding multipath gains.

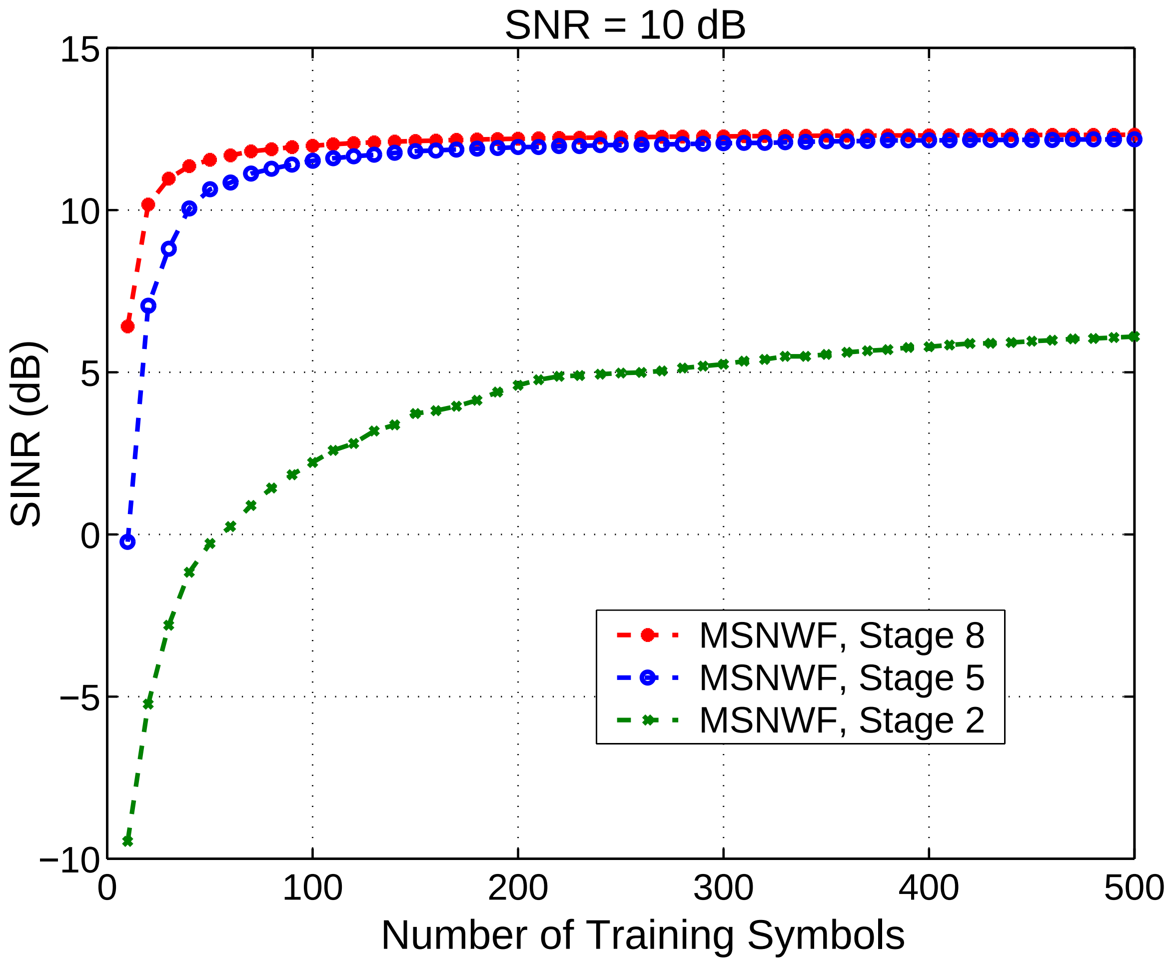
Thus the chip-level MMSE equalizer has the form

$$\mathbf{g}_c = \mathbf{R}_{xx}^{-1} \boldsymbol{\mathcal{G}}\, \mathbf{h}_m$$

Take projection of observed data vector onto a rank $2L_c \ll 2L$ subspace by forming

