Supplementary Materials

An Overview of the Theory of Phase Transition

NC-FET results have been interpreted by several different modeling approaches. The difference in the modeling approaches and the parameterization needed to fit the experiments sometimes lead to conflicting interpretations. The key challenge is to understand the role of domain dynamics. The exact problem is unsolved and potentially unsolvable, but a summary of the modeling approaches may be helpful.

The time dynamics of ferroelectric capacitor is often described by Ginzburg-Landau theory

$$M_0\, d^2P/dt^2 \;+\; L_0\, dP/dt \;=\; -\,dU/dP \qquad (1a)$$

where P is the net polarization, M_0 is the effective mass of the atoms being displaced, L_0 is phonon-induced "friction" and

$$U(P, V_{FE}, k) = \; U_{FE}(P, V_{FE}) + \; k\,(\nabla\, P)^2 \qquad (1b)$$

The key debate concerns the form of $U_{FE}(P, V_{FE})$ and if the domain dynamics can be used to renormalize M, L, and U_{FE}. Here, k is the domain coupling constant, which is related to spatial non-uniformity of P. We note that by definition:

$$\frac{dU_{FE}}{dQ} = \; V_{FE}, \text{or } C_F^{-1} = \frac{d^2(U_{FE})}{dQ^2}, \text{and } C_{FE} = \frac{dP}{dV_{FE}}$$

So that Eq. (1a) and (1b) can be written in variety of equivalent forms. The equation is solved subject to the boundary conditions of interfacial stress and charge/polarization continuity.

Four types of approximations have been used so far for solving the Ginzburg-Landau equations:

1. **Classical Landau-Khalatnokov (L-K) Model based on Renormalized Domain Dynamics:**
   In this formulation, Eq. (1b) is first written as

$$\begin{aligned} U_{FE}(P, V_{FE}) &= \; \alpha^*\, P^2 \; + \beta^*\, P^4 \; + \gamma^*\, P^6 - \; P\, V_{FE} + \; k\,(\nabla\, P)^2 \\ &\rightarrow \; \alpha P^2 \; + \beta\, P^4 \; + \gamma\, P^6 \; - \; P\, V_{FE}. \end{aligned} \qquad (2a)$$

   Here the polarization anisotropy factors are renormalized by including the $k(\nabla\, P)^2$ term either by using a two-domain approximation, or by fitting the experimental data. The empirical approach can also be used to renormalize M and L parameters by fitting time-dependent Fe-FET P-E data (by indirectly including the effect of k). The key to this analysis is that $\alpha < 0$ following the renormalization. Most transistor analysis (steady state and transient) are based on the renormalized L-K equation. In particular, the following equation is used to interpret the results in steady state

$$\frac{dU_{FE}}{dP} \; = \; 0 \; = \; 2\alpha\, P \; + \; 4\,\beta\, P^3 \; + \; 5\,\gamma\, P^5 - V_{FE}. \qquad (2b)$$

   The original L-K model suggests the second derivative of U_{FE} is positive for stable physical system. The NC-FET concept suggests a linear capacitor can stabilize the FE unstable NC state with the second derivative of U_{FE} negative.[5]

2. **KAI Model based on Explicit Time Approximation**: The second approach, proposed and published by Y.J. Kim et al.[19], involves assuming

$$M_0 = 0, U_{FE}(P, V_{FE}) = \alpha(k, V_{FE})P - P\,V_{FE}, \text{ and } L_0 \rightarrow L(V, k) \qquad (3a)$$

so that the effect of domain dynamics incorporated indirectly in α and L. Most important, here $\alpha > 0$. With these approximations, the resulting first order equation (i.e. $L\,dP/dt = \alpha\,(P_0 - P)$ is directly solvable. If $\alpha/L \equiv \left(\frac{t}{\tau}\right)^{\beta-1}$ in analogy to Avrami equation for crystal nucleation and growth, the solution of the corresponding equation

$$d\left(\frac{P}{P_0}\right)/dt = (t/\tau(V_{FE}, k))^{\beta-1}\,(1 - P/P_0)$$

yields the Weibull form

$$P(t) = P_0\,(1 - \exp\left(-\left(\frac{t}{\tau}\right)^{\beta}\right). \qquad (3b)$$

Merz's law makes the voltage-dependence explicit: $\tau = \tau_0 \exp(\alpha/V_{FE})$. The most important point is $\alpha > 0$ and there is no NC effect.

