Microwave Resonance of the Bubble Phases in 1/4 and 3/4 Filled High Landau Levels

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We have measured the diagonal conductivity, σ_{xx}, in the microwave regime of an ultrahigh mobility two dimensional electron system. We find a sharp resonance in $\mathrm{Re}[\sigma_{xx}]$ versus frequency when $\nu > 4$ and the partial filling of the highest Landau level, ν^*, is $\sim 1/4$ or $3/4$ and temperatures <0.1 K. The resonance appears for a range of ν^* from 0.20 to 0.38 and again from 0.64 to 0.80. The peak frequency f_{pk} changes from ~ 500 to ~ 150 MHz as $\nu^* = 1/2$ is approached. This range of f_{pk} shows no dependence on ν where the resonance is observed. The quality factor, Q, of the resonance is maximum at about $\nu^* = 0.25$ and 0.74. We interpret the resonance as due to a pinning mode of the bubble phase crystal.

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Two dimensional electron systems, 2DES, confined in ultraclean GaAs/AlGaAs heterostructures and subjected to perpendicular magnetic fields, B, show strongly anisotropic diagonal resistance at half integer filling factors, $\nu = 9/2,\ 11/2,\ 13/2, \ldots$ [1,2] for temperatures $T <$ 150 mK. The same studies find that minima appear in the diagonal resistance at partial fillings of $\nu^* \sim 1/4$ and $3/4$, where $\nu^* = \nu - [\nu]$, $\nu > 4$, and $[\nu]$ is the greatest integer less than ν. Concomitantly, the Hall resistance is quantized to the value of the adjacent integer quantum Hall effect plateau, and hence these states have been christened the reentrant integer quantum Hall effect (RIQHE) [3]. Previously constructed theories [4,5] made predictions with which these observations are consistent. In particular, it was proposed that the RIQHE was due to an isotropic solid phase of the 2DES—a regular triangular crystal lattice with two or more electron guiding centers per lattice site. This crystal, known as the "bubble" phase, would be insulating because of pinning by disorder.

At this time, the case for viewing the RIQHE as a manifestation of the bubble phase within the context of the theory rests (i) on its location between the stripe phase and the integer quantum Hall effect plateau and (ii) on the observed insulating behavior. In addition, one experimental study has seen nonlinear I-V data in 1/4 and 3/4 filled levels [6], a possible indication of a depinning transition. These data are consistent with a crystalline phase, but many phenomena, e.g., "breakdown" [7], can cause nonlinearities in transport. Thus, whether or not the 2DES forms a bubble phase at these fillings is still in question.

The observation of solid phases of the 2DES in GaAs heterostructures has so far focused on the Wigner crystal (WC) [8], at very high magnetic fields, $\nu < 1/5$. Like the bubble phase, the WC is a triangular lattice of electrons pinned by disorder and therefore insulating at low temperatures. One technique used to study the WC is to excite the pinning mode [9,10] of the lattice with microwave radiation at about 1 GHz. The pinning mode describes the oscillation of domains of WC, with length scales of many lattice spacings, within the disorder potential of the sample. These experiments [11–13] find a narrow resonance in the real part of the diagonal conductivity, $\mathrm{Re}[\sigma_{xx}]$, versus frequency, f.

In this Letter we present measurements of the microwave conductivity in the higher Landau levels, $\nu > 4$, of an ultrahigh quality 2DES. Our data show a strong resonance in $\mathrm{Re}[\sigma_{xx}]$ versus f, with $Q \sim 3$ around $\nu^* = 1/4$ and $3/4$ fillings when $T \leq 110$ mK and Q is the peak frequency over the full width at half maximum. The resonance is interpreted as a pinning mode. It follows that the resonance is direct evidence that the 2DES forms a solid around 1/4 and 3/4 fillings.

The sample used is a high quality 2DES grown by molecular beam epitaxy with mobility $\mu = 2.4 \times 10^7\ \mathrm{cm^2\,V^{-1}\,s^{-1}}$ and density $n = 3.2 \times 10^{11}\ \mathrm{cm^{-2}}$. The electrons are confined in a 300 Å quantum well, approximately 2000 Å beneath the surface. A metal film coplanar waveguide (CPW) [14] was evaporated onto the surface of the samples. The length, l, of the CPW is 2 mm and the slot width, w, is 20 μm. The overall geometry of the CPW is such that the line impedance $Z_0 = 50\ \Omega$ in the absence of the 2DES. The $\mathrm{Re}[\sigma_{xx}]$ is related to the transmitted power P by $\mathrm{Re}[\sigma_{xx}] = -\frac{w}{2lZ_0}\ln|P/P_0|$, where P_0 is the power that would be transmitted for $\sigma_{xx} = 0$. All the data presented here were measured using -80 dBm inserted at the top of the cryostat, although only a small fraction of this power is absorbed by the 2DES. Microwave signals propagating along the CPW couple capacitively to the 2DES. The microwave electric field, E_m, in the CPW is polarized perpendicular to the propagation direction and is mainly sensitive to the 2DES in the slot. Two microwave samples were patterned from this wafer: one in which E_m lies along the $\langle 110 \rangle$ crystal axis or