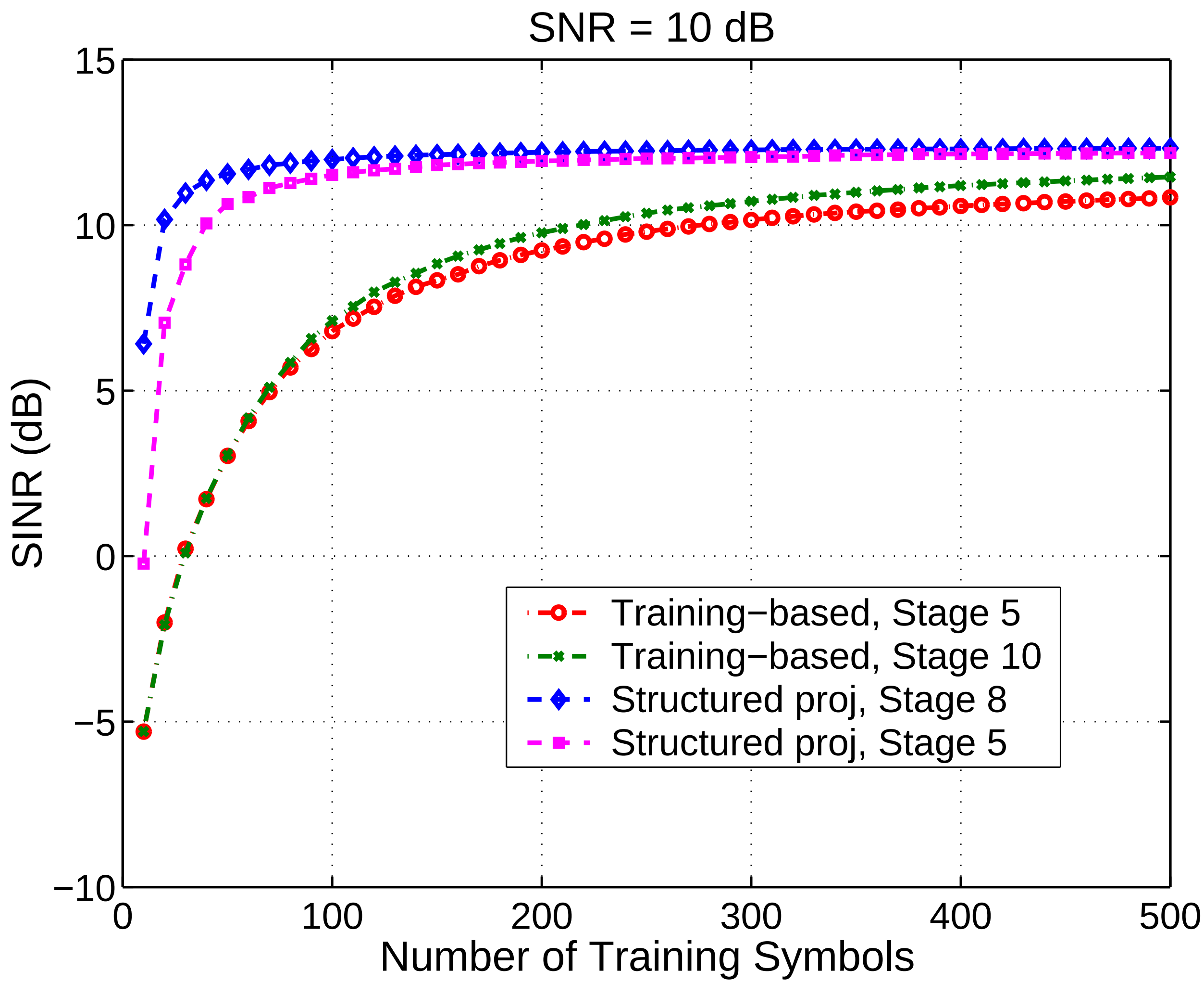
$$\mathbf{x}_r[m] = \boldsymbol{\mathcal{G}}^H \mathbf{R}_{xx}^{-1} \mathbf{C}_1^H[m] \tilde{\mathbf{y}}[m]$$

SINR vs. Time for Structured Projected Equalizers Arrival Times at Exact Chip Periods



