# Reduced-Rank Adaptive MMSE Equalization for the Forward Link in High-Speed CDMA

Samina Chowdhury and Michael D. Zoltowski
School of Electrical and Computer Engineering,
Purdue University, West Lafayette, IN 47907-1285.

J. Scott Goldstein
SAIC, 4001 N. Fairfax Drive, Suite 400
Arlington, VA 22203.

***Abstract*—The chip-level MMSE estimate of the (multi-user) synchronous sum signal transmitted by the base-station, followed by a correlate and sum, has been shown to perform very well in the CDMA forward link using orthogonal channel codes. In this paper, adaptive reduced-rank, chip-level MMSE estimation based on the Multi-stage Nested Wiener Filter (MSNWF) is presented. Our simulations show that, adaptive MSNWF operating in a very low rank subspace and using the pilot channel for training can achieve near full-rank performance at a fast convergence speed.

## I. Introduction

Chip-level downlink equalizers have been proposed to significantly increase the capacity for high-speed wireless communication links, such as cdma2000. In this case, the multi-path delay spread may span a significant portion of the symbol period, so that the orthogonality of the Walsh-Hadamard spreading codes on the downlink is lost and there is significant multi-user access interference (MAI). When many or all users are active in the cell, the BER curve of the standard RAKE receiver has been shown to flatten out at higher SNR [1]. Krauss and Zoltowski [1] derived a chip-rate MMSE equalizer that minimizes the mean-square error between the synchronous sum signal of all active users transmitted from a given base station and its estimate. The chip-level MMSE estimators with perfect channel knowledge was shown to significantly outperform both ZF and RAKE [1].

However, for high data rate applications, the multi-path delay spread may span several chips and so the MMSE equalizer would require computation of a large number of coefficients, and may take an unacceptably long time to converge in adaptive implementations. In this paper, we present a reduced-rank, chip-level MMSE estimator based on the Multi-Stage Nested Wiener Filter(MSNWF), first proposed by Goldstein and Reed [2]. Previously we showed that, with perfect knowledge of the channel statistics, the MSNWF requires only a small number of stages to achieve near full-rank MMSE performance over a wide SNR range [3]. In this paper, we simulate the performance of the MSNWF when the channel is unknown and the filter is adapted using a pilot channel and known pilot symbols. The SINR vs. time plot shows a convergence speed for low-rank MSNWF better than full-rank RLS and much faster than full-rank LMS. The superior performance of the MSNWF is further illustrated by simulated BER curves.

The results herein are for CDMA forward link with synchronous users, saturated loading, frequency selective fading, long code scrambling and with two antennas at the receiver. The channel is assumed to be unvarying with time, which might be valid only over a short time interval.

This research was supported by the Air Force Office of Scientific Research under grant no. F49620-00-1-0127.

## II. Data and Channel Model

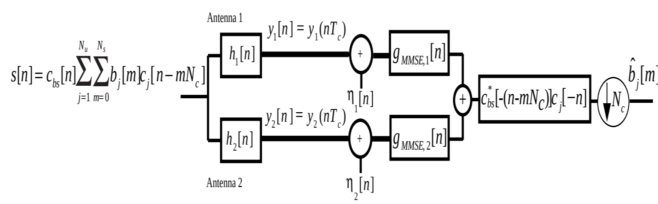



Fig. 1. Chip-level MMSE Equalization for CDMA downlink with 1 base-station.

The channel model is shown in Fig. 1. For the one base-station case, the impulse response for the $i$-th antenna channel between the transmitter and receiver (mobile station) is given by

$$h_i(t) = \sum_{k=0}^{N_m-1} h_i[k]p_{rc}(t-\tau_k) \qquad i = 1, 2. \tag{1}$$

where $p_{rc}(t)$ is the composite chip waveform ( including the matched low-pass filters on the transmit and receive end). The chip waveform is assumed to have a raised cosine spectrum. $N_m$ is the total number of delayed paths, i.e. 'multipath arrivals', some of which may have zero or negligible power, so that the channel impulse response is sparse.

