Application of Reduced-rank Chip-level MMSE Equalization to the Forward Link DS-CDMA with Frequency selective Multipath *

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Abstract

This paper deals with synchronous Direct-Sequence CDMA transmission using orthogonal channel codes in frequency selective multi-path, motivated by the forward link in 3G CDMA systems. The chip-level MMSE estimate of the (multi-user) synchronous sum signal transmitted by the base, followed by a correlate and sum, has been shown to perform very well in saturated systems. In this paper, reduced-rank, chip-level MMSE estimation based on the Multi-stage Nested Wiener Filter (MSNWF) first proposed by Goldstein and Reed is presented. Our simulations show that, with perfect channel knowledge, only a small number of stages is needed to achieve near full-rank MSE performance over a practical SNR range. This is valid for the "edge of cell" scenario, where two base-stations are contributing equal-power signals, as well as the single base-station case. Also, adaptive MSNWF operating in a very low rank subspace and using a dedicated pilot channel for training is shown to perform slightly better than RLS and significantly better than LMS.

1 Introduction

Chip-level downlink equalizers have been proposed to significantly increase the capacity for high-speed wireless communication links, such as cdma2000. In this case, the multipath delay spread may span a significant portion of the symbol period, so that the multipath propagation channel is frequency-selective. As a result the orthogonality of the Walsh-Hadamard spreading codes on the downlink is lost and there is significant multiuser access interference (MAI). When many or all users are active in the cell, the BER curve of the standard RAKE receiver was shown to flatten out at higher SNR [1]. Krauss and Zoltowski [1] derived a chip-rate MMSE equalizer that minimizes the mean-square error between the synchronous sum signal of all active users transmitted from a given base station and its estimate, using two antennas on the mobile receiver. In this derivation, the

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sum of the chip sequences from all the users is modeled as i.i.d random sequence, resulting in a "simple" chip-level equalizer that does not depend on the (Walsh-Hadamard) channel code,or the base-station dependent long code. The equalizer is followed by correlation with the desired user's spreading code and the output, downsampled by the spreading factor, gives the symbol estimate. The derived chip-level MMSE estimators with perfect channel knowledge was shown to outperform both ZF and RAKE [1].

However, for high data rate applications, the multi-path delay spread may span several chips and so even this "simple" MMSE equalizer would require computation of a large number of coefficients, and may take an unacceptably long time to converge in adaptive implementations. In this paper, we present a reduced-rank, chip-level MMSE estimator based on the Multi-Stage Nested Wiener Filter(MSNWF) of Goldstein and Reed [2]. This method does not require any knowledge of the eigenvectors of the channel covariance matrix, and so involves much less computation than either the Principal Components or the Cross-Spectral Components methods, the two most widely known reduced-rank techniques. Our simulations show that, with perfect knowledge of the channel statistics, the MSNWF requires only a small number of stages to achieve near full-rank MMSE performance over a wide SNR range. This would imply a rapid convergence in an adaptive implementation. We then use the adaptive algorithm developed by Honig and Goldstein [3] to simulate the performance of the MSNWF when the channel is unknown and the filter is adapted using a pilot channel and known pilot symbols. The SINR vs. time plot shows a convergence speed comparable to full-rank RLS and much faster than full-rank LMS. The superior performance of the MSNWF is further illustrated by simulated BER curves.

The results in this paper are for CDMA forward link with synchronous users, saturated loading, frequency selective fading and long code scrambling. The channel is assumed to be unvarying with time, which might be valid only over a short time interval.

2 Data and Channel Model

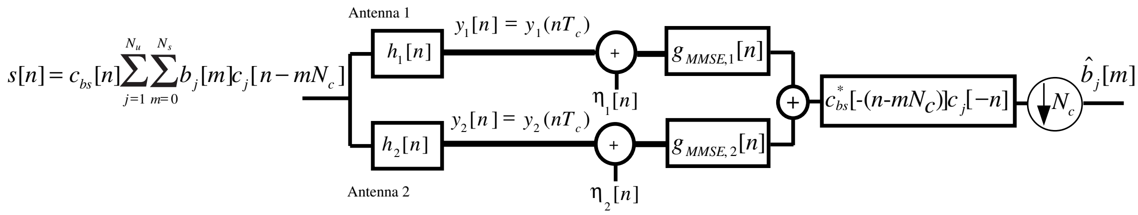


Figure 1: Chip-level MMSE Equalization for DS-CDMA downlink with 1 base-station.

