Low-Sample Performance of Reduced-Rank Power Minimization Based Jammer Suppression for GPS[1]

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Abstract

When wideband and narrowband interferences in a GPS system are stationary, a large number of data samples may be obtained to get a good estimate of the interference. However, the jamming environment may be one in which the narrowband jammers have the ability to change frequencies dynamically or the rapid dynamics of the aircraft during maneuvering causes arrival angles of wideband jammers to change. In either type of jamming environment, an interference suppression algorithm will only be effective if it can rapidly converge with a small sample size. We investigate the performance of reduced-rank interference suppression algorithms under conditions of low sample support. It is demonstrated that the Multi-Stage Nested Wiener Filter (MSNWF) outperforms other reduced-rank techniques in terms of suppressing both wideband and narrowband jammers under conditions of low sample support due to the optimal choice of the reduced-rank subspace effected by the MSNWF.

1 Introduction

GPS is known to provide significant force enhancement capability. This force enhancement capability has been demonstrated in every U.S. military operation since (and including) the Gulf War, but with this capability is a concern about the vulnerability of the GPS signal to jamming. The jamming threat is serious because of the physical design of the GPS system. The received power from the GPS satellites is approximately -157 dBW. Many jammers available on the arms market today either already cover the GPS frequencies, or can be modified to do so. A space-time preprocessing filter prior to the GPS correlators is one of several proposed methods for suppressing jammers.

The filter should also provide suppression capabilities against other types of interferers, including continuous wave (CW), swept CW, pulsed CW, phase shift keying (PSK), pseudo-noise signals (20 MHz bandwidth), and narrowband and wideband frequency modulated signals. The interferers may be located anywhere within or adjacent to the 20.46 MHz bandwidths centered at the GPS L1 frequency of 1575.42 MHz or the L2 frequency of 1227.60 MHz. The interferers may be distributed anywhere over 2π steradians of solid angle centered at zenith relative to the local horizontal plane of a GPS receiving antenna. The number of interferers may be as high as twenty.

In addition to providing interference suppression, the filter should allow reception of GPS satellite signals in a stressed environment by maximizing the signal-to-interference plus noise ratio (SINR)

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while minimizing distortion of the desired GPS signals for acquisition and tracking. This filter should furthermore be targeted for operation within the environmental requirements of the dynamics which characterize a high performance fighter aircraft.

2 Power Minimization Based Joint Space-Time Preprocessor

In the joint processing approach, each sample value input to the GPS receiver is formed from a linear combination of samples across both space and time. The space-time weights are realized through a tapped-delay line behind each digitized baseband antenna. The output of the preprocessor is then fed to a standard digital GPS receiver. The goal of the preprocessor is to suppress jammers as best as possible while simultaneously passing as many undistorted GPS signals as possible. Note that the anti-jam space-time filter will not be optimized for any one GPS satellite signal in terms of maximizing the SINR. The advantage of this approach is that the anti-jam space-time filter remains a separate component so that a standard digital GPS receiver may be employed [1],[2].

The criterion for determining the optimal set of space-time weights is premised on the fact that the respective power levels of the desired GPS signals are significantly below the noise floor and that the jammers that could have deleterious effects are above the noise floor. The goal then is to drive the power of the preprocessor output down to the noise floor. This approach serves to place point nulls at the respective angle-frequency coordinates of strong narrowband interferers and spatial nulls in the respective directions of broadband interferers.

In order for the GPS receiver to provide accurate navigation information, it is necessary to track the signals from at least four different GPS satellites. Given the parallax error associated with GPS satellites at near-horizon relative to the aircraft, it is generally desirable to track the respective signals from a larger number of GPS satellites, e.g., twelve. It is desired then that the preprocessor "pass" unaltered as many GPS signals as possible. Thus, the magnitude of the multidimensional Fourier transform of the space-time weights should be as flat (smooth) as possible in the spectral domain as a function of frequency and angular dimensions. The goal is to achieve a desired smoothness while simultaneously nulling both wideband and narrowband interferers under conditions of low sample support.