The transmitted 'sum' signal may be described as

$$s[n] = c_{bs}[n] \sum_{j=1}^{N_u} \sum_{m=0}^{N_s-1} b_j[m] c_j[n - N_c m] \qquad (2)$$

where $c_{bs}[n]$ is the base-station dependent long code, $b_j[m]$ is the bit/symbol sequence of the $j-th$ user, $c_j[n]$ is the $j$-th user's channel (short) code of length $N_c$, $N_u$ is the total number of active users, $N_s$ is the number of bit/symbols transmitted during a given time window.

The signal received at the $i-$th antenna (after convolving with a matched filter having a square-root raised cosine impulse response) is given by

$$y_i(t) = \sum_n s[n] h_i(t - nT_c) \qquad i = 1, 2 \qquad (3)$$

where $T_c$ is the duration of one chip.

## III. Chip-level MMSE Estimator

The chip-level MMSE equalizer is designed to minimize the mean-squared error between the multi-user synchronous sum signal, $s[n]$ and the sum of the equalizer outputs, as depicted in Fig. 1. Given the orthogonality of the channel codes, an estimate of the symbol, $\hat{b}_j[m]$ can be obtained via a correlate and sum with $c_j$ and the base-station dependent long code at the output of the chip-level MMSE equalizer, once per symbol.

Krauss and Zoltowski [1] assumed the sequence values for the multi-user sum signal to be i.i.d. random variables. Otherwise the covariance matrix of the sum signal $\mathbf{s}[n]$ is a complicated expression involving the Walsh-Hadamard spreading codes that varies from index to index. With this assumption, the covariance matrix of the signal is $E\{\mathbf{s}[n]^H \mathbf{s}[n]\} = \sigma_s^2 \mathbf{I}$ and the MMSE equalizer was shown to be

$$\mathbf{g}^c_{MMSE} = \{\sigma_s^2 \boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}} + \sigma_n^2 \mathbf{I}\}^{-1} \boldsymbol{\mathcal{H}}^H \boldsymbol{\delta}_{D_c} \qquad (4)$$

where $\boldsymbol{\delta}_{D_c}$ is a column vector of all zeros except 1 in the $(D_c + 1) - th$ position, $D_c$ is the combined delay of the equalizer and channel, $\sigma_s^2, \sigma_n^2$ are the signal and noise powers respectively, and $\boldsymbol{\mathcal{H}}$ is the $2N_g \times (L + N_g - 1)$ channel convolution matrix, $N_g$ is the length of the equalizer,

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \mathbf{H_1} \\ \vdots \\ \mathbf{H_2} \end{bmatrix}; \mathbf{H}_i = \begin{bmatrix} h_i[0] & 0 & \dots & 0 \\ h_i[1] & h_i[0] & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots \\ h_i[L-1] & h_i[L-2] & \ddots & h_i[0] \\ 0 & h_i[L-1] & \ddots & h_i[1] \\ \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & h_i[L-1] \end{bmatrix}^T.$$

Equation (4) has the form of the well-known Wiener-Hopf solution

$$\mathbf{w} = \mathbf{R_{xx}}^{-1} \mathbf{r_{dx}} \qquad (5)$$

where $\mathbf{R_{xx}}$ is the channel covariance matrix and $\mathbf{r}_{dx}$ is the cross-correlation vector.

In this paper, the delay $D_c$, $0 \leq D_c \leq N_g + L - 2$ that yields the smallest MMSE is calculated using the actual channel statistics and that $D_c$ is used in the simulations.

## IV. Reduced Rank Filtering with the Multi-Stage Nested Wiener Filter

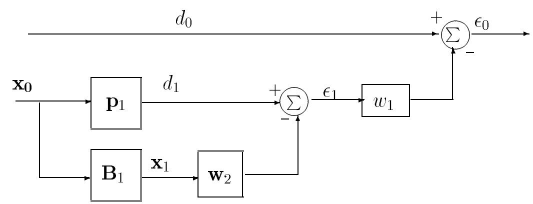



Fig. 2. Structure of the First Stage

Recently, reduced rank techniques have received attention in the context of adaptive linear equalization for Direct-Sequence Code-Division Multiple Access (DS-CDMA) systems. The received signal vector is projected onto a lower dimensional subspace, and the Wiener filter given by Equation (5) is constrained to lie in this subspace. This has the advantage of reducing the number of filter coefficients to be estimated, and so increases the speed of convergence dramatically for adaptive methods, if the subspace is chosen properly.

