

Homework Assignment #5
 Due Friday, December 8, 2023 by Midnight

Reading Assignment: Sections 10.1–10.5 (Sections 13.1–13.5 in First Edition.)

1. Consider a DMS to be encoded with source and destination alphabets $\mathcal{X} = \hat{\mathcal{X}} = \{0, 1, 2, 3, 4\}$. Assume that each of the five source letters are equi-probable and that the distortion matrix between \mathcal{X} and $\hat{\mathcal{X}}$ is

$$D = \begin{pmatrix} 0 & 1 & 2 & 2 & 1 \\ 1 & 0 & 1 & 2 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 2 & 1 & 0 & 1 \\ 1 & 2 & 2 & 1 & 0 \end{pmatrix}.$$

Consider the source code of length 2 consisting of the five codewords $(0, 0)$, $(1, 3)$, $(2, 1)$, $(3, 4)$, and $(4, 2)$. Calculate the resulting average distortion (per letter).

2. Let $A_U = \{0, 1\}$ and $A_V = \{0, 1, 2\}$ with distortion matrix

$$D = \begin{pmatrix} 0 & \infty & 1 \\ \infty & 0 & 1 \end{pmatrix},$$

and source statistics $\{1/2, 1/2\}$. Find the rate distortion function.

3. Consider a source with $\mathcal{X} = \hat{\mathcal{X}} = \{0, 1, 2\}$, source statistics $\{1/3, 1/3, 1/3\}$ and distortion matrix

$$D = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Find δ_{\min} and $R(\delta_{\min})$ for this source and distortion metric.

4. *Cover and Thomas*, Ch. 10, Prob. 1 (Ch. 13, Prob. 1 in First Edition).
5. *Cover and Thomas*, Ch. 10, Prob. 5 (Ch. 13, Prob. 5 in First Edition).
6. *Cover and Thomas*, Ch. 10, Prob. 8 (Ch. 13, Prob. 8 in First Edition).
7. Consider a random variable X with the triangle pdf shown below. Find the decision levels $\{x_0, x_1, x_2, x_3\}$ and the reconstruction levels $\{y_1, y_2, y_3\}$ for a 3-level quantizer with minimum mean-square error. What is the resulting mean-square error?