2.1 Dimensionality Reduction via Reduced-Rank Methods

The disadvantage of space-time processing relative to space-only processing is the large dimensionality of the space-time correlation matrix relative to the spatial correlation matrix. This translates into increased computational complexity and slower convergence. However, depending on the frequency and spatial distribution of the interferers, it may be possible to reduce the dimensionality. Reduction in dimensionality implies constraining the space-time weight vector to lie in a lower dimensional subspace.

Defining an $NM \times NM$ space-time correlation matrix $\mathbf{K}$ (formed from M antennas with N taps per antenna), the original power minimization problem from [5] is

$$\begin{array}{cc} \underset{\mathbf{h}}{\text{Minimize}} & \mathbf{h}^H \mathbf{K} \mathbf{h} \\ \text{subject to:} & \mathbf{h}^H \boldsymbol{\delta}_{NM} = 1 \end{array} \tag{1}$$

where $\boldsymbol{\delta}_{NM}$ is the $NM \times 1$ vector $\boldsymbol{\delta}_{NM} = [0, 1, .., 0, .., 0]^T$ where the 1 is located in the NM position of the vector. We now seek to force the space-time weight vector to be in a particular reduced dimension subspace. That is let $\mathbf{h} = \mathbf{T}\mathbf{h}_r$ where $\mathbf{T}$ is the dimensionality reducing transformation matrix. Substitution of $\mathbf{h} = \mathbf{T}\mathbf{h}_r$ into (1) allows one to rewrite the power minimization problem as

$$\begin{array}{cc} \underset{\mathbf{h}_r}{\text{Minimize}} & \mathbf{h}_r^H \mathbf{T}^H \mathbf{K} \mathbf{T} \mathbf{h}_r \\ \text{subject to:} & \mathbf{h}_r^H \mathbf{T}^H \boldsymbol{\delta}_{NM} = 1 \end{array} \tag{2}$$

Using the method of Lagrange multipliers, the solution to (2) may be found by solving

$$\mathbf{T}^H \mathbf{K} \mathbf{T} \mathbf{h}_r = \alpha \mathbf{T}^H \boldsymbol{\delta}_{NM} \tag{3}$$

where α is the Lagrange multiplier used to satisfy the unity weight constraint $\mathbf{h}_r^H \mathbf{T}^H \boldsymbol{\delta}_{NM} = 1$. It is easily shown that

$$\begin{array}{c} \text{Minimum} \\ \text{output power} \end{array} = \frac{1}{\boldsymbol{\delta}_{NM}^H \mathbf{T} (\mathbf{T}^H \mathbf{K} \mathbf{T})^{-1} \mathbf{T}^H \boldsymbol{\delta}_{NM}}. \tag{4}$$

Since $\mathbf{T}^H \mathbf{K} \mathbf{T}$ is Hermitian-symmetric, it follows that $(\mathbf{T}^H \mathbf{K} \mathbf{T})^{-1}$ is Hermitian-symmetric, so that α is real valued.

The reduced dimension transformation matrix $\mathbf{T}$ can be found by techniques such as the cross-spectral metric or principal-components. A brief overview of these methods is necessary to motivate the use of the MSNWF. The space-time matrix $\mathbf{K}$ can be spectrally decomposed as $\mathbf{K} = \sum_{i=1}^{NM} \lambda_i \mathbf{e}_i \mathbf{e}_i^H$, where λ_i are the eigenvalues of $\mathbf{K}$ indexed in descending order and $\mathbf{e}_i$ are the corresponding eigenvectors. One can then seek dimensionality reduction through the transformation $\mathbf{y}(n) = \mathbf{T}^H \mathbf{x}(n)$, where $\mathbf{T} = [\mathbf{e}_{i(1)} \vdots \mathbf{e}_{i(2)} \vdots \cdots \vdots \mathbf{e}_{i(D)}]$ is an $NM \times D$ matrix containing $D < NM$ eigenvectors of $\mathbf{K}$ and $\{i(1), i(2), ..., i(D)\}$ is a subset of the integers $\{1, 2, ..., NM\}$. Given that the columns of $\mathbf{T}$ are eigenvectors of $\mathbf{K}$, it follows that the $D < NM$ eigenvectors of $\mathbf{K}$ comprising $\mathbf{T}$ can be selected as those which maximize the cross-spectral metric [3],[4] defined as

