ECE600 Random Variables and Waveforms — Mark R. Bell
Spring 2024 — MSEE 336

Homework Assignment #1
Should be completed by Session 3

Reading Assignment: Sections 1-1, 1-2, 1-3, 2-1, and 2-2 of Papoulis

1. Reduce the following expressions to the simplest possible forms:
   (a) $(A \cap \overline{B}) \cup (B \cap \overline{A})$.
   (b) $(A \cap \overline{B}) \cap (A \cap B)$.
   Use DeMorgan's laws to show that:
   (c) $\overline{A \cap (B \cup C)} = (\overline{A} \cup \overline{B}) \cap (\overline{A} \cup \overline{C})$.
   (d) $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$.
2. Let $\{A_1, \ldots, A_n\}$ be a partition of the space $\mathcal{S}$, and define the family of sets $\{B_1, \ldots, B_n\}$ by
$$B_j = G \cap A_j, \quad j = 1, \ldots, n,$$
   where $G \subset \mathcal{S}$. Show that $\{B_1, \ldots, B_n\}$ is a partition of the set G.
3. Let $F_r = [0, 1/r)$, $r \in (0, 1]$. Find
$$\bigcup_{r \in (0,1]} F_r \qquad \text{and} \qquad \bigcap_{r \in (0,1]} F_r.$$
4. Prove that a finite set with n elements has 2^n distinct subsets.
5. Using the definitions of union, intersection, and complement, show that
   (a) $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
   (b) $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
6. Prove that if $A \cup B = A$ and $A \cap B = A$, then $A = B$.
7. Let $\mathcal{S}$ be the totality of all undergraduate students at Purdue University. Let A_1, A_2, A_3, A_4 be the sets of freshmen, sophmores, juniors, and seniors, respectively, let F denote the set of female students, and let H denote the set of students who are Indiana natives. Express in words each of the following sets:
   (a) $(A_1 \cup A_2) \cap F$.
   (b) $F \cap \overline{H}$.
   (c) $A_1 \cap \overline{F} \cap H$.
   (d) $A_3 \cap F \cap \overline{H}$.
   (e) $(A_1 \cup A_2) \cap H \cap F$.