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easy direction at $\nu = \frac{9}{2}$ and the other oriented at 90° to the first so that E_m lies along the $\langle 1\bar{1}0 \rangle$ or hard direction. The upper panel of Fig. 1 shows a schematic of the measurement circuit. The dark regions are the metallic gates. A third sample was cut adjacent to the other two and features eight diffused indium contacts at the corners and the center of the edges. Standard dc measurements at $T \sim$ 50 mK on this piece show strong anisotropic transport at $\nu = \frac{9}{2}$ and $\frac{11}{2}$ and a well formed RIQHE. All temperatures, T, were measured by a thermometer heat sunk directly to the same metal block as the sample.

In lower panel of Fig. 1, we plot Re$[\sigma_{xx}]$ versus B at 50, 150, 300, and 500 MHz. The four traces are offset for clarity and were acquired at 56 mK. Filling factors are marked with down triangles and a scale is given in the center of the figure. Note that E_m is along the easy direction. The data show well defined minima at integer $\nu = 8, 7, 6, 5,$ and 4 and a clear dip at $\nu = 3.5$. At 50 MHz, the transitions between the IQHE minima show a smooth increase in σ_{xx} as the center of a Landau level is approached. However, by 150 MHz, large peaks have developed at $\nu^* \approx 1/4$ and $3/4$ for $\nu > 4$. These are most prominent at 300 MHz where the peak at $\nu \approx 4.25$ is about 14 μS in height. In comparison, the $\nu = 4$ feature shows a change in σ_{xx} of about 2 μS. But these strong peaks have almost vanished in the 500 MHz data indicating that a resonance exists in σ_{xx} at $\nu^* = 1/4$ and $3/4$ for $\nu > 4$.

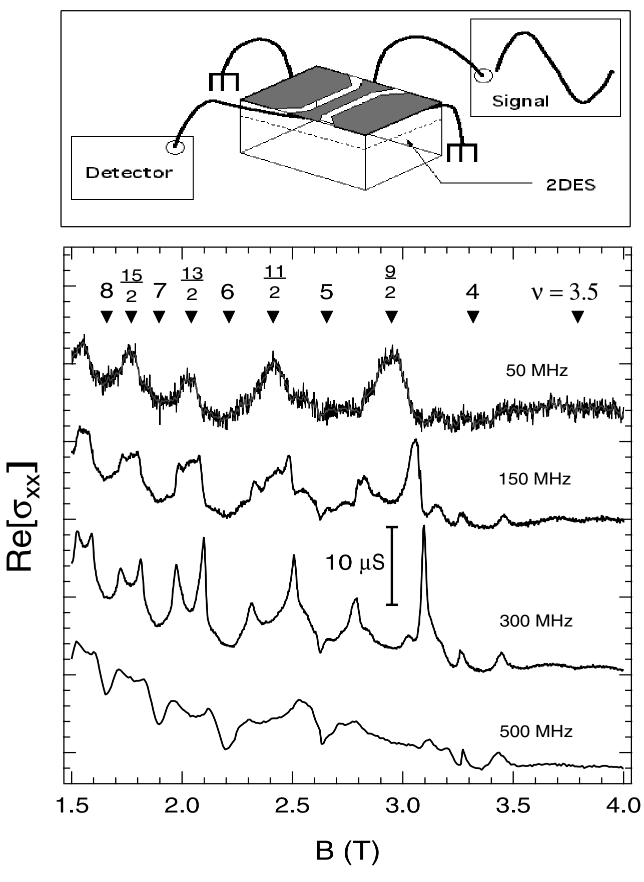

FIG. 1. Upper panel: A schematic of the measurement circuit. The dark regions represent the metallic gates. Lower panel: The real part of the diagonal conductivity, Re$[\sigma_{xx}]$, versus B magnetic field at 50, 150, 300, and 500 MHz. Filling factors are marked. The data were acquired at 56 mK with the microwave electric field, E_m, polarized along the $\langle 110 \rangle$ *easy* direction.