$$\boldsymbol{\delta}_{NM}^H \mathbf{T} (\mathbf{T}^H \mathbf{K} \mathbf{T})^{-1} \mathbf{T}^H \boldsymbol{\delta}_{NM} = \sum_{j=1}^{D} \frac{|\mathbf{e}_{i(j)}^H \boldsymbol{\delta}_{NM}|^2}{\lambda_{i(j)}} \tag{5}$$

The principal-components technique would instead select the D largest eigenvectors of $\mathbf{K}$ to form $\mathbf{T}$. Both techniques are quite computationally intensive since it is necessary to generate the eigenvectors of $\mathbf{K}$ before finding the reduced-dimensioned matrix $\mathbf{T}$ as well as compute $(\mathbf{T}^H \mathbf{K} \mathbf{T})^{-1}$. It was recently shown by [6] that the MSNWF generates a $\mathbf{T}$ that may be expressed as

$$\mathbf{T} = [\boldsymbol{\delta}_{NM} \vdots \mathbf{K} \boldsymbol{\delta}_{NM} \vdots \cdots \vdots \mathbf{K}^D \boldsymbol{\delta}_{NM}] \tag{6}$$

where again $\mathbf{T}$ is an $NM \times D$ matrix containing $D < NM$ vectors associated with the D stages of the MSNWF. This formulation leads to a simple computation of $\mathbf{T}$ as a function of the D^{th} stage chosen to truncate the MSNWF.

Once generating the particular $\mathbf{T}$ associated with each reduced-rank method, it is possible to explore the effects of sample support associated with each $\mathbf{T}$. It was shown in [5] that the MSNWF outperformed both cross-spectral and principal-components in terms of jammer suppression as a function of rank. It is now of interest to examine the jammer suppression performance of each rank reduced dimension method based on the sample support as illustrated in the following simulations.

3 Simulations

Two scenarios are presented to illustrate the performance of the reduced-rank MSNWF in terms of nulling both wideband and narrowband jammers while operating in a reduced-rank mode at low sample support. Consider $M = N = 7$. These definitions imply an $M = 7$ element equi-spaced linear array with $N = 7$ taps at each antenna. We constrain $h_1(0) = 1$ so that $x_1(n)$ (1^{st} tap behind 1^{st} antenna) is our reference signal. The other taps behind each antenna element form the column data vector $\mathbf{x}(n)$ entering the stages of the MSNWF as illustrated in Figure 1. Table 1 summarizes the values used in the first scenario. Five of the six jammers for this simulation are narrowband jammers with different angles of arrival (AOAs). In both scenarios, the narrowband jammers have different frequency offsets relative to the L1 frequency. Since we are assuming a $20MHz$ receiver bandwidth at each antenna, the noise floor was determined to be approximately -128 dBW after filtering at each antenna.

Figures 2 and 4 indicate the impressive power minimization of the MSNWF as a function of rank. Observe the exceptional power minimization performance of the multistage nested Wiener filter after stopping at only the 10^{th} and 6^{th} stages, respectively. Both figures illustrate that the principal-components and cross-spectral metric are unable to effectively null the jammers at lower ranks. Even when there are many wideband jammers, the MSNWF performance is exceptional. The eigenvector based methods do not account for the information in $\mathbf{k}_{dx}$ and thus require a higher rank to completely null the jammers. Note that the MSNWF is able to null the jammers effectively at low ranks with the added advantage of not requiring the computation of eigenvectors.