Goldstein and Reed [2] first formulated the MSNWF, which uses the information from the channel covariance matrix, $\mathbf{R_{xx}}$ and cross-correlation vector, $\mathbf{r}_{dx}$ to determine the bases of the lower-dimension subpace. To obtain the desired signal dependent rank reduction, at stage 1, we select the rank-one subspace $\mathbf{p_1} \in \mathcal{C}^{\mathbf{N}}$, which is maximally correlated with the desired signal $d_0$. This filter is given by

$$\mathbf{p}_1 = \frac{\mathbf{r}_{dx}}{||\mathbf{r}_{dx}||} \qquad \text{where } \mathbf{r}_{dx} = E[\mathbf{x}_0\, d_0^*] \qquad (6)$$

All of the information required for estimating $d_0$from $\mathbf{x}_0$ which is not in the direction of $\mathbf{r}_{dx}$ is contained in its $(N-1)$ dimensional nullspace. Therefore, the input process to the 2nd stage is $\mathbf{x}_0$ filtered with the operator $\mathbf{B_1}$, where $\mathbf{B_1}$ is a 'blocking' matrix such that $\mathbf{B_1 p_1} = \mathbf{0}$.

The observed vector process is thus decomposed by a sequence of nested filters $\mathbf{p_1}, \mathbf{p_2}, \dots, \mathbf{p_D}$, where $D$ is the order of the filter. The output is obtained by linearly combining the outputs of the various stages via the *scalar* weights $w_1, w_2, \dots w_{D-1}$, where the weights are chosen so that thge Mean-Square Error at each stage is minimized.

TABLE I

Training-based block MSNWF Algorithm

Block size = $N_t$ symbols.

- *Initialization:* $\mathbf{d_0} = \mathbf{b}(\mathbf{n})$ and $\mathbf{Y}_0 = \mathbf{Y}(n)$ where $\mathbf{b}$ is the length $N_t$ vector of the pilot symbols, $\mathbf{Y}(1) = [\mathbf{y}(1), \mathbf{y}(2), \cdots, \mathbf{y}(N_t)]$, are the received vectors.
- *Forward Recursion:* For $k = 1, 2, ..., D$:

$$\begin{aligned} \mathbf{c}_k &= \mathbf{Y}_{k-1} d_{k-1}^H \\ \delta_k &= ||\mathbf{c}_k|| \\ \mathbf{p}_k &= \mathbf{c}_k / \delta_k \\ d_k &= \mathbf{p}_k^H \mathbf{Y}_{k-1} \\ \mathbf{B}_k &= \mathbf{I} - \mathbf{p}_k \mathbf{p}_k^H \\ \mathbf{Y}_k &= \mathbf{B}_k \mathbf{Y}_{k-1} \end{aligned}$$

- *Backward Recursion:* For $k = D, ..., 1$, with $\epsilon_D = d_D$:

$$\begin{aligned} w_k &= (\epsilon_k d_{k-1}^H)/||\epsilon_k||^2 = \delta_k/||\epsilon_k||^2 \\ \epsilon_{k-1} &= d_{k-1} - w_k^* \epsilon_k \end{aligned}$$

This filter-bank structure is optimal in terms of reducing the MSE for a given rank, and if the decomposition is carried out for the full $N$ stages, then the multi-stage nested filter is exactly equivalent to the full-rank classical Wiener filter [2]. The MSNWF does not require any eigen-decomposition or inversion of the covariance matrix, and thus yields a significant reduction in complexity over the full-rank Wiener solution and other reduced-rank techniques.

## V. Application of MSNWF to CDMA Downlink

We use the class of training-based adaptive algorithms presented in [4] to investigate the performance of the MSNWF when the channel is unknown, with contribution from only one base-station. It is not possible to train the MMSE equalizer on the chip-rate multi-user synchronous sum signal signal as that would require the knowledge of all of the active channel codes and the transmitted symbols. Instead, we use the pilot channel of CDMA downlink for training.

