# Nature-Based Optimization of 2D Negative-Index Metamaterials

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## Introduction

In the optical frequency range (including the infrared and visible spectrum), various negative-index metamaterial (NIM) designs have recently been reported, in which noble metals are employed to realize a negative permittivity. Although the permittivity functions of various noble metals may be described accurately by simple Drude models at long wavelengths, they begin to deviate from such models at infrared (IR) and visible wavelengths. Therefore, it is desirable to use measured permittivity data for noble metals in the design process. Moreover, the optimization of a NIM geometry usually involves various parameters, so nature-based evolutionary algorithms are well-suited for this task. This paper applies two nature-based optimization techniques — the genetic algorithm (GA) [1] and particle swarm optimization (PSO) [2] — to the design of a two-dimensional IR-visible metamaterial using realistic bulk silver parameters by maximizing a properly-defined figure of merit.

## Genetic Algorithm and Particle Swarm Optimization

The GA is a powerful and robust multi-parameter optimization methodology based on the concepts of natural selection and survival-of-the-fittest in evolution. An initial population is randomly created where each 'individual' in the population represents a specific realization of the design under optimization in the pre-defined parameter space. A fitness value is assigned to each individual to quantify its performance in view of the desired characteristics being sought. A set of best-performing individuals in a generation are allowed to 'mate' to generate the next generation. This evolutionary process is repeated until convergence is ultimately achieved.

The PSO is one of the newest nature-based optimization techniques, which mimics the social behavior of animals and insects in groups. The evolution mechanism driving the GA optimization is competition among its population members, whereas PSO utilizes cooperation among the individuals in their respective groups. These individuals, i.e. particles, fly over a multi-dimensional parameter space and the position of each particle represents a solution for the optimization problem. Each particle also has its own velocity vector along with the position, from which its next position is determined. The velocity vector of each particle is updated based on the personal best location as well as the best location for the entire swarm, which allows information sharing and cooperation among particles. The position updates for the entire swarm are repeated until convergence is achieved.

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## Numerical Results

The two nature-based optimization methods (i.e., GA and PSO) are applied to the metamaterial design reported in [3] with the goal of optimizing its performance in the IR-visible spectrum. The cross-sectional unit cell geometry of the two-dimensional negative-index metamaterial (NIM) is illustrated in Fig. 1. Two strips of silver separated by a thin layer of alumina comprise a magnetic resonator. Thin silver films of thickness $t_f$ provide the negative permittivities needed for a negative index behavior. In the numerical simulations, constant refractive indices of 1.45, 1.62, and 1.483 were used for silica, alumina, and glass, respectively. Published experimental results [4] were used to represent the permittivity function of the silver. The metamaterial slab is illuminated by a plane wave normally incident from air with the electric field polarized in the $\hat{x}$ direction. For a specific design, the reflection and the transmission coefficients for the electric fields evaluated at the top and the bottom surfaces of the metamaterial are converted to effective material parameters via a well-established inversion procedure [5].

The goal of the optimization is to maximize the fitness, or to minimize the cost, defined by

$$\text{fitness} = -\text{cost} = \max_{\lambda} \left\{ -\frac{n'}{n''} \right\}, \tag{1}$$

where the wavelength is scanned over the specified search range of interest, and the effective index $n$ of the metamaterial is represented by $n = n' + in''$. Each of the four geometrical parameters $p$, $w$, $t$, and $d$ is constrained to vary between a pre-set minimum and maximum value. The two remaining parameters are set to the fixed values of $t_f = 20$ nm and $t_s = 20$ nm. The range of wavelength over which the maximum fitness is sought is set to be from 400 to 800 nm.

A specific implementation of GA, called the micro GA [6], was used with a population size of six for the optimization. A variation of the finite element-boundary integral (FE-BI) method [7] with periodic boundary conditions was employed for evaluating the fitness of each individual. For the PSO optimizations, the cost of a particle at a specific location was evaluated using the spatial harmonic analysis [8]. To make a fair comparison with the GA optimizations, a swarm of six particles was used for the PSO method.