The data in Fig. 1 allow us to piece together a rough picture of the frequency dependence Re$[\sigma_{xx}]$ at $\nu^* \sim 1/2$ for E_m along the easy direction. Focusing on $\nu \sim \frac{9}{2}$, Re$[\sigma_{xx}]$ decreases from a local maximum at 50 MHz to a broad minimum at higher frequencies. The $\nu = \frac{9}{2}$ peak drops at least 8 μS from 50 to 300 MHz but appears to stop changing as frequency is increased further. This trend is also followed by Re$[\sigma_{xx}]$ at $\nu \sim \frac{11}{2}$ and $\frac{13}{2}$. Data for Re$[\sigma_{xx}]$ with E_m polarized along the hard direction yield similar behavior at $f \sim 50$ MHz but appear to show anisotropy for frequencies from 200 to 1000 MHz. Further study of the frequency dependence about $\nu^* \sim 1/2$ is needed. The remainder of this Letter focuses on the resonance seen in Re$[\sigma_{xx}]$ at $\nu^* \sim 1/4$ and $3/4$.

The frequency dependence of ΔRe$[\sigma_{xx}] =$ Re$[\sigma_{xx}]$-Re$[\sigma_{\rm bg}]$ is measured with ν fixed near 4.25 by sweeping f from 50 MHz to 1 GHz. The subtraction of a background, $\sigma_{\rm bg}$, is necessary because the frequency response of the CPW and coaxial cables is not flat, although it is independent of B. We measure $\sigma_{\rm bg}$ at B well outside $\nu^* \sim 1/4$ and $3/4$ peak structures in Fig. 1 which shows flat frequency dependence with respect to $\nu = 3$.

In Fig. 2(a) we show the resonance in the ΔRe$[\sigma_{xx}]$ versus f at several ν from 4.20 to 4.33. $T \sim 56$ mK and E_m is along the easy direction. A weak peak is first visible at $\nu = 4.20$ (dotted line) where the peak frequency, $f_{\rm pk}$, is 434 MHz. The strongest peak (solid line) occurs at $\nu =$ 4.26, where $f_{\rm pk} = 301$ MHz and the conductance reaches nearly 14 μS. As the filling factor is further increased to $\nu = 4.33$, the resonance shrinks and $f_{\rm pk}$ is now only 166 MHz. Figure 2(b) plots $f_{\rm pk}$ versus ν on the left axis and Q versus ν on the right for ν from 4.20 to 4.36. $f_{\rm pk}$ decreases monotonically as the center of the level is approached. Over the same range, Q shows a maximum at $\nu \sim 4.25$. The range of ν of these data also shows that the resonance coincides with the RIQHE observed in Refs. [1,2]. Figure 2(c) shows the evolution of $f_{\rm pk}$ and Q with T from 70 up to 110 mK for the resonance at $\nu =$ 4.26. Q and $f_{\rm pk}$ both decrease as the temperature rises. The resonance disappears at temperatures larger than 110 mK.

Figure 3 presents a summary of $f_{\rm pk}$ versus ν^* (upper panel) and Q versus ν^* (lower panel) for data taken around $\nu = 4.25$, 4.75, 5.25, 6.25, and 6.75 resonances. Data from both polarizations of E_m are shown with the closed symbols representing measurements with E_m along the hard direction and open symbols with E_m along the easy. The majority of the data were recorded at $T \approx$