The channel model is shown in Figure 1. For the one base-station case, the impulse response for the i-th antenna channel between the transmitter and receiver (mobile station) is given by

$$h_i(t) = \sum_{k=0}^{N_m-1} h_i[k]p_{rc}(t-\tau_k) \qquad i = 1, 2. \tag{1}$$

where $p_{rc}(t)$ is the composite chip waveform (including the matched low-pass filters on the transmit and receive end). The chip waveform is assumed to have a raised cosine spectrum. N_m is the total number of delayed paths, i.e. 'multipath arrivals', some of which may have zero or negligible power, so that the channel impulse response is sparse.

The transmitted 'sum' signal may be described as

$$s[n] = c_{bs}[n] \sum_{j=1}^{N_u} \sum_{m=0}^{N_s-1} b_j[m] c_j[n - N_c m] \tag{2}$$

where $c_{bs}[n]$ is the base-station dependent long code, $b_j[m]$ is the bit/symbol sequence of the $j-th$ user, $c_j[n]$ is the $j-th$ user's channel (short) code of length N_c, N_u is the total number of active users, N_s is the number of bit/symbols transmitted during a given time window.

The signal received at the $i-th$ antenna (after convolving with a matched filter having a square-root raised cosine impulse response) is given by

$$y_i(t) = \sum_n s[n] h_i(t - nT_c) \qquad i = 1, 2 \tag{3}$$

where T_c is the duration of one chip.

For the sake of notational simplicity we assume a high enough chip rate so that the multipath delays are integer multiples of the chip time T_c, and that we sample at an offset such that the delay of the first path is zero.

2.1 Edge of cell/Soft Handoff

We consider the interference problem when the desired user is at the edge of a cell so that the total received signal at the mobile station is the sum of the contributions from two base-stations, plus noise.

$$y_i(t) = y_i^{(1)}(t) + y_i^{(2)}(t) + \eta_i(t) \quad i = 1, 2 \text{ antennas at receiver.} \tag{4}$$

where the superscript denotes the corresponding base-station and $\eta_i(t)$ is a noise process assumed white and gaussian prior to coloration by the receiver chip-pulse matched filter.

At each antenna, we oversample the signal $y_i(t)$ to obtain $y_{1i}[n] = y_i(nT_c)$ and $y_{2i}[n] = y_i(\frac{T_c}{2} + nT_c)$. These discrete-time signals have corresponding impulse responses $h_{1i}^{(k)}[n] = h_i^{(k)}(t)|_{t=nT_c}$ and $h_{2i}^{(k)}[n] = h_i^{(k)}(t)|_{t=\frac{T_c}{2}+nT_c}$ for base-stations $k = 1, 2$.

In the "soft-hand-off" mode, the desired user's data is transmitted simultaneously from the two base-stations. At the receiver, two equalizers are designed, one for each base-station. The output of each of the two chip-level equalizers is correlated with the desired user's channel code times the corresponding base-station's long code. These two signal estimates are averaged to get the symbol estimate for the soft-hand-off mode. In the "normal mode", the second base-station is considered interference.

3 Chip-level Minimum Mean-Square Error Estimator

The chip-level MMSE equalizer is designed to minimize the mean-squared error between the multi-user synchronous sum signal, $s[n]$ and the sum of the equalizer outputs, as depicted in Figure 1. Given the orthogonality of the channel codes, an estimate of the symbol, $\hat{b}_j[m]$ can be obtained via a correlate and sum with c_j and the base-station dependent long code at the output of the chip-level MMSE equalizer, once per symbol.

Krauss and Zoltowski [1] made some simplifying assumptions to present a chip-level MMSE equalizer that can be easily implemented. The sequence values for the multi-user sum signal are assumed to be i.i.d. random variables. Otherwise the covariance matrix of the sum signal $\mathbf{s}[n]$ is a complicated expression involving the Walsh-Hadamard spreading codes that varies from index to index. The i.i.d. assumption is valid if the (long) scrambling code is viewed as a random i.i.d. sequence and/or all users are active with equal power. With this assumption, the covariance matrix of the signal is $E\{\mathbf{s}[n]^H\mathbf{s}[n]\} = \sigma_s^2\mathbf{I}$ and the MMSE equalizer was shown to be

$$\mathbf{g}^c_{MMSE} = \{\sigma_s^2\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}} + \sigma_n^2\mathbf{I}\}^{-1}\boldsymbol{\mathcal{H}}^H\boldsymbol{\delta}_{D_c} \tag{5}$$

where $\boldsymbol{\delta}_{D_c}$ is a column vector of all zeros except 1 in the $(D_c+1)-th$ position, D_c is the combined delay of the equalizer and channel, σ_s^2, σ_n^2 are the signal and noise powers respectively, and $\boldsymbol{\mathcal{H}}$ is the $2N_g \times (L+N_g-1)$ channel convolution matrix, N_g is the length of the equalizer,