Dimension of Structured Projected Equalizer = 8

SINR vs. Time for Adaptive Equalizers using MSNWF



Generalized Arrival Times

When the multipath arrival times are not exact multiples of T_c

$$\mathbf{h}_i = \mathbf{G}\,\mathbf{h}_{c_i}$$

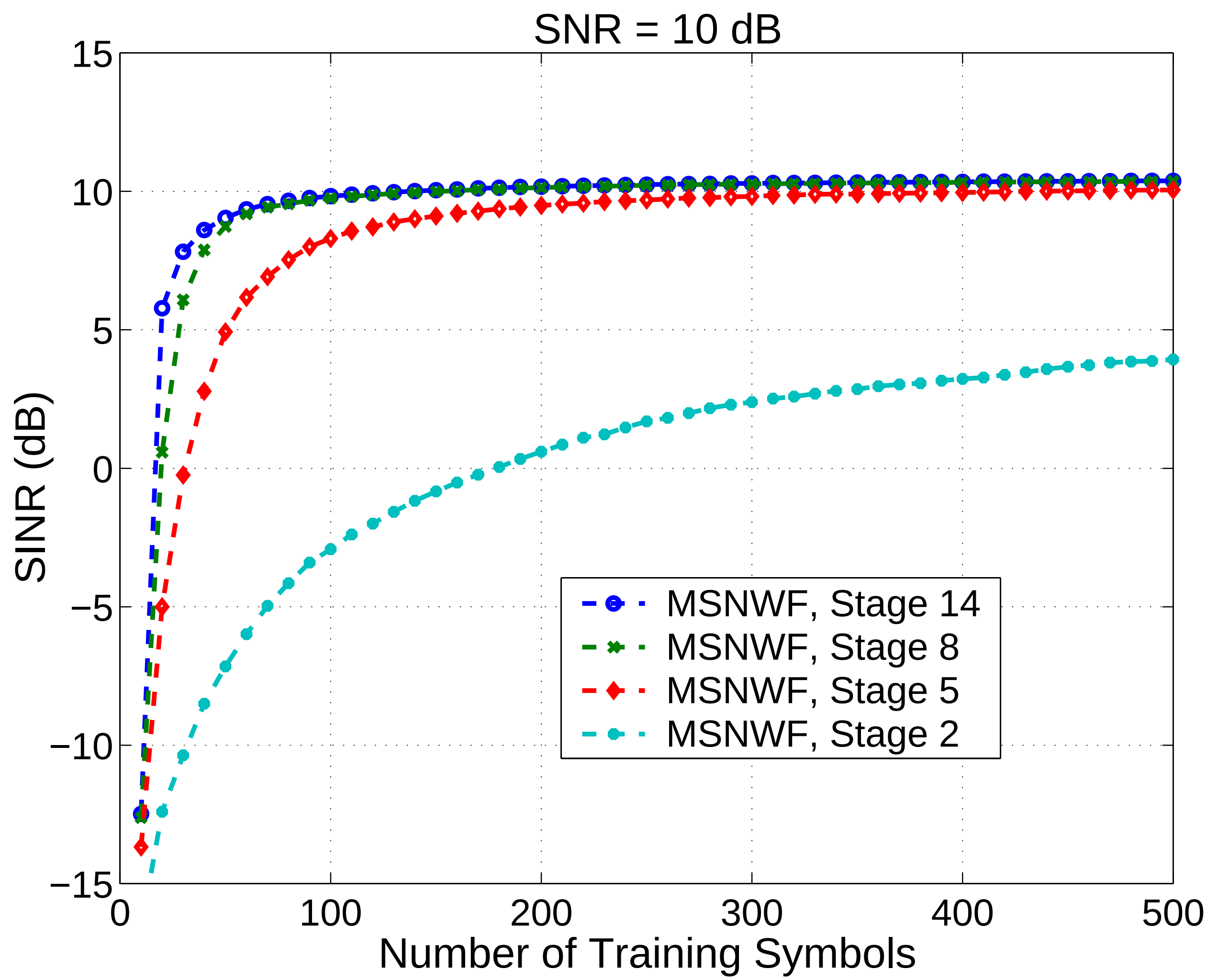
is only an approximate relation.

$\mathbf{G}_m$ now contains two consecutive columns of $\tilde{\mathbf{I}}\mathbf{G}$ for each multipath arrival — corresponding to $\lfloor \tau_k / T_c \rfloor$ and $\lceil \tau_k / T_c \rceil$.

Simulations

- 4 multipaths, one at 0, other 3 uniformly distributed within $10\,\mu s$, but at least $T_c = 0.27\mu s$ apart.
- Dimension of projected filter is now $2 \times 7 = 14$.

SINR vs. Time for Structured Projected Equalizers, Random Arrival Times



Future Work : Structured Projected Equalizers

- Incorporate delay estimation in a blind or semi-blind fashion
  - → Multipath delays change relatively slowly compared to the complex gains
  - → CDMA mobile receivers perform block serial search - the coherent correlations are combined in energy
  - → Synchronous sum signal has a gain of $10\log(64) \approx 18$ dB
  - → Multiple antennas at the receiver provide diversity
  - → If estimates are "noisy", we can take 2/3 consecutive columns of $\mathbf{G}$ centered on estimated delays
- Sample at twice chip-rate to improve performance with random arrival times
- Implement real-time, low-complexity estimation of $\mathbf{R}_{xx}^{-1}$

Space-Time Coding

* Transmit diversity scheme using multiple transmit antennas and spreading the user's symbols across time and space.
* We will investigate *space-time spreading* using multiple transmit and receive antennas without significantly increasing the processing complexity.
* Design linear receivers for space-time coding that will perform better than Rake.

Frequency-Domain Processing

- For systems operating at 2 GHz, there is a strong potential for time and frequency selectivity in the channels.
- In rapidly varying channels with high delay-spread,processing space-time block codes completely in the frequency-domain can be very effective
- Data-blocks are transformed into the frequency domain via FFT and then equalization is performed in the frequency domain.
- Advantages — very low complexity growth with block size N, faster convergence and robustness.

Decision-Directed/Multi-User Detection

- How to optimally use the ‘detected’ symbols to improve performance – “hard” vs. “soft” feedback.
- Successive and Parallel Interference Cancellation have been proposed to combat MAI.
- Multi-user detection combined with space-time processing yields substantial gain over single-user based methods.
- Iterative linear and non-linear MUD schemes approach optimum performance with reasonable complexity

Conclusions

- Reduced-rank MMSE equalizers obtained via MSNWF demonstrate near full-rank performance after only a few stages.
- The convergence speed of MSNWF is similar to full-rank RLS, and has better performance with low sample support.
- The block-adaptive MSNWF can be implemented with very low complexity
- The MSNWF is promising for time-varying channels.

Conclusions

- Structured equalizers exploit the sparseness of the multipath channel to substantially reduce the number of parameters.
- The convergence rate of structured MMSE equalizer was significantly better than unstructured MSNWF operating in a subspace of similar rank.
- Structured equalizer showed excellent convergence even when the underlying assumption was not accurate.