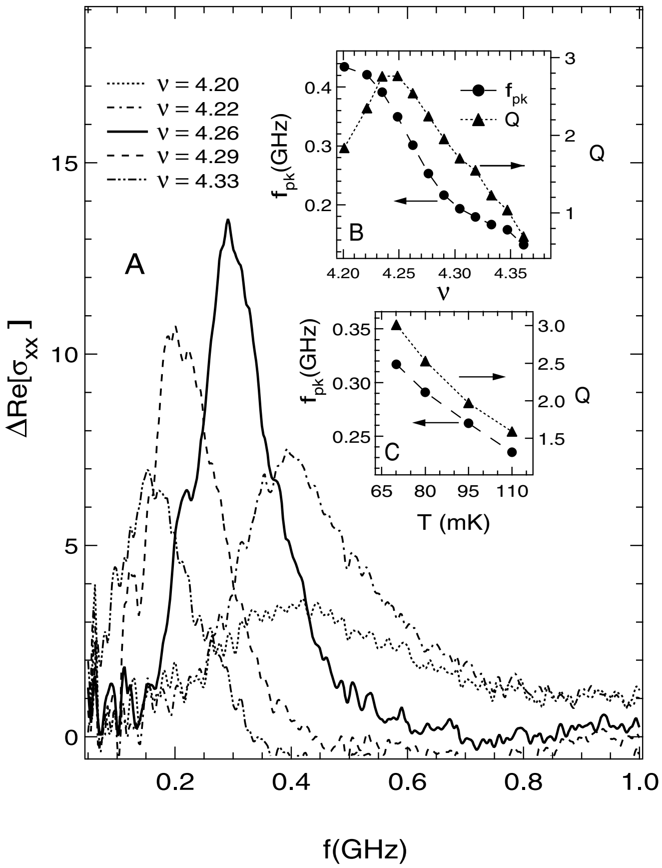

FIG. 2. (a) $\Delta\mathrm{Re}[\sigma_{xx}]$ vs f for several B fields in the vicinity $\nu = 4.25$. E_m is polarized along the *easy* direction. (b) The peak frequency, f_{pk}, vs ν on the left and Q versus ν on the right. (c) f_{pk} and Q vs T at $\nu = 4.26$.

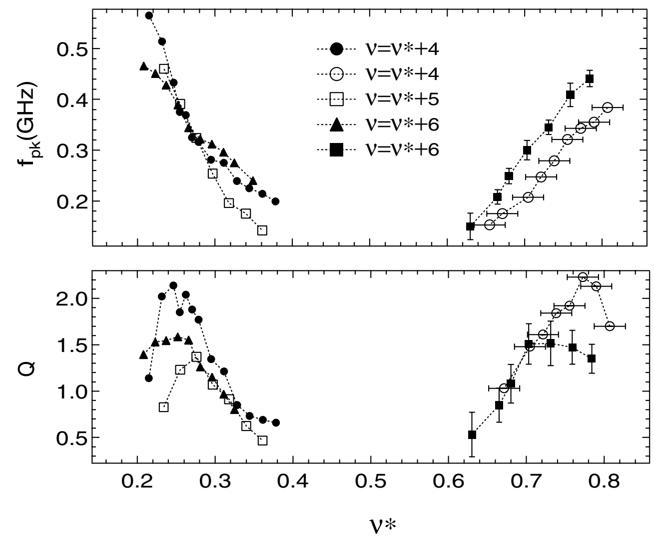

FIG. 3. Upper panel: f_{pk} vs ν^* measured for resonances around $\nu = 4.25$, 4.75, 5.25, 6.25, and 6.75. Filled symbols indicate measurements with E_m polarized along the *hard* direction. Open symbols indicate E_m along the *easy* direction. All measurements were performed at $T = 56$ mK except those around $\nu = 6.75$ (82 mK). Lower panel: Q vs ν^* from the same set of resonances. The legend is for both panels.

56 mK except the $\nu \sim 6.75$ data which were taken at $T \approx 82$ mK. The error in ν^* is ± 0.02 in both upper and lower panels and is shown on the $\nu \sim 4.75$ data. To ensure the consistency of ν throughout, the magnet was always stepped down between data points. Error bars for f_{pk} and Q are shown on the $\nu \sim 6.75$ data and are representative of the errors for all the data in Fig. 3.

In the upper panel of Fig. 3, f_{pk} decreases as the center, $\nu^* = 0.5$, of a Landau level (LL) is approached from either above or below. Around $\nu^* \sim 1/4$ the measured f_{pk} fall in a range from about 500 MHz near $\nu^* = 0.20$ to about 150 MHz near $\nu^* = 0.38$. Around $\nu^* \sim 3/4$, f_{pk} is within the same range of frequencies, with $f_{\mathrm{pk}} = 150$ MHz at $\nu^* = 0.63$ and $f_{\mathrm{pk}} = 441$ MHz at $\nu^* = 0.78$. Within experimental error, there is a remarkable similarity of f_{pk} at different N, where $N = ([\nu] - 2)/2$ is the LL index. Finally, the mirror image behavior of f_{pk} about the LL center is consistent with particle-hole symmetry.

The data shown in Fig. 3 for $\nu \sim 4.25$ where E_m is along the hard direction can be directly compared with those in Fig. 2(b) where E_m is along the easy direction. Within error, these two data sets for f_{pk} agree from $\nu^* = 4.24$ to 4.35. This observation fits the general trend of the f_{pk} data in Fig. 3 which do not show a dependence on the polarization of E_m. Overall, the resonance appears to be isotropic.