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix}\mathbf{H}_1\\ \vdots \\ \mathbf{H}_2\end{bmatrix} \qquad \mathbf{H}_i = \begin{bmatrix} h_i[0] & 0 & \dots & 0\\ h_i[1] & h_i[0] & 0 & 0\\ \vdots & \ddots & \ddots & \ddots\\ h_i[L-1] & h_i[L-2] & \ddots & h_i[0]\\ 0 & h_i[L-1] & \ddots & h_i[1]\\ \vdots & \ddots & \ddots & \ddots\\ 0 & 0 & 0 & h_i[L-1]\end{bmatrix}^T .$$

Equation (5) has the form of the well-known Wiener-Hopf solution

$$\mathbf{w} = \mathbf{R}_{xx}^{-1}\mathbf{r}_{dx} \tag{6}$$

where $\mathbf{R}_{xx}$ is the channel covariance matrix and $\mathbf{r}_{dx}$ is the cross-correlation vector.

The MMSE is given by

$$MSE = \sigma_s^2\{1 - \sigma_s^2\boldsymbol{\delta}_D^T\boldsymbol{\mathcal{H}}(\sigma_s^2\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}} + \sigma_n^2\mathbf{I})^{-1}\boldsymbol{\mathcal{H}}^H\boldsymbol{\delta}_D\} \tag{7}$$

In [1] and this paper, the delay D_c, $0 \le D_c \le N_g + L - 2$ that yields the smallest MMSE is calculated using the actual channel statistics and that D_c is used in all the simulations. In [1], [4] [5], Krauss and Zoltowski showed that the MMSE significantly outperforms the RAKE receiver. This is clearly illustrated in Figure 2, which plots the difference in SNR between the RAKE and MMSE receivers as a function of target uncoded (theoretical) BER. We refer to [5] for details of the simulation results. For both normal and soft hand-off modes of operation, the RAKE requires much more power than the MMSE receiver. This is more pronounced when soft hand-off is unavailable. So MMSE equalization would allow operation in SNR regions that would be impossible with RAKE receivers - especially when a large number of channel codes are active relative to the spreading factor.

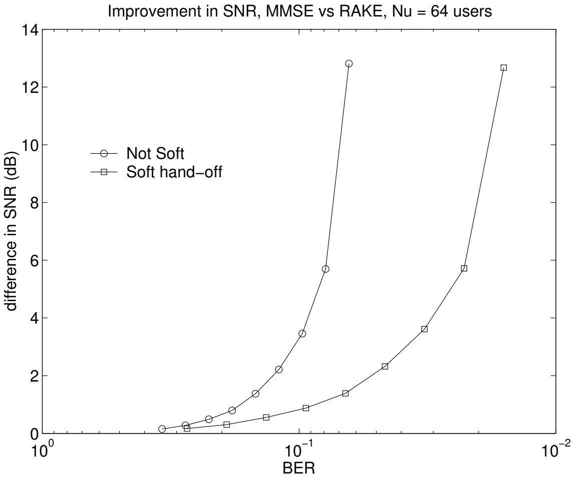


Figure 2: Improvement in SNR from RAKE to MMSE

4 Reduced Rank Filtering

Recently, reduced rank techniques have received attention in the context of adaptive linear equalization for Direct-Sequence Code-Division Multiple Access (DS-CDMA) systems. The received signal vector is projected onto a lower dimensional subspace, and the Wiener filter given by Equation (6) is constrained to lie in this subspace. This has the advantage of reducing the number of filter coefficients to be estimated, and so increases the speed of convergence dramatically for adaptive methods, if the subspace is chosen properly. But the overall MMSE for the reduced-rank filter may be higher than the MMSE for the full-rank filter.

The most widely used reduced rank techniques were based on eigen-decomposition of the channel covariance matrix, $\mathbf{R}_{xx}$. Let the full-rank problem be defined to be of dimension N. In the Principal Components method [6], the reduced rank filter is projected onto a D dimensional subspace with the largest energy. The basis of this subspace is the eigenvectors of $\mathbf{R}_{xx}$ with the D largest singular values. This method is very effective if the dimension of the signal subspace, K is less than D, but this is not true for CDMA with many active users.

A newer technique, known as Cross Spectral methods [7] chooses a set of D eigenvectors of $\mathbf{R}_{xx}$ such that the mean-squared error is minimized as a function of rank. It projects $\mathbf{w}$ onto a D-dimensional subspace using a 'cross-spectral metric' which is a measure of the energy projected along the $k-$th basis vector of the range of $\mathbf{R}_{xx}$ and chooses theeigenvectors with the D largest metrics. This technique can perform well for $D < K$ since it takes into account the energy in the subspace contributed by each user.