Figures 3 and 5 indicate the number of snapshots necessary to effectively null the jammers for each scenario. The power output for each snapshot was averaged over 250 trial runs via a Monte Carlo simulation. Each reduced-rank method used its respective ideal reduced dimension subspace matrix $\mathbf{T}$ in calculating the power output at each snapshot. Once the number of snapshots was equal to the rank for each reduced-rank method, the power output was calculated. Notice the MSNWF at rank 10 and 6 achieves the desired nulling performance with substantially less snapshots than the principal-components or cross-spectral methods. The dimension at which MSNWF achieves the noise floor is the same in both cases.

4 Conclusion

The MSNWF preprocessor was shown to exhibit exceptional nulling performance for both wideband and narrowband jammers at low sample support and low rank. The reduced dimension subspace selected by the MSNWF exhibits rapid convergence in rank and sample support implying adaptive null tracking in a dynamic jamming environment. The MSNWF preprocessor was shown to outperform both principal-components and cross-spectral metric while operating at a lower rank and sample support.

Table 1: Simulation Parameters

Jammer Type	SNR	AOA Ex.1	AOA Ex.2	Bandwidth
Wideband	-100 dBW	$20°$	$20°$	20 MHz
Wideband	-110 dBW	N/A	$0°$	20 MHz
Wideband	-100 dBW	N/A	$-20°$	20 MHz
Wideband	-100 dBW	N/A	$-40°$	20 MHz
Wideband	-110 dBW	N/A	$-60°$	20 MHz
Jammer Type	SNR	AOA	AOA	Frequency
Narrowband	-100 dBW	$60°$	$60°$	-10 MHz
Narrowband	-100 dBW	$15°$	N/A	-5 MHz
Narrowband	-100 dBW	$-10°$	N/A	0 MHz
Narrowband	-100 dBW	$-30°$	N/A	5 MHz
Narrowband	-110 dBW	$-55°$	N/A	10 MHz

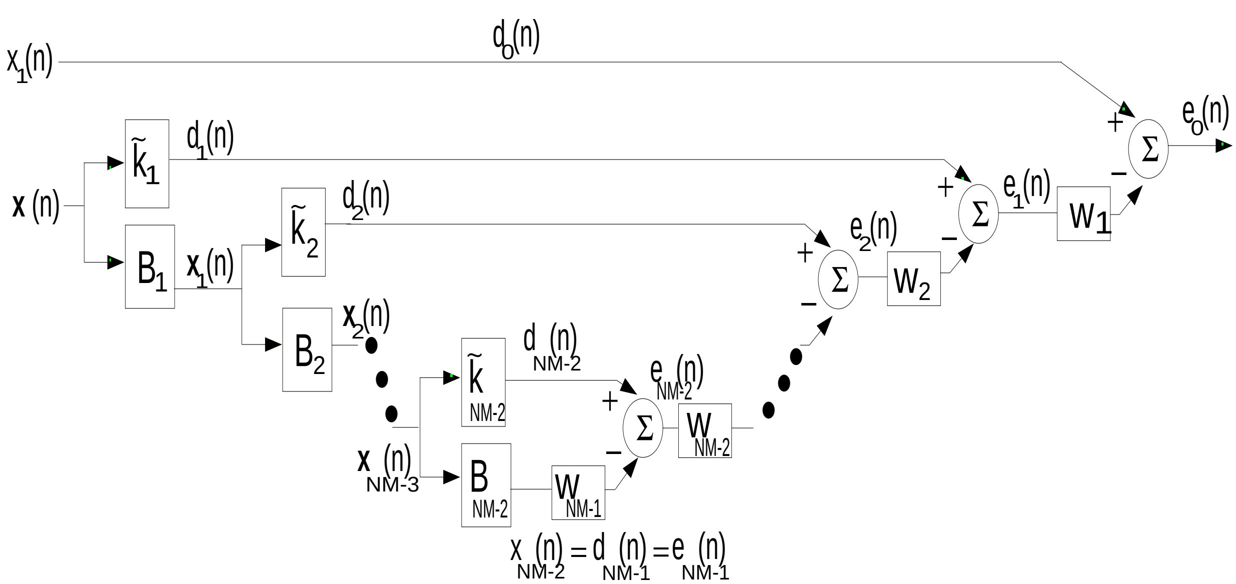

Figure 1. Nested chain of scalar Wiener filters for NM-1 joint space-time preprocessor.