3. **Preisach-Miller model involving Implicit Time Dependent Approximation** [20,22]**:**
In a slightly different approximation, one assumes that the steady state relationship between V_{FE} and P is given not by Eq. (2a), but rather by two branches of the P-V relationship (with positive α)

$$\frac{dU_{FE}}{dP} = 2\alpha P - V_{FE}, \text{where } P \equiv P_s\left[\tanh\left(w\,(V_{FE} \pm V_c) + \frac{\eta V_{FE}}{V_p}\right)\right]$$

$$\text{and } w = \sigma/(2V_c)\ln(P_S + P_R)/(P_S - P_R)$$

Here, P_S is the saturation polarization, P_R is the remnant polarization, V_c is the coercive voltage, V_p is the peak voltage for V_{FE}. Here, η and σ captures V_c distribution of FE domains and non-ferroelectric linear response of P.

$$L\frac{dP}{dt} = V_{FE} - 2\alpha P(t)$$

Or rewriting in the equivalent circuit format, with $V_A \rightarrow V_{FE}$, $C_{FE} = \frac{dP}{dV_{FE}}$, $\tau \equiv RC_{FE} \rightarrow LC_{FE}$, we find

$$\tau\,dV_{FE}/dt = V_A - V_{FE} \rightarrow L\frac{dP}{dt} = V_A - C_{FE}^{-1}P = V_A - 2\alpha P$$

$$C_{FE} = \frac{dP}{dV_{FE}} = (2\alpha)^{-1}$$

4. **Simplified Miller Model:** Finally, a quasi-static variant of the Miller Model (with $M_0 = 0, L_0 = 0$) is used in Ref. [70] where $\left(\frac{d^2U_{FE}}{dP^2}\right)^{-1} = C_{FE} = C_0 + \beta\,V_{FE}^n$. Parameter β and n are adjusted so that a double integration of the capacitance function would produce the Miller equation.

Interpretation of Experiments.

1. Time-dependent Analysis:

KAI, Miller, and modified Miller Models have been used to argue that the phase lag between applied voltage and domain flipping can also interpret the results reported in refs. [19–24,69,70]. The authors explain several features of the time-dependent data without invoking negative capacitance (i.e. $\alpha < 0, \beta < 0$). The model anticipates response time limited by domain dynamics and could be as slow as tens of MHz range.

Time-dependence can also be interpreted by the renormalized L-K model, where the domain dynamics with empirical mass and alpha parameters are applied. Once again, the response obtained is in the tens of MHz range.

A recent experiment shows optical evidence of much faster response[78] and direct pulse measurement with a single pulse switch time of 3.6 ns and a train of pulses with pulse width down to 100 ps[77]. The direct pulse response is limited by many parasitic effects of the measured device and the ultra-fast pulse might not be able to provide sufficient polarization charges for a full ferroelectric switch through inversion charges at a single ultra-short pulse time. It is not clear if these results can or should be interpreted by the generalized Landau framework.

2. DC Analysis:

The renormalized L-K model has been used to interpret the subthreshold slope, negative DIBL, and NDR reported by various groups[48,53,57]. Both 3D simulation of advanced transistors[66] and compact model for 3D and 2D transistors[67] have been proposed. The model has also been used to understand the scaling consequences (e.g. inclusion of quantum and parasitic capacitance, etc.) as well as new device design integrating various "Landau" devices (FE, AFE, MEMS, piezo)[99,100]. The reduction of noise with increasing dielectric thickness[68] is another example of such an analysis producing self-consistent results.

The KAI, Miller, and/or Modified Miller models interpret the steady state response by suggesting that all steady-state measurements are in-fact time-dependent, defined by the rate of measurement. Thus, they interpret the "DC" subthreshold slope as a consequence of time-dependent phase-lag.