4.1 Multi-Stage Nested Wiener Filter

Goldstein and Reed [2] first formulated the MSNWF, which uses the information from the channel covariance matrix, $\mathbf{R}_{xx}$ and cross-correlation vector, $\mathbf{r}_{dx}$ to determine the bases of the lower-dimension subpace that $\mathbf{w}$ is constrained to lie within. In contrast, the reduced-rank methods mentioned above utilised information from $\mathbf{r}_{dx}$ alone. The MSNWF algorithm is depicted in Figure 3. At each stage, a rank one basis is selected

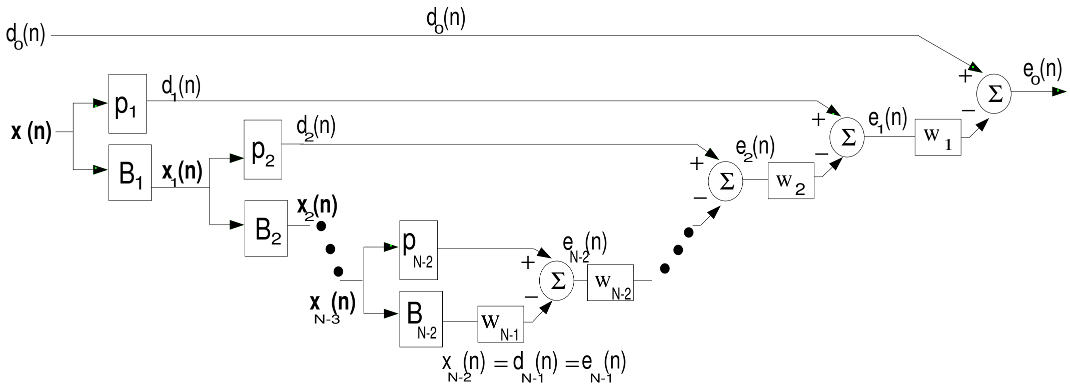


Figure 3: Structure of successive stages of the Multistage Nested Weiner Filter.

based on maximal correlation between the desired signal, d_0 and the observed signal $\mathbf{x}$. The detailed algorithm, as developed by Honig and Xiao [8] is shown in Table 1.

It follows that the matrix $\mathbf{T}_D = [\mathbf{p}_1\ \mathbf{B}_1\mathbf{p}_2\ ...\ \prod_{k=1}^{D-1}\mathbf{B}_k\mathbf{p}_D]$ forms an orthonormal basis for the reduced dimension subspace and that the reduced dimension $D \times D$ correlation matrix $\mathbf{T}_D^H\mathbf{R}_{xx}\mathbf{T}_D$ is tri-diagonal [8]. Honig and Xiao [9] have shown that if the MSNWF is terminated at stage D, then $\mathbf{w}$ is constrained to lie in the Krylov subspace spanned by $\{\mathbf{r}_{dx}, \mathbf{R}_{xx}\mathbf{r}_{dx}, \mathbf{R}_{xx}^2\mathbf{r}_{dx}, \ldots \mathbf{R}_{xx}^{D-1}\mathbf{r}_{dx}\}$.

Goldstein and Reed showed in [2] that if the decomposition is carried out for the full N stages, then the multi-stage nested filter is exactly equivalent to the full-rank classical Wiener filter. Furthermore, the filter-bank structure whitens the error residue at each stage, and compresses the colored portion of the observed data subspace and hence is optimal in terms of reducing the MSE for a given rank [10]. The MSNWF does not require any eigen-decomposition or inversion of the covariance matrix, and so represents a significant reduction in complexity over the full-rank Wiener solution and other reduced-rank techniques. This is very important for practical implementations, particularly if the rank one decomposition can be stopped after a few stages.

In [3], Honig and Goldstein applied adaptive MSNWF to the reverse link with asynchronous users, flat fading, no long code and training by pilot channel. Through theoretical analysis and extensive supporting simulations, MSNWF was shown to achieve near optimal SINR performance with a subspace of dimension roughly equal to $D = 8$ or less. This astounding result, and the superior performance of full-rank MMSE over RAKE motivated the application of MSNWF to chip-level MMSE Equalization for synchronous CDMA in frequency selective multipath.

5 Application of MSNWF to CDMA Downlink

Our first set of results solves for chip-level MMSE equalization based on Equation (5) when only one base-station is transmitting and finds the "ideal" MSNWF solution after various stages, assuming $\mathbf{R}_{xx}$ and $\mathbf{r}_{dx}$ are known (perfect channel estimation). The same simulations are also done for the edge of cell situation, when two base-stations of equal power are received at the mobile-station and the receiver uses soft handoff.