The lower panel of Fig. 3 plots Q versus ν for different LL and orientation of E_m. The separate measurements agree on the range of ν^* where resonances are seen. The data show the resonance is most developed for ν^* from 0.24 to 0.28 and ν^* from 0.71 to 0.77, where the maximum Q's occur. The smallest Q's occur near the center of the LL where $Q \sim 0.5$ is seen at $\nu^* = 0.36$ and 0.63. The Q of the resonance is a sharp function of ν^* with the maximally developed resonance occurring over a narrow range or ν^*. Again, we note agreement between different filling factors and polarizations of E_m and symmetry about the center of the LL.

The natural interpretation of these data is that the resonance is due to a pinning mode of the bubble phases around 1/4 and 3/4 filled Landau levels. The low observed $f_{\mathrm{pk}} \leq 550$ MHz means the energy of the mode is $h\nu/k_B \leq 26$ mK. But, an electron oscillating in a potential well that shallow would be ionized at $T > 50$ mK. Further, Q as high as 3 are measured. The collective motion of a large region of electrons, as in a WC domain, would average the disorder and allow high Q. Finally, the data are qualitatively similar to the pinning resonance observed in the high B WC phase [11–13].

The smooth change in f_{pk} with ν^* can be explained in the context of a pinned electron solid in which the density is steadily being changed. Increasing ν is equivalent to changing the density, n^*, of electrons that form the solid,

$n^* = n\nu^*/\nu$. In a weak pinning model [9,10], increasing n^* stiffens the WC domains and effectively softens the electron-disorder interactions which provide the restoring force [13]. Thus, around $\nu^* \sim 1/4$, increasing ν^* results in more electrons in the bubble phase and lower f_{pk}. Around $\nu^* \sim 3/4$, increasing ν^* means a bubble phase of holes becomes more dilute, softening the interbubble forces in relation to disorder, so that increasing f_{pk} is observed.

A comparison of the integrated oscillator strength, S, scaled by f_{pk} for the resonance shown in Fig. 2 agrees within a factor of 2 with the oscillator model of Fukuyama and Lee [9,15], $S/f_{pk} = n^* e\pi/2B$. For example, n^* runs from 1.6 to 2.7×10^{10} cm^{-2} where the resonance is seen around $\nu \sim 4.25$. At $n^* = 2.0 \times 10^{10}$ cm^{-2}, we measure $S/f_{pk} = 8$ μS and the model above gives 16 μS. This supports the idea that only the electrons in the uppermost LL participate in the bubbles and the identification of the resonances as the pinning mode of the bubble phase. However, where the resonance is less well developed, i.e., smaller Q and lower peak conductivity, the measured S/f_{pk} is less than 1/3 of the Fukuyama and Lee model. We note that integrating the experimental resonance data *underestimates* the oscillator strength because of the finite frequency range.

Using the density matrix renormalization group Shibata and Yoshioka [16] have predicted two-electron bubbles exist for $0.24 \leq \nu^* \leq 0.38$ for $4 < \nu < 6$. At fillings, $6 < \nu < 8$, they predict two electron bubbles from $0.18 \leq \nu^* \leq 0.25$ and three electron bubbles for $0.29 \leq \nu^* \leq 0.34$ followed by a transition to a stripe phase. Hartree-Fock calculations by the same authors give slightly wider ranges but predict discrete two and three electron bubbles when $4 < \nu < 6$ and two, three, and four electron bubbles for $6 < \nu < 8$. Reference [4] also uses Hartree-Fock to predict changes in the number of electrons per bubble and other numerical work [17] predicts bubbles at $\nu^* = 1/3$ and 1/4. Our data for Q in Figs. 2(b) and 3 is in rough agreement with the predicted range for bubbles. However, our data give neither a way to know the number of electrons per bubble nor a reason to suspect a change in that number within a given data set.

In summary, we have observed a sharp resonance in the real part of the microwave conductivity at $\nu^* = 1/4$ and 3/4 starting at $\nu = 4.25$ and going at least as high as $\nu = 6.75$. Changing the orientation of the microwave electric field with respect to the GaAs lattice has only minor effects on the resonance. Above $T \sim 110$ mK the resonance fades. The resonance is visible for a range of ν^* from 0.20 to 0.38 and again from 0.64 to 0.80. These ranges of ν^* coincide with observations of the RIQHE and theoretical discussions of the bubble phase. The data presented offer the most direct evidence to date that the reentrant insulating behavior of the 2DES around $\nu^* \sim 1/4$ and 3/4 in higher LL is caused by the formation of a crystalline bubble phase.

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