The MSNWF simulations are based on the "block-adaptive" training-based algorithm [4] for fixed subspace dimension, $D$, but without compressing the dimension of the filter $p_k$ by 1 at every stage. As we stop after only a few stages, this does not involve a significant increase in computation, but assures a stable blocking matrix $\mathbf{B_k}$. The algorithm is given in Table I.

## VI. Simulation Results

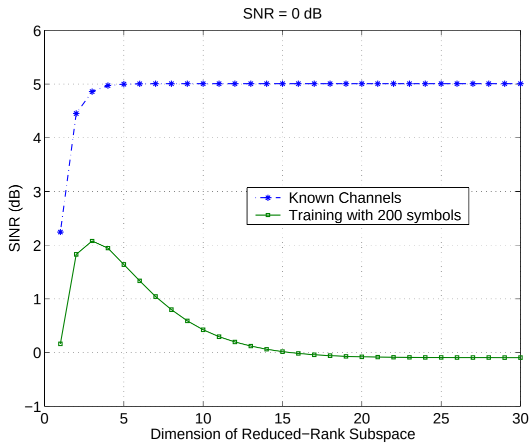
A wideband CDMA forward link was simulated similar to one of the options in the US cdma2000 proposal. The chip rate was 3.6864 MHz ($T_c = 0.27\mu s$), 3 times that of IS-95. The spreading factor was $N_c = 64$ chips per bit. Simulations were performed for a "saturated cell", i.e. all 64 possible channel codes are active. The data symbols were BPSK, and spread with one of 64 Walsh-Hadamard codes. All users were of equal power, and their signals were summed synchronously and then multiplied with a QPSK scrambling code of length 32678.

The channels were modeled to have four equal-power multi-paths, the first one arriving at 0, the last at $10\mu s$ (corresponding to about 37 chips) and the other two delays picked at random in between 0 and 37 chips. The multi-path coefficients are complex normal, independent random variables with equal amplitude. The arrival times at antenna 1 and 2 are the same, but the multi-path coefficients are independent.

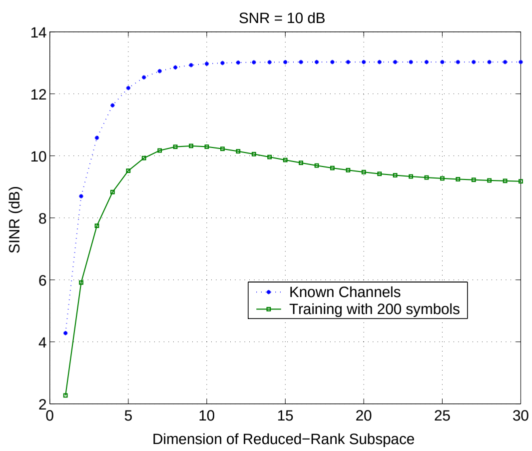
SNR is defined to be the ratio of the sum of the average powers of the received signals over all the channels, to the average noise power, after chip-matched filtering. The curves were generated by averaging over many different channels. Note that the abscissa in the graphs is the **post**-correlation SNR for *each* user which includes a processing gain of 10log(64) $\approx$ 18 dB. The output SINR is calculated using the formula derived by Krauss in [3] and plotted vs. rank of the reduced dimension subspace, $D$ in Fig. 3 at two different SNR's. For comparison, the SINR output for an "ideal" MSNWF, i.e. with perfect channel estimation, is included. At a low SNR of 0 dB, the SINR after 200 symbols for the adaptive MSNWF shows a distinct peak at a dimension of only 3! At SNR 10 dB, the peak is less prominent, but the SINR output goes down after stage 8 or so. This can be explained as the 'penalty' for learning the channels, i.e in the presence of significant noise, the advantage of the increased dimension of the filter is balanced by the need to adjust more coefficients.