Next we use the class of training-based adaptive algorithms presented in [3] to simulate

- *Initialization:* $d_0(n) = d(n)$ and $\mathbf{x}_0(n) = \mathbf{x}(n)$
- *Forward Recursion:* For $k = 1, 2, ..., D$:

$$\begin{aligned} \mathbf{p}_k &= E\{d_{k-1}^*(n)\mathbf{x}_{k-1}(n)\}/||E\{d_{k-1}^*(n)\mathbf{x}_{k-1}(n)\}|| \\ d_k(n) &= \mathbf{p}_k^H \mathbf{x}_{k-1}(n) \\ \mathbf{B} &= \mathbf{I} - \mathbf{p}_k \mathbf{p}_k^H \\ \mathbf{x}_k(n) &= \mathbf{B}\mathbf{x}_{k-1}(n) \end{aligned}$$

- *Backward Recursion:* For $k = D, D-1, ..., 1$, with $\epsilon_D(n) = d_D(n)$:

$$\begin{aligned} w_k &= E\{d_{k-1}^*(n)\epsilon_k(n)\}/E\{|\epsilon_k(n)|^2\} \\ \epsilon_{k-1}(n) &= d_{n-1}(n) - w_k^* \epsilon_k(n) \end{aligned}$$

It is easily shown that

$$\mathbf{p}_{k+1} = \frac{(\mathbf{I} - \mathbf{p}_k \mathbf{p}_k^H)\mathbf{R}_{k-1}\mathbf{p}_k}{||(\mathbf{I} - \mathbf{p}_k \mathbf{p}_k^H)\mathbf{R}_{k-1}\mathbf{p}_k||}$$

where

$$\mathbf{R}_{k+1} = (\mathbf{I} - \mathbf{p}_{k+1}\mathbf{p}_{k+1}^H)\mathbf{R}_k(\mathbf{I} - \mathbf{p}_{k+1}\mathbf{p}_{k+1}^H)$$

for $k = 0, 1, ..., D-1$, where $\mathbf{p}_1 = \mathbf{r}_{dx}$ and $\mathbf{R}_0 = \mathbf{R}_{xx}$.

Table 1: The basic MSNWF Algorithm

the performance of MSNWF when the channel is unknown, with contribution from only one base-station. Although the MMSE equalizer described in this paper estimates the chip-rate multi-user synchronous sum signal, it is not possible to train the equalizer on this signal as that would require the knowledge of number of active users, all of the active channel codes and the transmitted symbols. Instead, we use the pilot channel of CDMA downlink, which has a known code and known symbols. Frank and Visotsky [11] proved that for DS-CDMA with orthogonal spreading codes, the MMSE equalizer is identical for all channel codes within a multiplicative constant, so the pilot code can be used to train for any other channel code. After equalization, the "recovered" chip signal is arranged into length N_c vectors, correlated with the pilot channel code times the appropriate portion of the base-station long code and downsampled by the spread factor. This 'pilot symbol estimate' is used to train the chip-level equalizers.

The MSNWF simulations are based on the "block-adaptive" training-based algorithm [3] for fixed subspace dimension, D, but without compressing the dimension of the filter p_k by 1 at every stage. As we stop after only a few stages, this does not involve a significant increase in computation, but assures a stable blocking matrix $\mathbf{B}_k$. The algorithm is given in Table 2.

Block size = N_t symbols.

- *Initialization:* $\mathbf{d}_0 = \mathbf{b}$ and $\mathbf{Y}_0(n) = \mathbf{Y}(n)$ where
  $\mathbf{b}$ is the length N_t vector of the pilot symbols,
  $\mathbf{Y} = \left[\mathbf{y}(1), \mathbf{y}(2), \cdots, \mathbf{y}(N_t)\right]$,
  $\mathbf{y}(i)$ is obtained from the chip-rate "sum" signals, correlated with the corresponding long code chip signal and added over one symbol period.
- *Forward Recursion:* For $k = 1, 2, ..., D$:

$$\begin{aligned}
\mathbf{c}_k &= \mathbf{Y}_{k-1}\mathbf{d}_{k-1}^H \\
\delta_k &= ||\mathbf{c}_k \\
\mathbf{p}_k &= \frac{\mathbf{c}}{||\delta_k||} \\
\mathbf{d}_k &= p_k^H \mathbf{Y}_{k-1} \\
\mathbf{B}_k &= \mathbf{I} - \mathbf{p}_k \mathbf{p}_k^H \\
\mathbf{Y}_k &= \mathbf{B}\mathbf{Y}_{k-1}
\end{aligned}$$

- *Backward Recursion:* For $k = D, D-1, ..., 1$, with $\epsilon_D(n) = d_D(n)$:

$$\begin{aligned}
w_k &= (\epsilon_k \mathbf{d}_{k-1}^H)/||\epsilon_k||^2 = \delta_k/||\epsilon_k||^2 \\
\epsilon_{k-1}(n) &= \mathbf{d}_{n-1}(n) - w_k^* \epsilon_k(n)
\end{aligned}$$

Table 2: Training-based block MSNWF Algorithm

6 Simulation Results

A wideband CDMA forward link was simulated similar to one of the options in the US cdma2000 proposal [12]. The chip rate was 3.6864 MHz ($T_c = 0.27\mu s$), 3 times that of IS-95. Simulations were performed for a "saturated cell", i.e. all 64 possible channel codes are active. The spreading factor was $N_c = 64$ chips per bit. The data symbols were BPSK, and spread with one of 64 Walsh-Hadamard codes with length 64 for each user. All users were of equal power, and their signals were summed synchronously and then multiplied with a QPSK scrambling code of length 32678, similar to the IS-95 standard.