The GA optimization converged at generation 87 to a maximum fitness value of 3.25. The associated index of refraction was found to be $n = -0.810 + i0.249$ at $\lambda = 770$ nm. The index of refraction $n$ for the best individual is plotted with respect to $\lambda$ in Fig. 2(a). The metamaterial exhibits a negative index behavior between 0.75 and 0.81 $\mu$m, over which range $n''$ increases sharply. This sharp increase in the loss characteristic is closely related to the effective permeability transitioning from negative to positive values in association with a magnetic resonance. Convergence was achieved for the PSO with only 35 iterations to the minimum cost of $-3.23$, which was achieved at $\lambda = 780$ nm with the value of $n$ given by $n = -0.864 + i0.268$. The optimized geometrical values are very close to those of the GA-optimized design (see the caption of Fig. 2). The recovered index of refraction is shown in Fig. 2(b), which is very similar to the results plotted in Fig. 2(a). Finally, convergence curves for the GA and the PSO methods are compared in Fig. 3 for this design. It is observed that the PSO converges more quickly than the GA for this particular design example.

## Conclusion

Two nature-based optimization methodologies — the GA and the PSO techniques — were applied for optimizing an IR-visible negative index metamaterial design. These optimization methods are well-suited for metamaterial designs involving multiple parameters and material property data from measurements. Both optimizations lead to a NIM design with a negative index band around $\lambda \approx 790$ nm with the total thickness of roughly 240 nm. The best fitness achieved through both optimizations was found to be around 3.2.

## Acknowledgments

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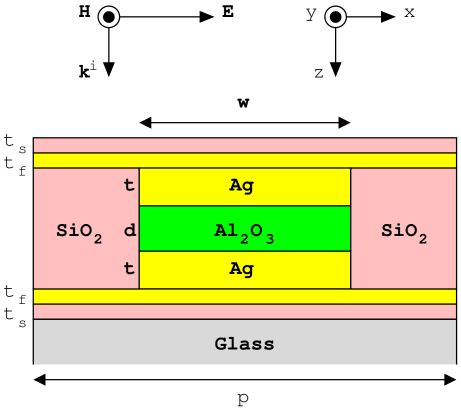
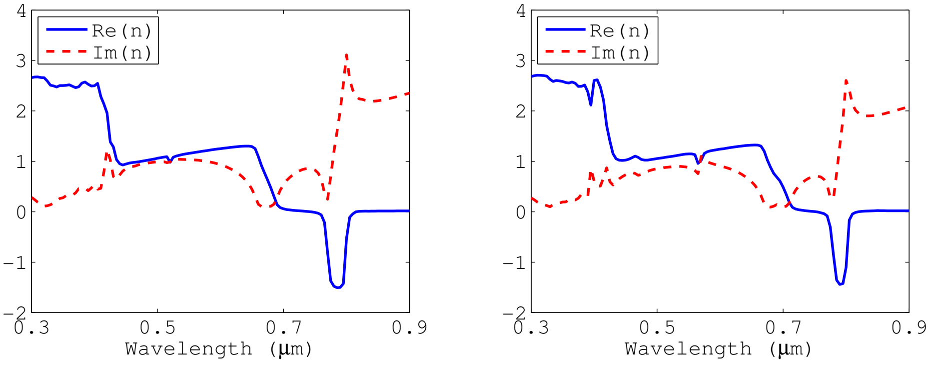


Figure 1: Unit cell geometry of a two-dimensional metamaterial.



(a)

(b)

Figure 2: The effective index $n$ corresponding to the optimized design: (a) $n$ for the GA-optimized design. The optimal geometrical parameters are equal to $p = 314.3$ nm, $w = 176.8$ nm, $t = 42.9$ nm, and $d = 72.1$ nm. (b) $n$ for the PSO-optimized design with $p = 328.7$ nm, $w = 168.0$ nm, $t = 45.8$ nm, and $d = 68.0$ nm.

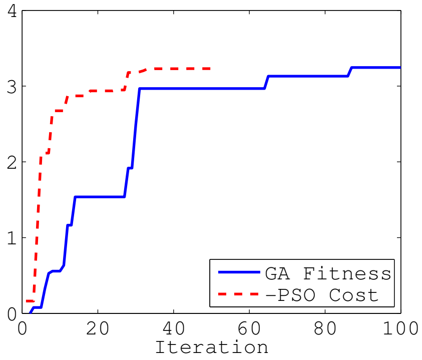


Figure 3: Comparison of the convergence properties of the two optimization methods. Each iteration corresponds to six fitness/cost evaluations for both methods.