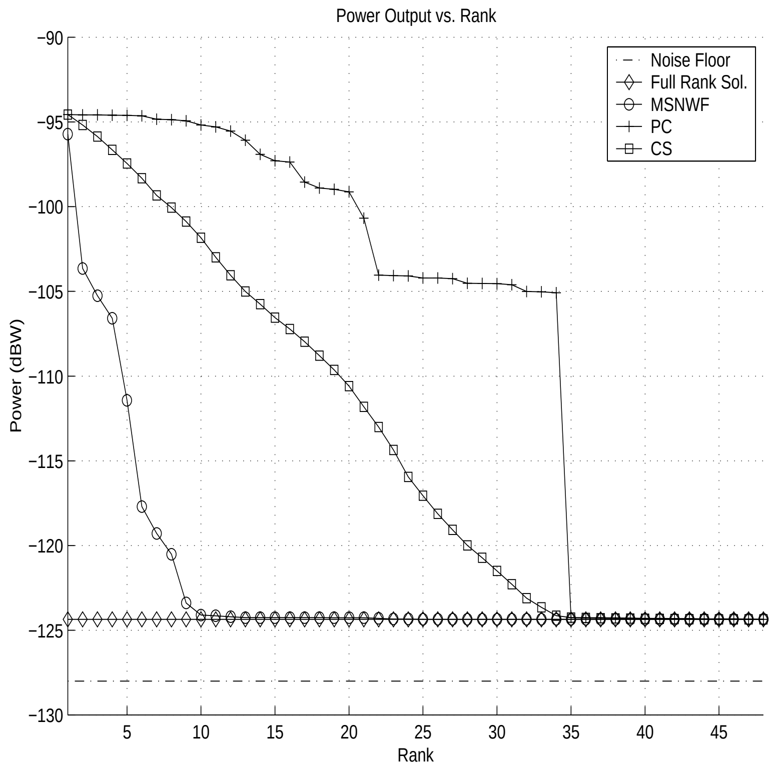

Figure 2. Power output versus Rank (Scenario 2)

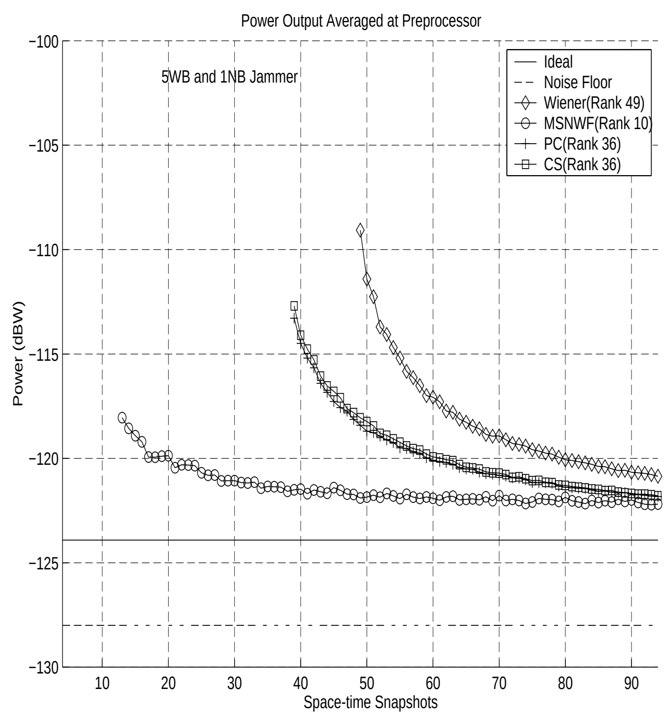

Figure 3. Power output versus Snapshots (Scenario 2)