The channels were modeled to have four equal-power multi-paths, the first one arriving at 0, the last at $10\mu s$ (corresponding to about 37 chips) and the other two delays picked at random in between 0 and 37 chips. The multi-path coefficients are complex normal, independent random variables with equal amplitude. The arrival times at antenna 1 and 2 are the same, but the multi-path coefficients are independent.

In the two base-station case, the channels are scaled so that the total energy from each of the two base-stations is equal at the receiver. Specifically,

$$\sum_{m=1}^{M} E\{|y_m^{(1)}[n]|^2\} = \sum_{m=1}^{M} E\{|y_m^{(2)}[n]|^2\}. \tag{8}$$

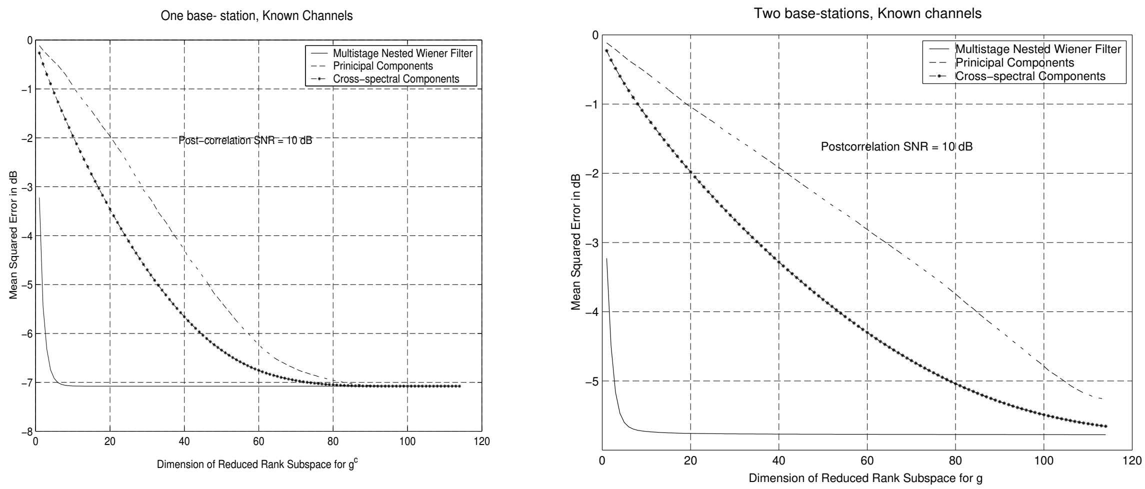
SNR is defined to be the ratio of the sum of the average powers of the received signals over all the channels, to the average noise power, after chhip-matched filtering. Since the spreading factor (number of chips per symbol) is equal to the number of users, and each user contributes the same amount of power, this chip signal SNR is equal to the post-correlation (or de-spread) symbol-rate SNR. The curves were generated by averaging over many different channels. Note that the abscissa in the graphs is the **post**-correlation SNR for *each* user which includes a processing gain of 10log(64) $\approx$ 18 dB.

Figure 4(a) plots the Mean-Square Error for the different reduced-rank methods as a function of the subspace dimension, D. The channel statistics and noise power are assumed to be known (i.e. perfect channel estimation). The dimension of the full space is 114 (the equalizer length is 57 at each of 2 antennas, as multipath delay spread is 37 chips and the chip pulse waveform is cut off after 5 chips at both ends). The MSE for MSNWF is seen to drop dramatically with D, and achieves the performance of the full-rank Wiener filter at dimension approximately 5! In contrast, the PC method takes longer than twice the delay spread, and the CS method does only slightly better.

Figure 5(a) displays the BER curves obtained with the MSNWF for different sizes of the reduced-dimension subspace. The channel statistics are assumed to be known perfectly, so these curves serve as an informative upper bound on the performance. For comparison, the BER curve for full-rank MMSE equalizer and the standard RAKE filter is also shown. It is observed that even a 2-stage reduced-rank filter outperforms the RAKE at all SNR's and only a small number of stages of the MSNWF are needed in order to achieve near full-rank MMSE performance over a practical range of SNR's. Indeed, the BER curve obtained stopping at stage 5 is nearly coincident with the full rank MMSE solution over the range of SNR actual systems operate within in practice. This implies a very significant reduction in computation complexity over the full-rank solution.