Fig. 4 shows the output SINR vs. time (in symbols) for the adaptive MSNWF at stages 5 and 10, and for the full-rank LMS and RLS Algorithms, at a fixed SNR of 10 dB. This plot shows the rapid convergence of MSNWF to the asymptotic SINR. As expected, the lower rank MSNWF converges slightly faster to a lower SINR. The RLS curve needs at least 114 time-samples to estimate the time-average of $\mathbf{R_{xx}}$, so at first it performs worse than even MSNWF of stage 5. Even asymptotically it does not beat the MSNWF of rank only 10! The LMS algorithm converges much slower and to a lower SINR.

The BER curves in Fig. 5 illustrate the performance of these equalizers after training with 200 symbols. At low SNR's, the BER for MSNWF stage 5 is actually slightly lower than the BER for stage 10 or 15. This is consis-




(a)




(b)

Fig. 3. SINR of MSNWF vs Dimension of Reduced-Rank Subspace

tent with the SINR curves of Fig. 3. Over a practical SNR range, in this adaptive implementation, the stage 5 MSNWF does better or almost as good as full-rank RLS! This improvement comes with much lower computational complexity than the RLS. The LMS algorithm is simpler, but performs quite poorly as its convergence is much slower.

It is noteworthy that in our simulations, the all the channels were of equal power whereas in IS-95, the pilot channel typically uses $20-25\%$ of the available power. This implies the MSNWF does not require a strong pilot signal for adaptation, so more power could be made available to the traffic channels.

## VII. Conclusions

We presented a training-based adaptive MSNWF for the CDMA downlink. The convergence rate for MSNWF operating in a very low-rank subspace was better than RLS and LMS. The BER performance showed improvement over the full-rank methods for practical SNRs.

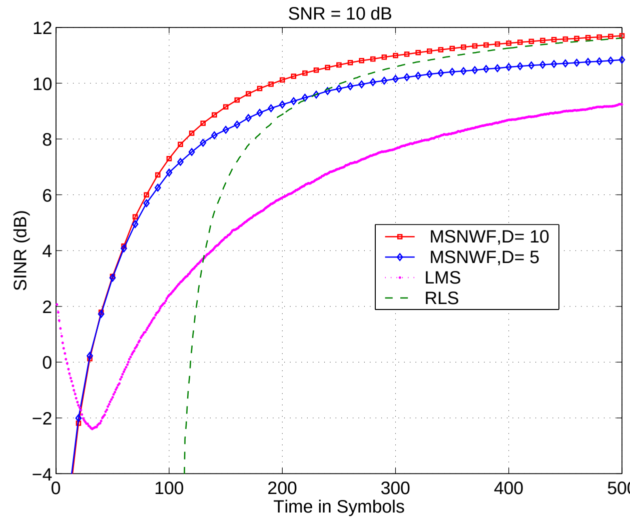



Fig. 4. Output SINR vs Time for Adaptive Chip-level Equalizers

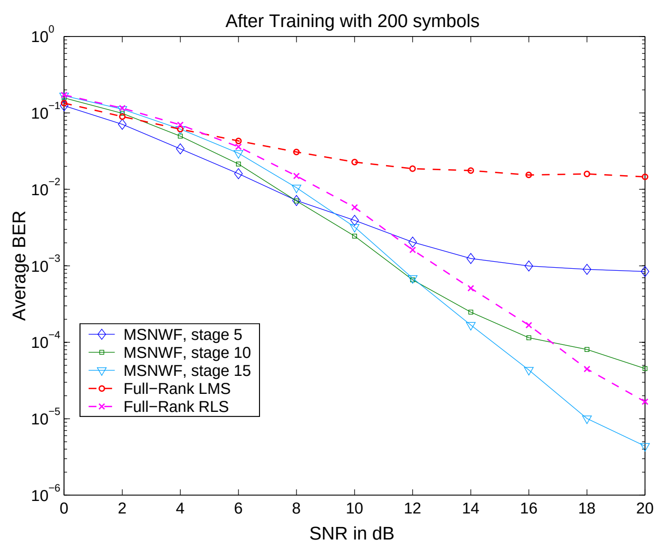



Fig. 5. BER for Adaptive Chip-level Equalizers

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