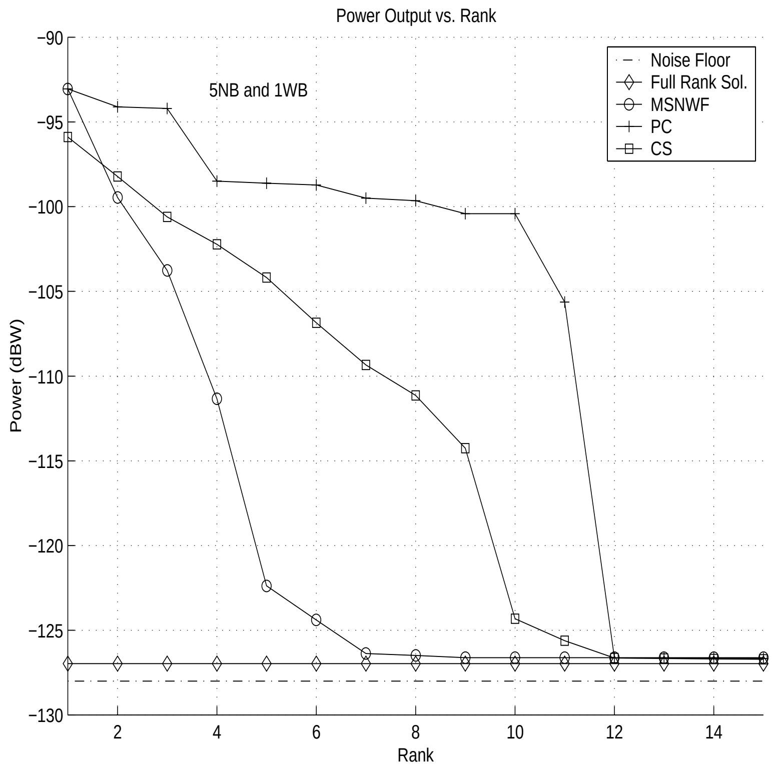

Figure 4. Power output versus Rank (Scenario 1)

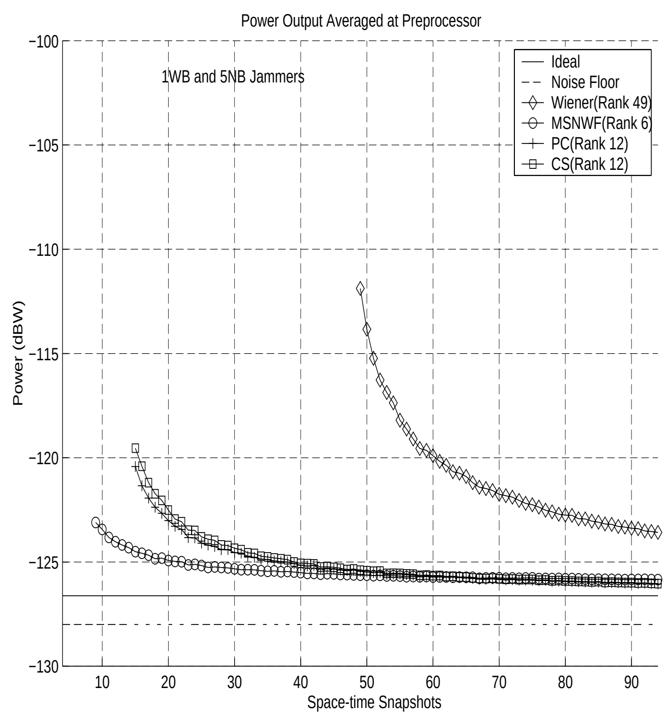

Figure 5. Power output versus Snapshots (Scenario 1)

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