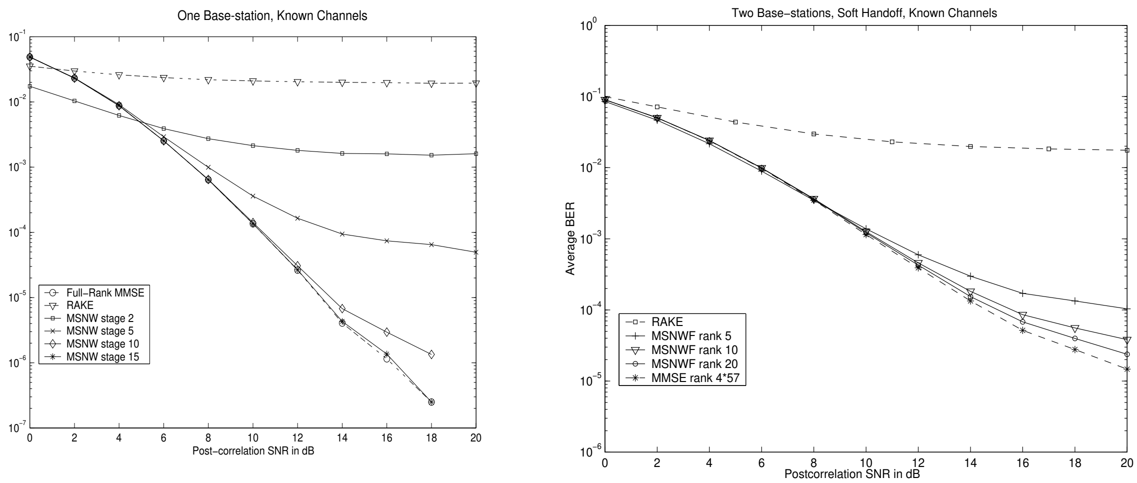
Figures 4(b) and 5(b) display the similar plots, but for the "edge of cell" scenario, with two base-stations contributing equal power signals at the receiver. In this case, there are 4 effective channels at the receiver, because we sampled the received signal at twice the chip-rate at each antenna. It can be shown that the two polyphase channels created from either antenna are nearly linearly dependent in the case of a sparse multipath channel as in our simulations. The dimension of the full space is 2 times 114, 228 which makes the full rank calculations very cumbersome. Amazingly, the MSE for MSNWF still goes down very steeply with rank and achieves the asymptotic value for subspace dimension of only 8 or so. Compared to the PC and CS methods, this is a huge difference in effectiverank reduction. In the BER plots of Figure 5(b) the bit error is calculated for the "soft handoff" mode. With perfect channel estimation, the MSNWF can achieve error rates similar to the full-rank MMSE over practical SNR range after stopping at stage as low as 5!

These plots suggest that MSNWF can achieve rapid adaptation in the case where the chip-level MMSE equalizer is adapted based on a dedicated pilot channel. Our adaptive simulations are for single base-station case only. The output SINR is calculated using the formula derived by Krauss in [4] and plotted vs. time (in symbols) in Figure 6 for the block-adaptive MSNWF at stages 5 and 10, and for the full-rank LMS and RLS Algorithms, at a fixed SNR. This plot shows the rapid convergence of MSNWF to the asymptotic SINR. As expected, the lower rank MSNWF converges slightly faster to a lower SINR. The RLS curve shows irregularities at the beginning because it needs at least 114 time-samples to estimate the time-average of $\mathbf{R}_{xx}$, so at first it performs worse than even MSNWF of stage 5. Even asymptotically it does not beat the MSNWF of rank only 10! The LMS algorithm converges much slower and to a lower SINR.



(a) (b)

Figure 4: MSE vs Rank of Reduced Dimension Subspace



(a) (b)

Figure 5: BER for Different Chip-level Equalizers for CDMA Downlink.

The BER curves in Figure 6 illustrate the performance of these equalizers after training with 200 symbols. At low SNR's, the BER for MSNWF stage 5 is actually slightly lower than the BER for stage 10 or 15. This can be explained as the 'penalty' for learning the channels, i.e. in the presence of significant noise, the MSNWF with the fewer coefficients to adjust performs better. But after about 9dB SNR, the signal power is sufficient for training, and the higher stages of MSNWF yield better approximation of the full-rank filter.

It is noteworthy that over a practical SNR range, in this adaptive implementation, the stage 5 MSNWF does better or almost as good as full-rank RLS! This improvement comes with much lower computational complexity than the RLS. The LMS algorithm is simpler, but performs quite poorly as its convergence is much slower.

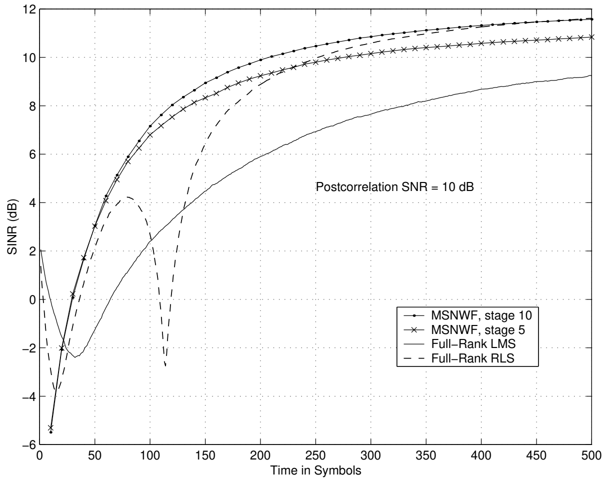


Figure 6: Output SINR vs Time for Adaptive Chip-level Equalizers for CDMA downlink with 1 base-station.

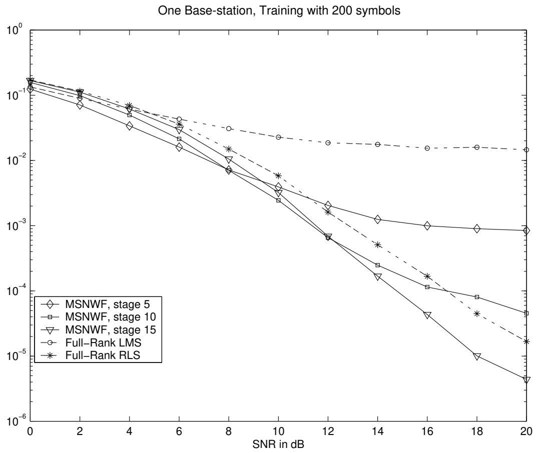


Figure 7: BER for Adaptive Chip-level Equalizers for CDMA Downlink.

References

[1] M. D. Zoltowski T. P. Krauss and G. Leus, "Simple MMSE Equalizers for CDMA Downlink to Restore Chip Sequence: Comparison to Zero-Forcing and RAKE", *in Intl. Conf. on Acoustics, Speech and Signal Processing*, Istanbul, Turkey,, 5-9 June, 2000.

[2] I. S. Reed J. S. Goldstein and L. L. Scharf, "A Multistage Representation of the Wiener Filter based on Orthogonal Projections", *IEEE Trans. Information Theory*, vol. 44, pp. 2943–2959, Nov. 1998.

[3] M. L. Honig and J. S. Goldstein, "Adaptive Reduced-Rank Residual Correlation Algorithms for DS-CDMA Interference Suppression".

[4] M. Zoltowski, T. Krauss and S. Chowdhury, "Chip-level MMSE Equalization for High-Speed Synchronous CDMA in Frequency Selective Multipath", *in Proceedings of SPIE*, vol. 4045, 27-28 April 2000.

[5] T. P. Krauss and M. D. Zoltowski, "MMSE Equalization Under Conditions of Soft Hand-Off", *in IEEE Sixth International Symposium on Spread Spectrum Techniques & Applications (ISSSTA 2000)*, NJIT, New Jersey, 6-8 Sept. 2000.

[6] X. Wang and H. V. Poor, "Blind Multiuser Detection", *IEEE Trans. on Information Theory*, vol. 44(2), pp. 677–690, March 1998.

[7] J. S. Goldstein and I. S. Reed, "Reduced-Rank Adaptive Filtering", *IEEE Trans. on Signal Processing*, vol. 43(2), pp. 492–496, Feb. 1997.

[8] M. L. Honig and W. Xiao, "Performance of Reduced-Rank Linear Interference Suppression for DS-CDMA", submitted to.

[9] M. L. Honig and W. Xiao, "Large System Performance of Reduced-rank Linear Interference Supression for DS-CDMA", *in Proc. Allerton Conf. on Comm.,Control and Computing*, UIUC,, Oct. 1999.

[10] *Reduced-rank intelligent signal processing with application to radar.*

[11] Colin D. Frank and Eugene Visotsky, "Adaptive Interference Suppression for Direct-Sequence CDMA Systems with Long Spreading Codes", *in Proceedings 36th Allerton Conf. on Communication, Control, and Computing*, pp. 411–420, Monticello, IL, 23-25 Sept. 1998.

[12] Telecommunications Industry Association, "Physical Layer Standard for cdma2000 Standards for Spread Spectrum Systems - TIA/EIA/IS-2000.2-A", TIA/EIA Interim Standard, March 2000.